

Khan Academy Study-Guide

This pre-SGPE math camp is designed to help dust off your math skills by utilizing the excellent software, videos, and problems sets from Khan Academy. The curriculum is designed for you to move at your own pace, and the entire course curriculum (all 8 modules plus the linear algebra supplement) is available via links below. The material below covers both pre-calculus and calculus. If you feel comfortable with pre-calculus and are just looking for a refresher of basic calculus concepts, start at Module 4. The course was designed with the expectation that you will be able to devote roughly 2 hours a day to studying.

Although you can use most of the materials on Khan Academy without setting up an account, creating a free account can help you keep track of your progress. Instructions for creating an account can be found on the [Khan Academy](#) website.

Module 1:

Suggested tasks for Module 1: Complete the Pre-Algebra Challenge on Khan Academy.

You may also find the following videos helpful:

- For Exponents: [Introduction to Exponents](#), [Integer Exponents 1](#), [Integer Exponents 2](#), [Exponent Properties 1](#), [Exponent Properties 2](#), [Exponent Properties 3](#), [Simplifying Expressions with Exponents](#), [Simplifying Expressions with Exponents 2](#), [Simplifying Expressions with Exponents 3](#).
- For Polynomials: [Multiplying and Dividing Monomials 1](#), [Multiplying and Dividing Monomials 2](#), [Brief Introduction to Polynomials](#), [Parts of the polynomial](#), [Working with polynomials1](#), [Working with polynomials 2](#), [Adding and Subtracting Polynomials 1](#), [Adding and Subtracting Polynomials 2](#), [Adding and Subtracting Polynomials 3](#), [Multiplying Polynomials1](#), [Multiplying Polynomials 2](#), [Multiplying Polynomials 3](#), [Special Products of Polynomials 1](#), [Special Products of Polynomials 2](#), [Special Products of Polynomials 3](#), [Special Products of Polynomials 4](#), [Special Products of Polynomials 5](#).
- For Factoring: [Factoring and the Distributive Property](#), [Factoring and the Distributive Property 2](#), [Factoring and the Distributive Property 3](#), [Factoring and the Distributive Property 4](#), [Factoring trinomials with a common factor](#), [Factoring Trinomials by Grouping 1](#), [Factoring Trinomials by Grouping 2](#), [Factoring Trinomials by Grouping 3](#), [Factoring Trinomials by Grouping 4](#), [Factoring Trinomials by Grouping 5](#), [Factoring Trinomials by Grouping 6](#), [Factoring Special Products 1](#), [Factoring Special Products 2](#), [Factoring Special Products 3](#).
- For Radicals: [Simplifying Radical Expressions 1](#), [Simplifying Radical Expressions 2](#), [Simplifying Radical Expressions 3](#), [Fractional Exponent Expressions 1](#), [Fractional Exponent Expressions 2](#), [Fractional Exponent Expressions 3](#).

- For Fractions: [Simplifying Rational Expressions 1](#), [Simplifying Rational Expressions 2](#), [Simplifying Rational Expressions 3](#), [Multiplying and Dividing Rational Expressions 1](#), [Multiplying and Dividing Rational Expressions 2](#), [Multiplying and Dividing Rational Expressions 3](#), [Adding and Subtracting Rational Expressions 1](#), [Adding and Subtracting Rational Expressions 2](#), [Adding and Subtracting Rational Expressions 3](#).
- For Order of Operations: [Order of Operations](#), [Order of Operations 2](#).

Additional, more basic, videos can be found in the Developmental Math playlist on Khan Academy Website. Relevant videos have similar titles to the above.

Module 2:

Suggested tasks for Module 2: Begin the Algebra Challenge on Khan Academy. If you are very industrious and already pretty good at maths, you may be able to finish the Algebra Challenge on Khan Academy in Module 2.

You may also find the following videos helpful...

