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Abstract

We document distinctive empirical features of wage pass-through in Germany that are consistent with Thomas-Worrall wage contracting in the presence of both idiosyncratic and nonstationary aggregate productivity components. These empirical features are hard to reconcile with the predictions of search models based on period-by-period Nash bargaining over match surplus and with the predictions of financial models where risk neutral firms may costlessly shield risk averse workers from idiosyncratic shocks (Guiso, Pistaferri et al. 2005).

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Summary

We construct a simple model in which risk-neutral firms can hire risk-averse workers on a competitive market by offering long-term contingent contracts. Each firm can hire at most one worker, and is subject to an exogenous level of productivity which consists of an idiosyncratic and a common (aggregate) component. There is complete information. Either party can, after observing the current productivity level, quit the relationship. For a worker this involves joining a new firm, and receiving the current market (lifetime-) utility. The firm's outside option is zero profits. We study theoretically and in simulations the pass-through from firm productivity to wages. Wages increase only when the worker's participation constraint binds, in which case her outside option determines future lifetime utility within the firm. Symmetrically the wage falls only when the firm's participation constraint binds, so that future expected profits are zero. Idiosyncratic shocks to a firm's productivity only affect wages when they are negative (i.e., smaller than in the previous period holding the aggregate level constant) because an idiosyncratic positive movement in productivity does not tighten the worker's participation constraint. On the other hand a positive movement in aggregate productivity may increase wages by improving the worker's outside option, while a negative one may reduce wages if the firm's participation constraint binds. Finally the overall effect of aggregate productivity movements on wages within firms is expected to be muted rather than one for one. Using matched employer-employee data from Germany and proxying for productivity with estimated firm value-added data, we estimate pass-through of asymmetric idiosyncratic and aggregate changes in these proxies to wages. We get results broadly in line with the theoretical and simulation predictions. The empirical features we find are at odds with the predictions of a model based on period-by-period Nash bargaining over match surplus suggesting that long-term implicit contracts may play an important role, although it should be emphasised that they do not directly speak to the issue of whether the labour market is competitive or whether contracts are the outcome of ex-ante bargaining. They are also inconsistent with the predictions of financial models where firms may costlessly shield risk averse workers from diversifiable (idiosyncratic) shocks (see Guiso, Pistaferri and Schivardi, 2005).

1 Introduction and Overview

How does a firm's performance impact the wages of its workers? Inability to commit to contracts creates a tension between a firm's ability to insure risk-averse workers and the desire to preserve viable matches. This leads to interesting features of wage setting which have been taken to the data most notably by Lagakos & Ordonez (2011). Other work has looked at pass-through to wages from the point of view of rent sharing, for example Lemieux et al. (2009) and Card et al. (2018). Recent work has included search frictions with on the job search, worker effort and stochastic heterogenous match and worker productivity (see for example Balke & Lamadon (2020)).

In this paper we extend the literature to consider separately the impact on wages of non stationary *aggregate* and stationary *idiosyncratic* productivity. We analyse a parsimonious (three economic parameters) model – an extension of Thomas & Worrall (1988) – that has highly distinctive predictions; pass through of a firm's idiosyncratic productivity to wages will be sign asymmetric and aggregate pass through within jobs will be relatively small. In an empirical application we show that these features are strongly present in German data. As with much of the recent literature our results are at odds with the predictions of macro search models with period- by-period Nash bargaining and with the predictions of financial models where risk neutral firms may costlessly shield risk-averse workers from idiosyncratic shocks (Guiso et al. (2005)).

The outline of the paper is as follows. The next section describes our data, shows how we obtain measures of aggregate and idiosyncratic firm productivity and then presents regression results detailing pass-through of these components to wages. Section 3 establishes that our empirical findings are consistent with a general equilibrium model of limited commitment wage contracting. The section develops the model and its main results. We offer a partial characterisation of equilibrium wage dynamics with further properties established via numerical simulations of the model. Under a variety of realistic parameter scenarios the model readily mirrors the stylised facts of pass-through we find in the German data. Section 4 concludes.

