

Macroeconomics Lecture 3

SGPE Summer School

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Introduction

Questions

- What is the interest rate?
- What is the real interest rate
- How are investments determined?
- How is consumption determined?
- How is the real interest rate determined in equilibrium?

Chs 3-4

Interest rate

Price of money

i_t Nominal interest between period t and $t + 1$

$1 + i_t$ The price of money today in terms of money tomorrow. If I borrow one euro today, I will have to pay it back with interest tomorrow.

$\frac{1}{1+i_t}$ Discounting factor which is the price of money tomorrow in terms of money today. How much I have to set aside today to get one euro next year. Used to evaluate future incomes.

Inflation:

$$\pi_{t+1} = \frac{P_{t+1} - P_t}{P_t} = \frac{\Delta P_{t+1}}{P_t} \implies \frac{P_{t+1}}{P_t} = 1 + \pi_{t+1}$$

Real Interest and Intertemporal Price

A consumer chooses between buying/consuming a product today or tomorrow

The price of the product P_t and nominal interest i_t

Consume one unit less today:

- We save P_t today
- In the next period, we can consume $\frac{P_t(1+i_t)}{P_{t+1}} = \frac{1+i_t}{P_{t+1}/P_t}$
- This is the price of consumption today in terms of consumption tomorrow

Intertemporal price

Real interest

$$1 + r_{t+1} = \frac{1 + i_t}{P_{t+1}/P_t} = \frac{1 + i_t}{1 + \pi_{t+1}}$$

Real interest shows how much more we have to reduce consumption next year.

E.g., Example: If $r = 0.1$. We consume 10 units more this year we must reduce consumption by 11 units next year to pay back the loan.

Approximation of real interest rate:

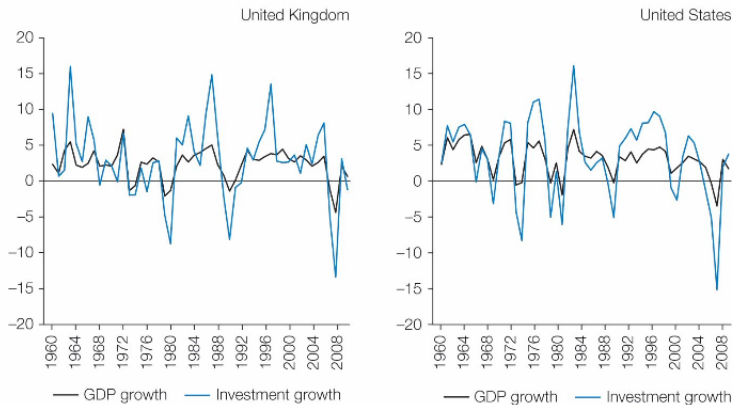
$$1 + r_{t+1} = \frac{1 + i_t}{1 + \pi_{t+1}} \approx 1 + i_t - \pi_{t+1} \implies r_{t+1} = i_t - \pi_{t+1}$$

Investment

Why is it important to understand what determines investments?

- In the long run: Investments affect the capital stock and thereby the level of income
- In the short run: Investments affect demand and investments are very volatile

Fig. 3.2 *Growth rates for private investment and GDP*



Important Questions

What determines investments in the long run?

What determines investments in the short run?

Capital Accumulation



Investment in period t is I_t and depreciation of capital in the same period is δK_t

$$K_{t+1} - K_t = \underbrace{I_t - \delta K_t}_{\text{Net Investment}}$$

Desired Capital Stock

If we desire a capital stock of K_{t+1}^d to be used the next period we have to invest

$$I_t = \underbrace{K_{t+1}^d - K_t}_{\text{New Investment}} + \underbrace{\delta K_t}_{\text{Replacement investment}}$$

...but what factors determine K_{t+1}^d ?

