Problem Set 4 – Producer Theory

1. Write the equation for the marginal product of capital for each of the following production functions.

   i) Q = K + L
   ii) $Q = 4K^{0.5} L$
   iii) $Q = 5L^{0.5}K - L$

2. Using the production function from part (c) in Question 1, assume that capital is fixed at two units. At what point does MPL reach zero?

3. Leann′ s Telecommunication firm production function is given by $Y(K,L) = 200(KL)^{\frac{2}{3}}$, where K is the number of Internet servers and L is the number of labor hours she uses. Does this production function exhibit increasing, constant or decreasing returns to scale? Holding the number of Internet servers constant at 8, is the marginal product of labor increasing, constant or decreasing as more labor is used?

4. Draw a graph showing a set of isoquants that depict capital and labour to be perfect complements (not substitutable at all) in a production function that exhibits constant returns to scale. Be sure to label the input and output levels on the isoquants.

5. For each of the following production functions, sketch a representative isoquant. Calculate the marginal product for each input, and indicate whether each marginal product is diminishing, constant, or increasing. Also calculate the marginal rate of technical substitution for each function. Also indicate whether the function exhibits constant, increasing, or diminishing returns to scale.

   i) $F(L, K) = L K^3$
   ii) F (L, K) = L + 3K
   iii) $F(L, K) = (\min\{L, K\})^{1/3}$

6. The world production of food is determined by the function $F = (M)(L)^{\frac{1}{2}}$ where F is food, M is the world supply of arable land and L is the world supply of labor (population).

   a. What is the marginal product of labor? At what point does this production function exhibit diminishing marginal returns to labor? If the world supply of arable land is fixed, what does continual population growth imply for the amount of food available per person?

b. Calculate the marginal product of land. As the world population increases, does land become more or less productive?
c. New technologies make farming on once un-farmable land possible. If the supply of arable land and the supply of labor increase by the same amount, what will happen to the world supply of food?

7. Homer′ s boat manufacturing plant production function is $Y(K,L)=K^{\frac{1}{5}}L^{\frac{1}{2}}$ where K is the number of hydraulic lifts and L is the number of labor hours he employs. Does this production function exhibit increasing, decreasing or constant returns to scale? At the moment, Homer uses 20,000 labor hours and 50 hydraulic lifts. Suppose that Homer can use any amount of either input without affecting the market costs of the inputs. If Homer increased his use of labor hours and hydraulic lifts by 10%, how much would his production increase? Increasing the use of both inputs by 10% will result in Homer′ s costs increasing by exactly 10%. If Homer increases his use of all inputs by 10%, what will increase more, his production or his costs? Given that Homer can sell as many boats as he produces for \$75,000, does his profits go up by 10% with a 10% increase in input use?

8. Consider the production function $f(K,L)=2L^{\frac{1}{4}}K^{\frac{1}{4}}$.

   a. Find the associated (long run) total, average, and marginal cost curves.
   b. Sketch the total, average, and marginal cost curves.

9. In an industry where there are increasing returns to scale over wide ranges of output, would you expect to find a few relatively large firms or many relatively small firms? Why?

10. You run a cost-minimizing firm with production function f(L,K) = [min{L,K}]1/3 , where L is labour and K is capital. Assume that you are a price-taker in the input markets: you pay w for each unit of labour you hire and r for each unit of capital (where w and r are set exogenously), and face no costs other than those from labour and capital.

a) Assuming that you can freely choose both labour and capital (i.e., the "long-run problem"), derive expressions for your cost-minimizing conditional input demands, L* (r, w, Q) and K* (r, w, Q). Confirm that the conditional input demand functions are "homogeneous of degree zero" in w and r; that is,

$$\begin{cases} L^*(tr,tw,Q) = L^*(r,w,Q) \\ K^*(tr,tw,Q) = K^*(r,w,Q) \end{cases} \quad \textit{for all} \quad t > 0$$

b) What will happen to your conditional demand for labour if there is an increase in the wage rate, assuming that r and Q remain the same? Explain in one sentence why your answer makes intuitive sense.

c) Use your answers from (a) to write down an expression for your total cost function TC(r, w, Q). Is this function "homogeneous of degree one" in w and r; that is, does TC (tr, tw, Q) = t * TC (r, w, Q)?

11. Let α and β be the labour and output elasticities respectively of a Cobb-Douglas production function (The exponent attached to L and the exponent attached to K).

i) Show that the production function is homogenous of degree $\alpha + \beta$.
ii) If we multiply both of our inputs by x, show that the elasticity of output with respect to x is $\alpha + \beta$.

12. Prove that the following production function has a constant elasticity of substitution.

$$q = (L^\rho + K^\rho)^{\frac{1}{\rho}}$$

13. A function is said to be homogenous of degree γ if $f\ (xL,\ xK) = x^\gamma f\ (L,\ K)$ where x is a positive constant. If your production function is $q = K^\alpha L^{1-\alpha}$, show that the marginal product of labour and the marginal product of capital are homogenous of degree 0. What is the interpretation of this?