Problem Set 9 – Uncertainty

1. Suppose you are selling kites on the boardwalk at the New Jersey shore as your summer job. Every fourth person that passes you selects a kite at random and buys it. One-half of your kites sell for $8, and the other half sell for $4. If 100 people per hour pass by, what is your expected revenue for six hours work? What are the variance and standard deviation of revenue per person from people who bought kites?

2. Many states run lotteries of various kinds. One of the most popular is a daily number game where an individual buys a $1 ticket with a number of their choosing between 000 and 999. The payoff is typically $500. Would a strictly risk-neutral person buy such a ticket? What payoff would ensure that at least some people who are risk averse would buy a ticket?

3. What is the Arrow-Pratt measure of risk aversion for somebody who has a utility U = ln(W)?

4. Suppose you face the following lottery. You can earn 1 of 3 possible grades in this class: an "A", a "C", or an "F", with the following probabilities:

$$\pi_A = \frac{2}{10}, \pi_C = \frac{6}{10}, \pi_F = \frac{2}{10}$$

Your current wealth (w) is $400. If you receive an "A", you gain (e.g. I pay you) $500. However, if you get an "F", you lose (e.g. you pay me) $300. If you receive a "C", you DO NOT GAIN OR LOSE anything. Assume your utility function, defined over wealth, is $U(w) = (w)^{0.5}$.

   i) What is your expected utility (EU)? [Hint: be sure to calculate your total wealth in each "state".]
   ii) What is the certainty equivalent level of wealth ($w*$), that is, the guaranteed payoff at which a person is "indifferent" between accepting the guaranteed payoff and their expected utility from (a)?

5. Consider three levels of wealth W = 1,000, W = 0, and W = 2,000. We know U (0), U (1,000), and U (2,000). How can we tell if somebody is risk averse, risk neutral or risk loving from this information?

6. What would be the price of fair insurance for a $20,000 motor home for one year, assuming that during that year there is a 0.02% chance that it will be destroyed in an accident, leaving a $3000 salvage value and no chance of any partial loss? Assume that the owner keeps the salvage value.

7. A city is considering how much to spend to hire people to monitor its parking meters. The following information is available to the city manager:

   a) Hiring each meter monitor costs $10,000 per year.
   b) With one monitoring person hired, the probability of a driver getting a ticket each time he or she parks illegally is equal to 0.25. With two monitors, the probability of getting a ticket is 0.5. With three monitors, the probability is 0.75. Currently we have two monitors hired and the current ⋯ne for overtime parking is $20.

   i) Assume first that all drivers are risk-neutral. What parking ⋯ne would you levy, and how many meter monitors would you hire (1,2,3, or 4) to achieve the current level of deterrence against illegal parking at the minimum cost?
   ii) Now assume that drivers are highly risk-averse. How would your answer to (a) change?
   iii) What if drivers could insure themselves against the risk of parking fine? Would it make good public policy to permit such insurance? Note this question is not necessarily related to uncertainty.

8. Suppose it costs a corporation an additional $500 in transactions costs to have two executives fly separately rather than together. If they were both killed, they would lose $5 million in profits. Given that a probability of any single flight crashing of 0.000000432, should a risk-neutral firm separate the executives?

9. Suppose Natasha's utility function is given by $U = (10W)^{0.5}$, where W is wealth in thousands

   i) Is Natasha risk loving, risk neutral, or risk averse? Explain.
   ii) Suppose that Natasha is currently earning an income of 40,000 (W = 40) and can earn that income next year with certainty. She is offered a chance to take a new job that offers a 0.6 probability of earning $44,000 and a 0.4 probability of earning $33,000. Should she take the job?
   iii) Suppose in (b), would Natasha be willing to buy insurance to protect against the variable income associated with the new job? If so, how much would she be willing to pay for the insurance?

10. Consider a lottery with three possible outcomes: £ 125 will be received with probability 0.2, £ 100 will be received with probability 0.3, and £ 50 will be received with probability 0.5.
    i) What is the expected utility of the lottery?
    ii) What is the variance of the outcomes?
    iii) What would a risk-neutral person pay to play the lottery?

11. Show that a risk-averse person will fully insure if insurance is actuarially fair.

12. Lori, who is risk averse, has two pieces of jewelry, each worth £ 1,000. She wants to send them to her sister in Thailand. She is concerned about the safety of shipping them. She believes that the probability that a given shipment of jewelry won't arrive is p. Is her expected utility higher if she sends the articles together or in two separate shipments?

13. John is coming back from the insurance broker, where he has just taken out insurance on his house. On his way home, he decides to pop into the corner store and buy a lottery ticket. Can his behavior be explained by expected utility theory? If so, what should be the shape of his utility function?