- Equations: [Simple Equations](#), [Equations 2](#), [Equations 3](#), [Equations 4](#), [Solving Equations 1](#), [Solving Equations 2](#), [Solving Equations with the Distributive Property](#), [Solving equations with the distributive property 2](#), [Solving Radical Equations 1](#), [Solving Radical Equations 2](#), [Solving Radical Equations 3](#), [Solving Rational Equations 1](#), [Solving Rational Equations 2](#), [Solving Rational Equations 3](#), [Applying Rational Equations 1](#), [Applying Rational Equations 2](#), [Applying Rational Equations 3](#).
- Cartesian Coordinate System: [Plotting \(x,y\) relationships](#), [Quadrants of Coordinate Plane](#), [Graphing using X and Y intercepts](#), [Algebra: graphing lines 1](#).
- Linear Equations and Graphs: [Basic Linear Function](#), [Exploring linear relationships](#), [Recognizing Linear Functions](#).
- Slopes: [Algebra: Slope](#), [Algebra: Slope 2](#), [Algebra: Slope 3](#), [Slope of a line](#), [Slope Example](#).
- Intercepts: [X and Y intercepts](#), [X and Y intercepts 2](#).
- Slope-Intercept Form: [Graphing a line in slope intercept form](#), [Converting to slope-intercept form](#), [Point Slope form](#).
- Determining the Equation of a Straight Line: [Algebra: Equation of a line](#), [Equation of a line](#).
- Concepts and Definitions: [Functional Relationships 1](#), [Testing if a relationship is a function](#), [Domain and Range 1](#), [Domain and Range 2](#), [Exploring nonlinear relationships](#).
- Graphing Functions: [Graph Parabola](#), [Graph piecewise function](#), [Graph Logarithm](#), [Graph rational function](#)
- The Algebra of Functions: [Introduction to functions](#), [Introduction to functions 1](#), [Functions Part 2](#), [Functions Part 3](#), [Functions Part 4](#).
- Solving Quadratic Equations: [Solving Quadratic Equations by Factoring](#), [Solving Quadratic Equations by Factoring 2](#), [Solving Quadratic Equations by Factoring 3](#), [Quadratic Functions 1](#), [Quadratic Functions 2](#), [Completing the Square 1](#),

[Completing the Square 2](#), [Completing the Square 3](#), [Completing the Square 4](#), [Quadratic Formula 1](#), [Quadratic Formula 2](#), [Quadratic Formula 3](#).

- Facilitating Non-linear Graphing: See videos on quadratics.

Module 3:

Suggested tasks for Module 3: Finish the Algebra Challenge on Khan Academy.

You may also find the following videos helpful...

- Introduction: [Systems of equations](#), [System of equations 2](#).
- Graphical Solutions: [Solving systems by graphing](#), [Solving systems by graphing 2](#), [Solving systems by graphing 3](#), [More Solving Systems by Graphing](#).
- Elimination and Substitution Methods: [Solving systems by substitution 1](#), [Solving systems by substitution 2](#), [Solving systems by substitution 3](#), [Solving systems by elimination](#), [Solving systems by elimination 2](#), [Solving systems by elimination 3](#).

Module 4:

Suggested tasks for Module 4: Complete the rest of the material on Khan Academy (i.e. the remaining exercises following completion of the Algebra Challenge).

You may also find the following videos helpful:

- Limits: [Introduction to Limits \(HD\)](#), [Introduction to Limits](#), [Limit Examples \(part 1\)](#), [Limit Examples \(part 2\)](#), [Limit Examples \(part3\)](#), [Limit Examples \(part 4\)](#), [Squeeze Theorem](#), [Proof: \$\lim \(\sin x\)/x\$](#) , [More Limits](#), [Formal Definition of Limits 1](#), [Formal Definition of Limits 2](#), [Formal Definition of Limits 3](#), [Formal Definition of Limits 4](#).
- The Derivative: [Calculus: Derivatives 1 \(new HD version\)](#), [Calculus: Derivatives 1.1\(numerical\)](#), [Calculus: Derivatives 2 \(new HD version\)](#), [Calculus: Derivatives 2.5 \(new HD version\)](#), [Calculus: Derivatives 1](#), [Calculus: Derivatives 2](#), [Calculus: Derivatives 3](#).
- Rules of Differentiation: [The Chain Rule](#), [Chain Rule Examples](#), [Even More Chain Rule](#), [Product Rule](#), [Quotient Rule](#), [Proof: \$d/dx\(x^n\)\$](#) , [Proof: \$d/dx\(\sqrt{x}\)\$](#) , [Proof: \$d/dx\(\ln x\) = 1/x\$](#) , [Proof: \$d/dx\(e^x\) = e^x\$](#) , [Proofs of Derivatives of \$\ln\(x\)\$ and \$e^x\$](#) .
- Implicit Functions: [Implicit Differentiation](#), [Implicit Differentiation \(part 2\)](#), [More implicit differentiation](#), [More chain rule and implicit differentiation](#).