2 Estimating Productivity Pass-through to Wages

2.1 Overview

Using administrative data from Germany we analyse the pass through to wages of aggregate and idiosyncratic productivity allowing the latter to have sign asymmetric effects. We find that the pass through of idiosyncratic firm productivity is indeed asymmetric; positive changes have a small and insignificant effect whilst negative changes have larger effects that are highly significant. We also find that aggregate pass-through is substantially below unity for our sample of stayers[1]. In an additional exercise we show that persistent components of productivity impact wages more than transient ones. Finally we relate these findings to the existing empirical literature.

Our focus on the above data moments is motivated by a limited commitment model which allows for variations in both stationary idiosyncratic and nonstationary aggregate job productivity. Firms (in the theory "firms" are separate constant returns to scale, henceforth "CRS", jobs) set wages which in equilibrium are characterised by upper and lower bounds. These bounds are determined by the current productivity state within the firm (which impacts the firm's participation constraint) and within the economy (which also impinges on the worker's participation constraint). However wages themselves will be determined not only by the bounds but also by the level of the lagged wage in relation to the bounds; if the lagged wage lies above the current upper bound then wages must fall but if it lies below then wages will not change. This makes each firm's wage dependent on its own (idiosyncratic) history. These considerations suggest that responses to changes in firm productivity will be sign-asymmetric.[2] The intuition – established formally below – is as follows. If the firm's idiosyncratic productivity falls, the upper bound will fall as the job has become less viable. If the fall is severe enough the bound will fall sufficiently far to exclude the pre-existing wage and the wage needs to fall for the relationship to survive. By contrast upticks in idiosyncratic productivity – which will not affect a worker's outside option – will be absorbed by the firm and not passed through to wages. Hence asymmetric pass-through may be expected here. Crucially, the asymmetry arises because it is the history of productivity states that matters for wages. In other models where the wage

[1] We do not estimate pass through for job switchers or those transitioning from unemployment/non-employment to employment given the substantial sample selection issues in relation to worker quality.

[2] By contrast, the theory does not predict asymmetric responses from movements in aggregate productivity. We do not test this empirically however because to do so would require more than the 24 annual data points we have in our data.

depends only on the current state such as those based on continuous Nash bargaining over surplus, this asymmetry would not arise.

2.2 Empirical Model

Following the discussion above we wish to examine separate pass-through effects to wages of idiosyncratic and aggregate productivity respectively, allowing for asymmetric effects in the former.[3] To begin with we need a model of firm productivity which under the assumption of CRS[4] we take to be output per worker. We adopt the following structure,

$$\Delta \mathbf{a}_{jt} = \Delta \mathbf{y}_t + \Delta \mathbf{r}_{jt}$$

where $\mathbf{a}_{jt}, \mathbf{y}_t$ and $\mathbf{r}_{jt}$ are the log of firm j's total, aggregate and idiosyncratic productivity in year t respectively and Δ is the first difference operator. This is a simple short-run decomposition that takes capital as fixed. It is motivated in part by the desire for tractability when we take it to the theory and in part by data limitations (we do not observe a firm's capital stock). Additionally however, it is easy to show that under CRS and fully adjustable capital then output per worker measures labour augmenting productivity. One natural estimate of $\Delta \mathbf{y}_t$ in this simple set up might be the within year cross firm unweighted average of $\Delta \mathbf{a}_{jt}$. However the firms in our sample account for a small proportion of those in the economy as a whole. Furthermore the uncensored data is unrepresentative and heavily skewed towards manufacturing. We therefore use (the change in the log of) GDP per worker employed as an estimate of $\Delta \mathbf{y}_t$.

