Problem Set 4 – Producer Theory

1. Write the equation for the marginal product of capital for each of the following production functions.

   i) $Q = K + L$
   ii) $Q = 4K^{0.5} L$
   iii) $Q = 5L^{0.5}K – L$

2. Using the production function from part (c) in Question 1, assume that capital is fixed at two units. At what point does MPL reach zero?

3. Suppose inputs are only substitutable at two units of labor for every one unit of capital. What would be the equation for the production function? What is the average and marginal product of labor in this case?

4. Draw a graph showing a set of isoquants that depict capital and labour to be perfect complements (not substitutable at all) in a production function that exhibits constant returns to scale. Be sure to label the input and output levels on the isoquants.

5. For each of the following production functions, sketch a representative isoquant. Calculate the marginal product for each input, and indicate whether each marginal product is diminishing, constant, or increasing. Also calculate the marginal rate of technical substitution for each function. Also indicate whether the function exhibits constant, increasing, or diminishing returns to scale.

   i) $F(L, K) = L K^3$
   ii) $F(L, K) = L + 3K$
   iii) $F(L, K) = (\min\{L, K\})^{1/3}$

6. In an industry where there are increasing returns to scale over wide ranges of output, would you expect to find a few relatively large firms or many relatively small firms? Why?

7. The original production function is $Q = 10K^{0.4}L^{0.5}$. A technological change occurs that alters the production function to $Q = 15K^{0.4}L^{0.7}$. Is this an example of neutral technological change? Why or why not?

8. Suppose output is produced according to the production function $Q = M^{0.5}K^{0.5}L^{0.5}$, where M is materials. Does this production function exhibit decreasing, increasing, or constant returns to scale? Show using an example.

9. If your production function is $q = 5L + L^2K^{0.5} - L^3$ and capital is fixed at $K = 4$, prove that the marginal product of labor equals the average product of labor at the maximum of the average product of labor.

10. Let α and β be the labour and output elasticities respectively of a Cobb-Douglas production function (The exponent attached to L and the exponent attached to K).

    i) Show that the production function is homogenous of degree $\alpha + \beta$.
    ii) If we multiply both of our inputs by x, show that the elasticity of output with respect to x is $\alpha + \beta$.

11. Prove that the following production function has a constant elasticity of substitution.

$$q = (L^{\rho} + K^{\rho})^{\frac{1}{\rho}}$$

12. A function is said to be homogenous of degree γ if $f(xL, xK) = x^{\gamma} f(L, K)$ where x is a positive constant. If your production function is $q = K^{\alpha}L^{1-\alpha}$, show that the marginal product of labour and the marginal product of capital are homogenous of degree 0. What is the interpretation of this?