Module 5:

Suggested tasks for Module 5: By now you should have completed all of Khan Academy material from Pre-Algebra all the way through basic Calculus. If you have not, keep working on it!

You may also find the following videos helpful:

- Increasing and Decreasing Functions: [Behaviour of functions](#), [Behaviour of functions](#).

- Relative Extrema: [Maxima Minima Slope Intuition](#), [Calculus: Maximum and minimum values on an interval](#), [Extreme Value](#), [Min-max points](#) .
- Inflection Points: [Inflection Points and Concavity Intuition](#).
- Curve Sketching: [Calculus: Graphing Using Derivatives](#), [Calculus Graphing with Derivatives Example](#), [Graphing with Calculus](#).
- Optimization of Functions: [Optimization with Calculus 1](#), [Optimization with Calculus 2](#), [Optimization with Calculus 3](#), [Optimization Example 4](#).
- Economic examples: [Optimizing profit at a shoe factory](#), [Minimizing sum of squares](#).

Module 6:

Suggested tasks for Module 6: To gain some appreciation of the power of the exponential function, watch the following 8 part video series: [Part 1](#), [Part 2](#), [Part 3](#), [Part 4](#), [Part 5](#), [Part 6](#), [Part 7](#), [Part 8](#)

You may also find the following videos helpful:

- Exponential Functions: [Exponential Growth Functions](#), [Exponential Growth](#)
- Logarithmic Functions: [Introduction to Logarithms](#).
- Properties of Exponential and Logarithmic Functions: [Introduction to logarithm properties](#), [Introduction to logarithm properties \(part 2\)](#).
- Interest Compounding: [Introduction to interest](#), [Interest \(part 2\)](#), [Introduction to compound interest](#), [e and compound interest](#).

Module 7:

For a more thorough and advanced treatment of single variable calculus (including both differential and integral calculus), we recommend two courses from MIT's OpenCourseWare. ([Single variable calculus](#) and [Calculus with applications](#))

You may also find the following videos helpful:

- Integration: [The Indefinite Integral or Anti-derivative](#).
- Rules for indefinite integrals: [Indefinite integrals \(part 2\)](#), [Indefinite Integration \(part 3\)](#), [Indefinite Integration \(part 4\)](#), [Indefinite Integration \(part 5\)](#).
- Area under a curve: [Riemann Sums](#), [Riemann Sums and Integrals](#).
- Definite Integrals: [Introduction to definite integrals](#), [Definite integrals \(part 2\)](#), [Definite Integrals \(part 3\)](#), [Definite Integrals \(part 4\)](#), [Definite integral with substitution](#).
- Integration by substitution: [Another u-substitution example](#), [\(2^{ln x}\)/x Antiderivative Example](#).
- Integration by parts: [Integration by Parts](#) , [Integration by Parts 2](#), [Integration by Parts 3](#) , [Integration by Parts 4](#) .
- Present value of a cash flow (very basic): [Introduction to Present Value](#), [Present Value 2](#), [Present Value 3](#), [Present Value 4 \(and discounted cash flow\)](#).

Module 8:

For a more advanced, but very thorough, treatment of multivariable calculus, check out [MIT OpenCourseWare](#). In particular, see Part C of Topic 2 (Partial Derivatives) for a discussion of Lagrange multipliers and constrained optimization. Week 5 quiz has exercises on how to use Lagrange multipliers to help you work through the topic.