The previous discussions suggest we should investigate separately the effects of changes in aggregate and idiosyncratic productivity respectively, and for the latter distinguish positive and negative changes. We therefore estimate the following regression equation

$$\Delta \mathbf{w}_{ijt} = \alpha + \delta \Delta \mathbf{y}_t + \gamma^{+} \Delta \mathbf{r}^{+}_{jt} + \gamma^{-} \Delta \mathbf{r}^{-}_{jt} + \mathit{controls} + \mathit{error} \tag{1}$$

where $\Delta \mathbf{r}^{+}_{jt}$ ($\Delta \mathbf{r}^{-}_{jt}$) is equal to the first-differenced idiosyncratic productivity when it is positive (negative) and is equal to zero otherwise. The controls are quartics in worker-firm tenure and age (proxies for firm specific and non firm specific human capital respectively).

[3] In the theory a firm is a single job but of course in reality firms play host to a number of jobs. Extending the theory to allow groups of workers with identical productivity is easily achieved and does not change the characterisation of equilibrium wages.

[4] CRS is not an innocuous assumption of course but there is a substantial body of previous work (e.g., Basu & Fernald (1997); Syverson (2004*a*); Syverson (2004*b*)) which shows that it offers a good medium run approximation for production conditions in many plants particularly in the manufacturing sector.

We do not treat (1) as a causal relationship and nor do we try and explicitly map its parameters into those that underpin the primitives of the theory (or vice versa). Instead we treat the estimates as data moments that will form stylised facts about the pass-through of idiosyncratic and aggregate productivity components to wages. We then examine the extent to which a limited-commitment model of wage contracting may account for these stylised facts by comparing these data moments with their counterparts obtained via model simulation.

Finally, in order to examine the potentially differential impact of persistent productivity changes on wages (Guiso et al. (2005)) we re-estimate (1) using changes taken over three years rather than one; taking longer differences averages out temporary components and increases the contribution of persistent components to the variance of the productivity measures in (1). Henceforth and purely for reasons of brevity we refer to our productivity measures merely as “productivity”.

2.3 The Data

We form a matched panel dataset of workers and German establishments by merging information from the LIAB firm survey and BeH. The former is a survey of establishments for the years 1993 to 2018 that contains our productivity measure for Germany: value added (output) per employed worker. The data contains information on establishments' workers and in particular their wages, tenure, gender, experience, age and occupation. The BeH is a well used administrative worker-establishment dataset so we offer only outline detail here (for more information on matters such as top coding, etc., see for example Snell et al. (2018)).

The BeH is organised by spells of continuous work at an establishment. We collate these spells for each year of our basic sample to obtain an estimated hourly wage of each full time worker in each of the surveyed establishments. Whilst actual hours worked are not documented, there is evidence that the variation in weekly hours of full time workers in Germany is fairly minimal (see for example Snell et al. (2018)). Worker tenure, measured in days, is obtained by adding the number of days worked ignoring periods of absence because of, e.g., maternity or sickness.

Whilst we have data on establishments, decisions on wages are almost certainly made at the firm level and this raises some interesting issues. We might expect that a firm would implement the same CRS technology across all of its establishments thus equalising their costs. If so each firm would merely be a scaled up version of each of its establishments. An additional interesting issue worthy of empirical examination is the extent to which multi-establishment firms or large firms are able to diversify movements in idiosyncratic productivity across workers (or rather jobs) better than single establishment or small ones can and therefore more able to offer their workers more insurance. Interestingly we find that the standard deviation of (changes in the log of) our value added per worker measure is as high in large (high employment) establishments as in small. This supports the idea that productivity shocks occur predominantly at establishment rather than worker level. Whilst in our theory we assume that productivity shocks occur at the worker level, all results go through for multi-worker establishments provided that all workers receive the same productivity realisations. In the light of this discussion we refer to establishments as "firms".