Desired Capital Stock Con't

Assumptions:

- The same basket of goods is used for consumption and investment
- Capital can be bought at the price of P_t
- The company finances its investments by borrowing money from households at interest i_t
- Capital bought during period t can be used for production in period $t + 1$

Desired Capital Stock Con't

What is the effect on profit period $t + 1$ of an increase in K_{t+1} by one unit?

Period t : The company borrows and invests. No effect on profit.

Period $t + 1$:

$$MPK_{t+1} \times MR_{t+1} - (1 + i_t)P_t + (1 - \delta)P_{t+1}$$

The company should invest until the profit increase from an additional unit of capital equals zero

Symmetry Again

Consider again a symmetric equilibrium where all companies set the same price:

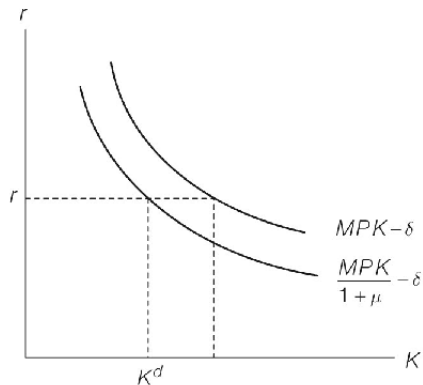
$$\begin{aligned}MPK_{t+1} \times \frac{P_{t+1}}{1 + \mu} - (1 + i_t)P_t + (1 - \delta)P_{t+1} &= 0 \\ \frac{MPK_{t+1}}{1 + \mu} - (1 + i_t)\frac{P_t}{P_{t+1}} + (1 - \delta) &= 0\end{aligned}$$

Since $1 + r_{t+1} = (1 + i_t)/(P_{t+1}/P_t)$ we have

$$\underbrace{\frac{MPK_{t+1}}{1 + \mu} - \delta}_{\text{Real net value of an extra unit of capital}} = \underbrace{r_{t+1}}_{\text{Real interest}}$$

Long run demand for capital

Fig. 3.3 *The long-run demand for capital when employment is on the natural level*



In the Cobb-Douglas case we can derive:

$$K^d = \left(\frac{\alpha}{(1 + \mu)(r + \delta)} \right)^{\frac{1}{1-\alpha}} EN^n$$

Demand for capital depends on:

- Real interest (-)
- Rate of depreciation (-)
- Technology (+)
- Size of mark-up (-)
- Employment in long run equilibrium (+)

How does the demand for capital in the long run differ from the demand for capital in the short run?

In the short run, demand, production and employment can be higher or lower than their equilibrium levels

We want to know how demand for capital depends on aggregated demand

Short run demand for capital

Cobb-Douglas: Demand for capital is proportional to expected production/demand

$$K^d = \frac{\alpha}{(1 + \mu)(r + \delta)}$$

where Y^e is the expected aggregated demand in the next period

This gives us an investment function

$$I = K^d - K + \delta K = \frac{\alpha}{(1 + \mu)(r + \delta)} Y^e - K + \delta K$$

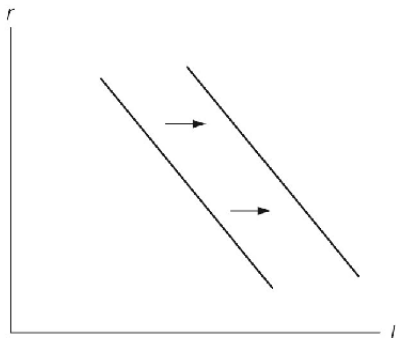
Thus we get an investment function

$$I = I(r, Y^e, K)$$

so that investment depends on three factors:

- Expected real interest rate, r (-)
- Expected future aggregate demand, Y^e (+)
- Size of the existing capital stock, K (-)

Fig. 3.4 *The effect on investment of an increase in aggregate demand*



To understand volatility in investment let us use our model

Let $\alpha = 1/3$, $\delta = 0.07$, $\mu = 0.10$ and $r = 0.05$

$$K^d = \frac{\alpha}{(1+\mu)(r+\delta)} Y^e \approx 2.5Y^e$$

So expectations matter!