Problem Set 5 – Production costs

1. Is it possible for a firm to use a production process that is economically efficient that is NOT technologically efficient?

2. [True or False, explain your answer.] I paid $25 for the materials to make these flower arrangements, and sold them at the craft fair for $25, so I just broke even."

3. Some questions about short-term and long-term costs:

   i) What is the relationship between the long-run average cost curve and the short-run average cost curve? Please show graphically.
   ii) What algebraic condition describes a firm that is at an output level that maximizes its profits, given its capital in the short-term?
   iii) What two algebraic conditions describe a firm that is at a capital level that minimizes its costs in the long-term?
   iv) If a firm is characterized by short-run marginal cost that is greater than long-run marginal cost and short-run average cost greater than long-run average cost, how should it change its capital level in the long-run to minimize costs?

4. Duane breeds parrots for a living. He has discovered that the production function for parrot chicks (Q) is:

$$Q = K^{0.5}L^{0.5}$$

where K is capital (for example nest boxes, cages and the like) and L is parrot food. The marginal products of capital and labor are as follows:

$$MP_K = 0.5K^{-0.5}L^{0.5}$$
$$MP_L = 0.5K^{0.5}L^{-0.5}$$

The price of K is $8 and the price of L is $2.

   a. What type of production function is this?
   b. Does this production function exhibit constant, increasing or decreasing returns to scale? Explain.
   c. What is the average product of capital?
   d. Does capital obey the "law of diminishing returns?" Explain.
   e. Suppose that Duane wants 144 parrot chicks, how much K and L should be employed to minimize costs, and what is the cost of producing 144 parrot chicks?

f.   Suppose that Duane is faced with the same problem as in (f) except that he has a fixed amount of K. In fact, K = 16. How much L should be employed to minimize costs, and what is the total cost?

5. A firm's total cost function is given by the equation TC = 4000 + 5Q + 10Q². Write an expression for each of the following cost concepts:

> a. Total Fixed Cost
> b. Average Fixed Cost
> c. Total Variable Cost
> d. Average Variable Cost
> e. Average Total Cost
> f. Marginal Cost

Determine the quantity that minimizes average total cost. Demonstrate that the predicted relationship between marginal cost and average cost holds.

6. Suppose a firm's average cost curve is described by the equation $AC = 2q^2 - 16q + 90$. At what output level does the marginal cost curve cross the average cost curve?

7. Acme Container Corporation produces egg cartons that are sold to egg distributors. Acme has estimated this production function for its egg carton division:

$$Q = 25L^{0.6}K^{0.4},$$

where Q = output measured in one thousand carton lots, L = labor measured in person hours, and K = capital measured in machine hours. Acme currently pays a wage of $10 per hour and considers the relevant rental price for capital to be $25 per hour. Determine the optimal capital-labor ratio that Acme should use in the egg carton division.

8. If input prices are $w = 3$, and $r = 2$, and $q = 10KL$, what is the least cost input combination required to produce 60 units of output? How would input usage change if output is increased to 240 units? Sketch the solutions on a graph.

9. Davy Metal Company produces brass fittings. Davy's engineers estimate the production function represented below as relevant for their long-run capital-labor decisions.

$$Q = 500L^{0.6}K^{0.8},$$

where Q = annual output measured in pounds, L = labor measured in person hours, K = capital measured in machine hours. The marginal products of labor and capital are:

$$MP_L = 300L^{-0.4} K^{0.8} \quad MPK = 400L^{0.6} K^{-0.2}$$

Davy's employees are relatively highly skilled and earn $15 per hour. The firm estimates a rental charge of $50 per hour on capital. Davy forecasts annual costs of $500,000 per year, measured in real dollars.

a) Determine the firm's optimal capital-labor ratio, given the information above.

b) How much capital and labor should the firm employ, given the $500,000 budget? Calculate the firm's output.

c) Davy is currently negotiating with a newly organized union. The firm's personnel manager indicates that the wage may rise to $22.50 under the proposed union contract. Analyze the effect of the higher union wage on the optimal capital -labor ratio and the firm's employment of capital and labor. What will happen to the firm's output?

10. Two firms currently produce the goods q_1 and q_2 separately. Their cost functions are $C(q_1) = 25 + q_1$, and $C(q_2) = 35 + 2q_2$. By merging, they can produce the two goods jointly with costs described by the function $C(q_1, q_2) = 45 + q_1 + q_2$. Are there scope economies in this case that would justify the merger?

11. Ricardo produces widgets, using as inputs labor (L) and machines (K). His production function is given by the following equation:

$$q = 10K^{\frac{2}{3}} + L^{\frac{1}{2}}$$

i) What type of returns to scale (increasing/constant/decreasing) does Ricardo's production function exhibit?

At the end of last year, Ricardo bought his only machine for $1,000. He will use this machine for 5 years, after which the machine will have no value. Ricardo will calculate depreciation linearly (depreciation will be

20% of the total value of the machine per year). This machine has no other use besides Ricardo's production of widgets, and, at this moment, Ricardo cannot buy any more machines.

ii) What is Ricardo's annual fixed cost of production? Is the fixed cost sunk or not? Explain.

iii) What is Ricardo's demand for labor as a function of the quantity he wants to produce annually?

iv) Assuming that wage equals 1, what is Ricardo's annual total cost function?

12. Murray Manufacturing Company produces pantyhose. The firm's production function is given as:

$$Q = 5LK,$$

where Q = pairs of pantyhose, L = labor measured in person hours, and K = capital measured in machine hours. Murray's labor cost, including fringe benefits, is $20 per hour, while the firm uses $80 per hour as an implicit machine rental charge per hour. Murray's current budget is $64,000 per month to pay labor and capital.

(a) Given the information above, determine Murray's optimal capital/labor ratio.

(b) Using the Lagrangian technique, determine the quantities of labor and capital that will allow the firm to maximize output given their budgeted input expenditure. What is the firm's output?

(c) Again using the Lagrangian technique, demonstrate the duality in production and cost theory.

13. A firm's marginal cost function is as follows

$$MC = 3q^2 - q$$

What is the firm's average variable cost of producing 10 units of output?