Turning to the LIAB, we use its survey data to estimate each firm's value added for the year, deflate by a CPI deflator and divide by the number of full-time (equivalent) workers in that year. Whilst the survey documents the amount of intermediate inputs used by the firm it does not offer data on inventory changes. There is also a concern that intermediates themselves are poorly estimated by the survey's responder. We take up the issue of measurement error below in section 2.5. There we use an IV method which under certain assumptions would give approximately consistent estimates in some of the sectors we look at. We then use these estimates to calibrate the likely size of the measurement error variance.

In terms of reliability of the LIAB its documentation claims that once a firm is selected, that firm is rigorously pursued each year to answer each question. Despite this the data on value added is heavily censored with larger firms being more responsive than smaller ones. This raises a concern in the regression context that unobservables relevant to the determination of wages may drive the probability of censorship and cause "bias"[5]. However below we adopt a first differenced specification and regress the wage growth of stayers on productivity growth. It is somewhat comforting then that if the relevant unobservables driving censorship are time invariant (and hence vanish under differencing) then they will cause no issues. In terms of representativeness the tendency for larger firms to respond more regularly to the survey questions suggests that if there is pass-through heterogeneity across firms then our estimates will be more representative of large than small firms.

[5]As we have already noted, we do not attempt to identify any deep parameters but merely try and estimate data moments. "Bias" here and henceforth then refers to a deviation from the moments that would obtain from data free of measurement error.

Finally we identify six separate and mutually exhaustive sectors:- 1) Mining, Agriculture, etc., 2) Manufacturing, 3) Utilities, 4) Construction, 5) Retail, and 6) Non retail services. This allows us to examine parameter heterogeneity and as we explain later the extent and impact of measurement error.

2.4 Measurement Error

Our proxy for firm value added is reported sales multiplied by one minus the reported proportion of intermediates used in production. As with any survey data, measurement errors will be present. A further issue is that we do not have data on the proportion of sales met by changes in the inventory of final goods. We can however make some headway to assess the likely biases in OLS estimates caused by these two problems if we make further assumptions.

First of all we assume that sales are reported without error (or at least with negligible error). Sales are directly reported in a firm's accounts and the manager/respondent is likely to both understand this quantity and know its value well. By contrast the proportion of intermediates used is not directly reported in the accounts. It has to be estimated by the respondent and is therefore likely to be reported with considerable error. We have no access to accounting data with which to calibrate measurement error in this item. To deal with this we follow a standard approach and assume that the errors are classical in nature – i.e., are uncorrelated with the true level of intermediate inputs (and other regressors/controls). If these assumptions hold then we can re-estimate (1) using sales to form instruments for the value added proxies.[6] Then, it is easy to show that in sectors/cases where final goods inventory changes are relatively unimportant - viz. utilities, construction, retail and non retail services - the IV estimates will be consistent whilst the OLS estimates will be biased towards zero.[7] The intuition for the consistency of IV is as follows (details are available on request). In sectors where inventory changes are negligible the use of sales as a proxy for value added leads to a measurement error equal to the intermediate inputs themselves. By contrast the measurement error in our original value added proxy (sales minus reported intermediates) is just the measurement error in the reported intermediates. These two proxies for value added have respective measurement

[6] Specifically we use Δs^{+} and Δs^{-} as instruments for Δr^{+}, Δr^{-} where $\Delta s^{+} = \mathbf{1}_{\Delta s>0}.\Delta s$, $\Delta s^{-} = \mathbf{1}_{\Delta s<0}.\Delta s$ and where Δs is the change in the log of sales minus Δy.

[7] Note that in sectors where final goods inventories do matter, the classical measurement error assumptions are untenable; measurement error here would include inventory changes and these are almost certainly correlated with changes in value added (although the sign of this relationship is unclear).

errors that – under the classical assumption – are uncorrelated. In these circumstances an IV estimator instrumenting one proxy with the other will lead to consistent parameter estimates (see for example the discussion on multiple measures in Bound et al. (2001)). In addition and again where we believe inventories to be unimportant we may compare the IV estimates to their OLS counterparts to get an estimate of the variance of the (change in the log of) “true” value added - a quantity that will be useful for our calibration exercise.

