## Problem Set 7 - General equilibrium theory

1. Suppose the government were to impose a $20 \%$ tax on newspapers. What other markets (both product market and input market) should be included in a general equilibrium analysis?
2. Suppose the only two meat dishes consumed in the United States are flounder and beef. What are the likely effects in these markets of a rumor of a major water pollution problem discovered in the commercial fishing waters? Assume that production resources cannot be shifted between sectors.
3. The industry analysts have long recognized that there is a high degree of complementarity between automobile tires and gasoline. A recent study done by an automobile industry trade group estimated the following supply and demand functions:

$$
\begin{aligned}
& Q_{D}^{T}=5,250,000-12,500 P^{T}-750,000 P^{G} \\
& Q_{S}^{T}=350,000+11,750 P^{T} \\
& Q_{D}^{G}=80,500,000-30,000,000 P^{G}-2,500 P^{T} \\
& Q_{S}^{G}=35,000,000+15,000,000 P^{G}
\end{aligned}
$$

where $\mathrm{QD}^{\mathrm{T}}$ and $\mathrm{Qs}^{\mathrm{T}}$ refer to quantities of tires demand and supplied each month measured in sets of four, $\mathrm{Q}_{\mathrm{D}}{ }^{\mathrm{G}}$ and $\mathrm{Q}_{5}{ }^{\mathrm{G}}$ refer to quantities of gasoline demanded and supplied each month measured in gallons, PG is the price of gasoline per gallon, and $\mathrm{P}^{\mathrm{T}}$ is the price per set of four tires.
i. Calculate the equilibrium price and quantity that will prevail in both the tire and gasoline markets. (Hint: recall that $Q_{D}$ must equal $Q_{s}$ in each market.)
ii. Assume that a recession causes the demand curve for gasoline to shift leftward as follows: $\mathrm{Q}_{\mathrm{D}}{ }^{G}=76,000,000-30,000,000 \mathrm{P}^{G}$ $2,500 \mathrm{P}^{\mathrm{T}}$. Calculate the initial impact of this change in demand on the gasoline and tire markets. (You need calculate only one change in $P$ and $Q$ for each market.)
iii. Discuss the changes that will occur after the initial round to move each market back to a stable equilibrium. Your answer to part (c) requires no calculations, but graphs would help convey your understanding of the process.
4. Two individuals, Fred and Helen, in an economy with no production, each have the utility function $U=10 X Y$. Prices of both $X$ and $Y$ are set
at $\$ 1$. Initial endowments for Fred are 10 units of $X$ and 6 units of $Y$. Helen has 8 units of X and 12 units of Y . Show that this initial endowment is not on the contract curve.
5. Two individuals, A and B, are free to engage in trade of clothing and food. Initially, A has 12 units of clothing and 9 units of food, and B has 8 units of clothing and 11 units of food. The individuals have the following utility functions in clothing C and food F :

$$
\begin{aligned}
& U_{A}=0.15 Q_{C} \times Q_{F} \\
& U_{A}=0.08 Q_{C} \times Q_{F}
\end{aligned}
$$

where QF represents units of food, QC represents units of clothing, and U represents utility. Determine if a mutually beneficial trade is possible between A and B. If so, who would trade for what?
6. In a competitive economy, the marginal cost of producing $X$ is $M C_{X}=$ $20+5 q_{x}$. The marginal cost of producing $Y$ is $M C_{Y}=10+10 q_{\gamma}$. Prices are $p_{X}=5$, and $p_{Y}=5$. What linear relationship describes the proportion in which the goods should be produced?
7. Suppose Jenna and Karen both regard peanut butter and jelly as perfect complements at a 1:1 ratio. Show using an Edgeworth box diagram that if Jenna receives 10 peanut butter and no jelly, and Karen receives 10 jelly and no peanut butter, after trading, they will each end up with 5 units of each.
8. Olga and Vanessa have an endowment of red wine and white wine. Vanessa is not a classy woman and views red wine and white wine as perfect substitutes in a one to one ratio. Olga, however has a utility function for red and white wine which is

$$
U=\max [R, W]
$$

That is, if Olga consumes red and white wine together, her utility will decrease. If Olga and Vanessa trade, what will the contract curve look like?
9. Sarah and Jane are two representative individuals living in an economy that produces two goods, X and Y . Sarah's and Jane's utility functions are given as:

$$
\begin{aligned}
& U^{S}=100 X^{0.5} Y^{0.5} \\
& U^{J}=50 X^{0.4} Y^{0.6}
\end{aligned}
$$

The market determined prices of $X$ and $Y$ are $\$ 10$ and $\$ 20$, respectively. Current outputs are 58 units of $X$ per time period and 36 units of $Y$. Jane's current income is $\$ 600$ per time period, while Sarah's income is $\$ 700$ per time period.
a. Write expressions for Sarah and Jane's marginal rates of substitution.
b. Determine the quantities of $X$ and $Y$ that Sarah and Jane should consume in equilibrium.
c. Do the values calculated in part (b) satisfy the conditions for equilibrium in exchange? Explain using numbers.
d. Examine your answers in parts (b) and (c). If equilibrium has not been achieved, what would be necessary to reach equilibrium? If equilibrium has been achieved, comment on the process by which equilibrium was reached.
10. Suppose total economic welfare $W$ is measured as suggested by Rawls; $W=\min \left(U_{1}, U_{2}\right)$. Show that maximizing $W$ by this criterion results in the same final allocation as an egalitarian scheme where each individual gets an equal share.
11. Refer to the diagram below to answer this question.


The Edgeworth Box diagram above refers to input usage in the automobile industry and the computer chip industry. The contract curve is given by $\mathrm{O}_{\mathrm{A}} \mathrm{O}_{\mathrm{c}}$. All points on this curve correspond to input prices being equivalent across industries. For example, the line segment AC indicates the input price ratio at point $B$ in the two industries. Also, line segment DF indicates the input price ratio at
point E in the two industries. The diagram below presents the production possibilities frontier for automobiles and computer chips.


The labeled points in the production possibilities frontier diagram correspond to the points A, B, E, and F in the Edgeworth Box diagram above. Which points in the production possibilities frontier diagram correspond to which points in the Edgeworth Box diagram? Why?
12. The U.S. and Mexico can produce the amounts of computer chips and textiles indicated in the table below with one unit of land. Each country has 10 units of land. Does either country have an absolute advantage? Which country has a comparative advantage in chip production? Which country has a comparative advantage in textile production? Is it possible for the two countries to benefit from trade?

| Country | Chips | Textiles |
| :---: | :---: | :---: |
| U.S. | 1,000 | 100 |
| Mexico | 300 | 50 |

13. Suppose we are in an exchange economy in which two traders have Cobb-Douglas utility functions. The two goods are H and G where total endowments are $G=100$ and $H=50$ : What are the equilibrium prices (note it is easier to normalize the price of one good to 1 )? [This is a difficult question, but remember you need to set supply equal to demand in order to find the price].

$$
\begin{aligned}
U_{a} & =\left(G_{a}\right)^{\frac{1}{4}}\left(H_{a}\right)^{\frac{3}{4}} \\
U_{b} & =\left(G_{b}\right)^{\frac{1}{2}}\left(H_{b}\right)^{\frac{1}{2}}
\end{aligned}
$$

