Problem Set 2 – Consumer Theory

1. Suppose a consumer's utility derived from consuming bananas is described by the function $U = 10X + 3X^2 - 1/3X^3$

   i) Make a table showing total and marginal utility for X from 0 to 7 units.
   ii) Would this individual ever choose to consume more than 7 units? Explain.

2. Explain what information is contained in the slope of an indifference curve? Why are these curves typically convex to the origin? [Bonus: Show in graph a person's utility function such that he will be indifferent between an education voucher and cash.].

3. In each of the following examples, a consumer purchases just two goods: X and Y. Based on the information in each of the following parts, sketch a plausible set of indifference curves (that is, draw at least two curves on a set of labelled axes, and indicate the direction of higher utility). Also, write down a utility function U (X, Y) consistent with your graph. Note that although all these preferences should be assumed to be complete and transitive (as required for utility representation), not all will be monotone.

   i) Nick enjoys bagels x and coffee y, and consuming more of one makes consuming the other more enjoyable.
   ii) Sean loves chocolate chip ice cream x, but he hates broccoli y.
   iii) Irina likes blueberry muffin x, and neither likes nor dislikes lemon banana tart y.
   iv) Anna always buys three white tank tops x for every pair of jeans y.
   v) Yulia likes both peanut butter x and jelly y, and always gets the same additional satisfaction from an ounce of peanut butter as she does from two ounces of jelly.

4. Suppose a consumer has an income of \$500 and faces prices $P_x = 5$ and $P_z = 10$

   i) Write the equation for the budget constraint.
   ii) Draw the budget constraint, placing good *X* on the horizontal axis.
   iii) What is the slope of budget constraint?
   iv) Suppose income decreases to \$300. Draw the new budget constraint.

5. Calculate the optimal consumption combination for each of the following utility functions:

i) $U = 30 X_1 X_2$
ii) $U = 30 X_1^{\alpha} X_2^{1-\alpha}$
iii) $U = X_1 + 2X_2$
iv) $U = \min \{ X_1 , X_2 \}$

6. Suppose a consumer has income of \$120 per period, and faces prices $P_X = 2$ and $P_Z = 3$. Her goal is to maximize her utility, described by the function $U = 10X^{0.5}Z^{0.5}$. Calculate the utility maximizing bundle (X^*, Z^*) using the Lagrangian method.

7. A consumer's preferences are representable by the following utility function:

$$U (X, Y) = X^{0.5} + Y.$$

   i) Obtain the MRS of the consumer at an arbitrary point (X^*, Y^*), where $X^* > 0$ and $Y^* > 0$.
   ii) Suppose the price of the second good (y) is 1, and the price of the first good (x) is denoted by $p > 0$. If the consumer's income is $m > 0$, obtain the optimal consumption bundle of the consumer (in terms of m and p). [Hint: Try to cover both cases, one in which m is relatively low, and the other in which m is relatively high.].

8. Suppose Carmela's income is \$100 per week, which she allocates between sandwiches and books. Sandwiches cost \$2 each. Books cost \$10 each if she purchases between 1 and 5 books. If she purchases more than 5 books in a week, the price falls to \$5 for the 6th book and all subsequent books. Draw the budget constraint. Is it possible that Carmela might have more than one utility-maximizing solution?

9. Confirm that if a consumer's utility function is described by $U = 2X + Z$, and prices are $P_X = 2$ and $P_Z = 1$, there is no unique utility maximizing solution regardless of income level. What does this tell you about X and Z as commodities? (Hint: draw a graph showing a budget constraint and indifference curve using the information provided.)