You may also find the following videos helpful:

- Partial derivatives: [Partial Derivatives](#), [Partial Derivatives 2](#), [Partial Derivatives 3](#),
- Double Integrals: [Double Integral 1](#), [Double Integrals 2](#), [Double Integrals 3](#), [Double Integrals 4](#), [Double Integrals 5](#), [Double Integrals 6](#).
- Triple Integrals: [Triple Integrals 1](#), [Triple Integrals 2](#), [Triple Integrals 3](#).

Linear Algebra Guide:

Knowledge of linear algebra will come in very handy throughout the core Micro, Macro, and Econometrics courses on the SGPE. As such, we have taken the time to put together a "crash course" in linear algebra for those interested. The course takes advantage of the excellent Khan Academy videos on Linear Algebra. For those more advanced students, I provide links to Prof. Gilbert Strang's video lectures on the sub-topics of Linear Algebra that are particularly relevant for economists (and that will be of use for your MSc).

The following videos from Khan Academy may be of use:

- Definition and Terms: [Introduction to matrices](#), Linear Algebra: [Transpose of a Matrix](#), Linear Algebra: [Introduction to Vectors](#), Linear Algebra: [Vector Examples](#), Linear Algebra: [Transpose of a Vector](#).
- Addition and Subtraction of Matrices: [Sums and Scalar Multiples of Linear Transformations](#).
- Scalar Multiplication: [More on Matrix Addition and Scalar Multiplication](#).
- Vector Multiplication: [Vector Dot Product and Vector Length](#), Linear Algebra: [Cross Product Introduction](#), [Dot and Cross Product Comparison/Intuition](#).
- Multiplication of Matrices: [Matrix multiplication \(part 1\)](#), [Matrix multiplication \(part 2\)](#), [Matrix Vector Products](#), [Matrix Vector Products as Linear Transformations](#), [Linear Transformations as Matrix Vector Products](#), [Matrix Product Associativity](#), [Distributive Property of Matrix Products](#).
- Row Operations: [Matrices: Reduced Row Echelon Form 1](#), [Matrices: Reduced Row Echelon Form 2](#), [Matrices: Reduced Row Echelon Form 3](#).
- Determinants and Linear Independence: [Linear Combinations and Span](#), Linear Algebra: [Introduction to Linear Independence](#), [More on linear independence](#), [Span and Linear Independence Example](#).
- Third-order Determinants: [Linear Algebra: 3x3 Determinant](#), [Linear Algebra: nxn Determinant](#).
- Inverse Matrices: [Inverse Matrix \(part 1\)](#), [Inverting matrices \(part 2\)](#), Linear Algebra: [Deriving a method for determining inverses](#), Linear Algebra: [Example of](#)

[Finding Matrix Inverse](#), Linear Algebra: [Formula for 2x2 inverse](#), [Solving Systems of Equations with Matrices 1](#), [Solving Systems of Equations with Matrices 2](#).

For more advanced students, the following lectures from Prof. Gilbert Strang (MIT) cover the linear algebra that will be used on the SGPE:

- Lecture 1: [The geometry of linear equations](#)
- Lecture 2: [Elimination with matrices](#)
- Lecture 3: [Multiplication and inverse matrices](#)
- Lecture 4: [Transposes, permutations, and spaces](#)
- Lecture 5: [Column space and null space](#)
- Lecture 6: [Solving \$Ax=0\$](#)
- Lecture 7: [Solving \$Ax=b\$](#)
- Lecture 8: [Independence, basis, and dimension](#)
- Lecture 9: [The four fundamental sub-spaces](#)
- Lecture 10: [Orthogonal vectors and subspaces](#)
- Lecture 11: [Projections onto sub-spaces](#)
- Lecture 12: [Projection matrices and least-square \(linear regression\)](#)
- Lecture 13: [Properties of determinants](#)
- Lecture 14: [Determinant formulas and cofactors](#)
- Lecture 15: [Eigenvalues and eigenvectors](#)
- Lecture 16: [Symmetric matrices and positive definiteness](#)
- Lecture 17: [Positive definiteness and minima](#)
- Lecture 18: [Linear transformation and their matrices](#)