SGPE Summer School
Macro Problem Set 4

Economic Growth II
Mankiw, Chapter 8

2^{nd} August 2013

Short Answer Questions

(1) What data would you need to determine whether an economy has more or less capital than in the Golden Rule steady state?

(2) How does endogenous growth theory explain persistent growth without the assumption of exogenous technological progress? How does this differ from the Solow model?

Problems

Question 1

In the United States, the capital share of GDP is about 30 percent, the average growth in output is about 3 percent per year, the depreciation rate is about 4 percent per year, and the capital–output ratio is about 2.5. Suppose that the production function is Cobb–Douglas, so that the capital share in output is constant, and that the United States has been in a steady state. (For a discussion of the Cobb–Douglas production function, see Chapter 3.)

(a) What must the saving rate be in the initial steady state? [Hint: Use the steady-state relationship, $sy = (\delta + n + g)k$].

(b) What is the marginal product of capital in the initial steady state?

(c) Suppose that public policy raises the saving rate so that the economy reaches the Golden Rule level of capital. What will the marginal product of capital be at the Golden Rule steady state? Compare the marginal product at the Golden Rule steady state to the marginal product in the initial steady state. Explain.

(d) What will the capital–output ratio be at the Golden Rule steady state? (Hint: For the Cobb–Douglas production function, the capital–output ratio is related to the marginal product of capital.)

(e) What must the saving rate be to reach the Golden Rule steady state?

Question 2

Prove each of the following statements about the steady state of the Solow model with population growth and technological progress.

(a) The capital–output ratio is constant.

(b) Capital and labor each earn a constant share of an economy's income. [Hint: Recall the definition $MPK = f(k+1)–f(k)$.]

(c) Total capital income and total labor income both grow at the rate of population growth plus the rate of technological progress, $n + g$.

(d) The real rental price of capital is constant, and the real wage grows at the rate of technological progress g. (Hint: The real rental price of capital equals total capital income divided by the capital stock, and the real wage equals total labor income divided by the labor force.)

Question 3

Two countries, Richland and Poorland, are described by the Solow growth model. They have the same Cobb–Douglas production function, $F(K, L) = AK^a L^{1-a}$, but with different quantities of capital and labor. Richland saves 32 percent of its income, while Poorland saves 10 percent. Richland has population growth of 1 percent per year, while Poorland has population growth of 3 percent. (The numbers in this problem are chosen to be approximately realistic descriptions of rich and poor nations.) Both nations have technological progress at a rate of 2 percent per year and depreciation at a rate of 5 percent per year.

(a) What is the per-worker production function $f(k)$?

(b) Solve for the ratio of Richland's steady-state income per worker to Poorland's. (Hint: The parameter α will play a role in your answer.)

(c) If the Cobb–Douglas parameter a takes the conventional value of about 1/3, how much higher should income per worker be in Richland compared to Poorland?

(d) Income per worker in Richland is actually 16 times income per worker in Poorland. Can you explain this fact by changing the value of the parameter α? What must it be? Can you think of any way of justifying such a value for this parameter? How else might you explain the large difference in income between Richland and Poorland?