2.5 The Estimates

The results from estimating (1) by OLS for the complete sample are in Table 1 below.

OLS and IV Estimates of (1)									
	OLS(Δ^1)			**IV**(Δ^1)					
	δ	γ^+	γ^-	δ	γ^+	γ^-	$\gamma^-_{IV}/\gamma^-_{OLS}$	N_{fy}	N_w
All	.212	$-.002$	.008	.203	.003	.027	3.25	96,458	5,732,574
	(.041)	(.004)	(.004)	(.039)	(.006)	(.006)			
Sector 1	.035	$-.001$	.012	.042	.013	.032	2.75	3,801	104,417
	(.103)	(.005)	(.006)	(.100)	(.023)	(.010)			
Sector 2	.302	$-.003$	.007	.283	.003	.027	3.85	35,979	4,079,112
	(.065)	(.007)	(.007)	(.061)	(.010)	(.008)			
Sector 3	.022	.003	.031	$-.090$	.022	.052	1.68	1,291	148.055
	(.027)	(.006)	(.017)	(.070)	(.027)	(.055)			
Sector 4	$-.088$	.001	.009	$-.080$	.008	.022	2.45	11,826	253.404
	(.053)	(.003)	(.003)	(.050)	(.008)	(.007)			
Sector 5	.153	$-.002$	.006	.160	.003	.019	3.17	14,587	269,055
	(.071)	(.002)	(.002)	(.072)	(.003)	(.007)			
Sector 6	$-.051$	$-.002$	.005	$-.05$	$-.005$	.013	2.60	29,170	878,531
	(.045)	(.003)	(.002)	.050	(.005)	(.004)			
OLS and IV Estimates of (1) in 3rd Differences									
	OLS(Δ^3)			**IV**(Δ^3)					
	δ	γ^+	γ^-	δ	γ^+	γ^-			
All	.287	.003	.020	.300	.010	.053		42,236	
	(.042)	(.003)	(.008)	(.007)	(.012)	(.010)			

Table 1: Estimates of Pass-through

Notes: Standard errors - in brackets - are clustered by firm[8]. Δ^3 denotes a 3 year difference

N_{fy} is the number of firm-years and N_w is the number of wage observations.

[8]Given the presence of macro aggregates, clustering by year may have been preferable but having only 24 years militates against this. Nonetheless it is comforting to note that clustering by year makes very little difference to our standard errors and inferences.

Looking at the OLS estimates for the entire sample ("All") we see that aggregate pass through is significant but the effect is small and in particular well below unity. On the idiosyncratic side there is a significant effect of negative productivity movements but wholly insignificant effects of positive ones. Qualitatively the OLS results accord with the heuristic intuition above that anticipated the predictions of the theory. When we estimate separately for 6 sectors we see that the effects of idiosyncratic productivity are consistent across the economy; downward movements are significant whilst upward ones are quantitatively far smaller (often negative) and wholly insignificant. The effect of aggregate productivity is not consistent across the sectors however and is only significantly positive in two sectors. This may be a result of poor precision[9]; the only variation in the regressand is year to year so precision requires a large number of within year data points to average out non macro effects in wages such as idiosyncratic movements in human capital. However it is comforting to note that re-estimating by SUR we cannot reject the hypothesis that the pass through parameters are constant across sectors. One other thing that stands out from these estimates is their size; the coefficients on idiosyncratic productivity are quantitatively very small compared with those found in other studies (see for example the survey in Card et al. (2018)). This may well be a result of measurement error in our value added proxies. The standard deviation of the measured change in log idiosyncratic productivity is .53 and whilst firm level productivity does tend to be volatile this figure is considerably higher than found in other studies. This backs up our previous concerns that our productivity measure could be heavily contaminated by measurement error.

