SGPE Summer School
Macro Problem Set 3

Economic Growth I
Mankiw, Chapter 7

1^{st}August 2013

Short Answer Questions

(1) In the Solow model, how does the saving rate affect the steady-state level of income? How does it affect the steady-state rate of growth?

(2) Might a policymaker choose a steady state with more capital than in the Golden Rule steady Golden Rule level of capital state? With less capital than in the Golden Rule steady state? Explain your answers.

Problems

Question 1

Country A and country B both have the production function

$$Y = F(K, L) = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

(a) Does this production function have constant returns to scale? Explain.

(b) What is the per-worker production function, $y = f(k)$?

(c) Assume that neither country experiences population growth or technological progress and that 5 percent of capital depreciates each year. Assume further that country A saves 10 percent of output each year and country B saves 20 percent of output each year. Using your answer from part (b) and the steady-state condition that investment equals depreciation, find the steady-state level of capital per worker for each country. Then find the steady-state levels of income per worker and consumption per worker.

(d) Suppose that both countries start off with a capital stock per worker of 2. What are the levels of income per worker and consumption per worker? Remembering that the change in the capital stock is investment less depreciation, use a calculator or a computer spreadsheet to show how the

capital stock per worker will evolve over time in both countries. For each year, calculate income per worker and consumption per worker. How many years will it be before the consumption in country B is higher than the consumption in country A?

Question 2

Consider an economy described by the production function:

$$Y = F(K, L) = K^{0.3}L^{0.7}.$$

(a) What is the per-worker production function?

(b) Assuming no population growth or technological progress, find the steady-state capital stock per worker, output per worker, and consumption per worker as a function of the saving rate and the depreciation rate.

(c) Assume that the depreciation rate is 10 percent per year. Make a table showing steady-state capital per worker, output per worker, and consumption per worker for saving rates of 0 percent, 10 percent, 20 percent, 30 percent, and so on. (You will need a calculator with an exponent key for this.) What saving rate maximizes output per worker? What saving rate maximizes consumption per worker?

(d) (Harder) Use calculus to find the marginal product of capital. Add to your table the mar- ginal product of capital net of depreciation for each of the saving rates. What does your table show?

Question 3

One view of the consumption function is that workers have high propensities to consume and capitalists have low propensities to consume. To explore the implications of this view, suppose that an economy consumes all wage income and saves all capital income. Show that if the factors of production earn their marginal product, this economy reaches the Golden Rule level of capital.

(Hint: Begin with the identity that saving equals investment. Then use the steady-state condition that investment is just enough to keep up with depreciation and population growth and the fact that saving equals capital income in this economy.)

Question 4

Consider how unemployment would affect the Solow growth model. Suppose that output is produced according to the production function

$$Y = K^a[(1-u)L]^{1-a}$$

where K is capital, L is the labor force, and u is the natural rate of unemployment. The national saving rate is s, the labor force grows at rate n, and capital depreciates at rate d.

(a) Express output per worker ($y = Y/L$) as a function of capital per worker ($k = K/L$) and the natural rate of unemployment. Describe the steady state of this economy.

(b) Suppose that some change in government policy reduces the natural rate of unemploy- ment. Describe how this change affects output both immediately and over time. Is the steady-state effect on output larger or smaller than the immediate effect? Explain.