Following the arguments of section 2.4 we re-estimated (1) using sales to construct instruments for the value added proxies therein. The results are in columns 5 to 7 of Table 1. If the assumptions about measurement error made earlier are correct then the IV estimates of the $\gamma' s$ in sectors where inventories play no (or little) role - in particular utilities, construction, retail and non retail services (henceforth "no inventory" sectors) - will be consistent (or approximately consistent). We would also expect estimates of the $\gamma' s$ to be larger in magnitude than their OLS counterparts. We see that the latter is borne out in a striking way; IV estimates of the key γ^- parameters are two to three times their OLS counterparts. The γ^+'s are also somewhat larger but remain wholly insignificant. By contrast the parameters on aggregate productivity are virtually unchanged and display the same characteristics as their OLS counterparts.

[9]Although it appears to be significantly perverse in two other sectors this significance disappears when we switch to clustering by year - something we discussed above. These are the only two cases where switching the clustering to years changes the result of a coefficient's significance.

Turning to the results on three year first differences, here we see much larger pass through estimates of all coefficients. This is another anticipated result of our theory. Intuitively, persistent shocks are "larger" than transient ones because they will reoccur with a high chance. Consequently they would be expected to have a greater impact.

We can exploit the consistency of IV in the "no inventory" sectors to get a better idea of the standard deviation of "true" idiosyncratic productivity (henceforth "s"). The results from this exercise are indicative so we outline them here relegating a full exposition to the annex. For each "no inventory" sector we use the IV estimates, a guessed value of s and a simplified version of (1) to generate synthetic data on wages and productivity. We then re-estimate (1) using OLS and iterate over different values of s until we find the value that generates the OLS estimates found in the data. The estimated s values were .34, .39, .35 and .37, respectively. Although these results are not definitive they strongly suggest that for the calibration exercise below a value of s of around .35 is more appropriate than the .53 reported above.[10] We summarise by saying that there is strong evidence in German data for significant pass-through from negative idiosyncratic productivity changes to wages but little to no evidence of pass-through to wages of positive ones. Aggregate pass-through to wages (for stayers) is substantially and significantly below unity. Persistent components are passed through with larger coefficients in both countries. In section 3 below we assess the ability of Thomas-Worrall contracting to reproduce the empirical features we have identified here.

2.6 Relationship to the Empirical Literature

As noted already there is now a huge body of empirical work examining the extent and nature of pass through from a firm's performance to its wages. Here we focus only on those papers that examine asymmetric pass-through. (See Card et al. (2018) for an excellent summary and overview of the broader literature). There are two recent (and concurrently written) papers in this vein. Using Danish data Chan et al. (2020) extract measures of idiosyncratic firm productivity from a dynamic production function using a nonparametric approach. They find the pass-through to wages of negative idiosyncratic TFP shocks is larger than for positive ones but only if a Heckman correction for selection

[10] In the simple bivariate "y=βx+u" case the bias is just -β times the ratio of the variance of measurement error to the variance of the measure. Using this as a rule of thumb to back out estimates of σ gives similar numbers to those obtained here via numerical simulation.

is employed.[11] Rather than offering a simple macroeconomic explanation of the canonical features of the data as we do their focus is on explaining the heterogeneity in pass through across firm types. In particular they model firms that are heterogeneous with respect to market power, size, pecuniary benefits to workers and productivity levels. On the worker side there are worker specific shocks to the value of non employment. A paper whose theoretical approach is more related to ours - a theory based on recursive labour contracts - is that of Azzalini (2023). Using Swedish data from 2004-18 he finds asymmetric pass through from idiosyncratic value added per worker to wages of the kind we document here. However a key difference is that he finds this asymmetry only exists in the years of the Great Recession. He develops a model of directed search with recursive contracts to explain this phenomena. Whilst this is an interesting finding, it is unclear how much of it is down to the focus on one, possibly very special, economic episode - the Great Recession. Whilst our data also includes the Great Recession it spans many more years and our analysis does not revolve around the Great Recession.

Other papers have examined the sign of asymmetric pass-through. For example, Juhn et al. (2018) find some degree of asymmetry. They find that negative changes in firm performance have a marginally greater impact on wages than positive. However their results are hard to compare directly with ours because they use firm revenue rather than value added (output) and do not distinguish between aggregate and firm specific shocks.

3 A Non-Stationary Model of Wage Contracting

In this section we outline a version of firm-employee wage contracting without commitment. We then show that the empirical stylised facts presented above are intrinsic features of such contracting.

Time is discrete, $t = 0, 1, 2, \dots$. There is a fixed large number of infinitely-lived identical workers. There is free entry of firms at each date, each of which can employ at most one worker. Productivity at a firm j at time t is

$$a_{jt} = \hat{y}_t y_t r_{jt}$$

[11] We do not interpret our estimates as causal parameters as these authors do. Their need to control for selection effects to deliver the asymmetry may be down to the high labour mobility that exists in Denmark; average firm tenure is just over one half of what it is in Germany.

where $\hat{y}_t$ and y_t are aggregate shocks and r_{jt} is an idiosyncratic shock to firm j. $\hat{y}_t \in \widehat{\mathcal{Y}}$ is assumed to follow a geometric random walk, $\hat{y}_t = \xi_t \hat{y}_{t-1}$ where $\xi_t \in \{\xi_1, \ldots, \xi_{\hat{Y}}\} =: \Xi$ with distribution $\Phi_{\hat{Y}}$; both $y_t \in \mathcal{Y}$ and each $r_{jt} \in \mathcal{R}$ follow independent Markov chains where $\mathcal{Y}$, $\mathcal{R} \subset \mathbb{R}_{++}$ are finite sets, with $\mathcal{S} = \widehat{\mathcal{Y}} \times \mathcal{Y} \times \mathcal{R}$ the state space.

Firms are assumed to be risk-neutral and workers risk-averse with per-period utility given by $u(w)$, where w is the wage received (they can neither borrow nor save); $u(\cdot)$ is assumed to be differentiable and strictly concave. Both workers and firms discount future payoffs with discount factor β, $0 < \beta < 1$.

Assume an exogenous separation rate of $1 - \sigma$, $0 < \sigma < 1$, whereupon the firm exits, and the worker starts a new match.[12]

The time t shocks are observable at the beginning of the period. We assume that there is an outside option, available to any worker at t whose value $\chi(\hat{y}, y)$ depends only on the aggregate state at t, and after observing the current state a worker can leave and take the outside option and the firm can costlessly exit.

Consider a bilateral match formed at time t, with aggregate shock $(\hat{y}_t, y_t)$ and initial idiosyncratic shock r_{jt} known, so that the current state relevant to the match is $s_t \equiv (\hat{y}_t, y_t, r_{jt})$. Firm j and the worker agree on a wage contract $(w_\tau(h_\tau))_{\tau=t}^{\infty}$, $w_\tau(h_\tau) \geq 0$, where $h_\tau \equiv (s_t, s_{t+1}, \ldots, s_\tau)$.[13] The value of the contract to the worker at each date $\tau \geq t$, after observing the current state, is

$$U_\tau(h_\tau) = E\left[\sum_{t'=\tau}^{\infty} \beta^{t'-\tau}\sigma^{t'-\tau} u\left(w_{t'}\left(h_{t'}\right)\right) + \sum_{t'=\tau+1}^{\infty} \beta^{t'-\tau}\sigma^{t'-\tau-1}(1-\sigma)\chi\left(\hat{y}_{t'}, y_{t'}\right) \mid h_\tau\right],$$

where the second summation captures the assumption that after a separation the worker gets the outside option value. The corresponding firm value is

$$V_\tau(h_\tau) = E\left[\sum_{t'=\tau}^{\infty} \beta^{t'-\tau}\sigma^{t'-\tau}\left(a_{t'}\left(h_{t'}\right) - w_{t'}\left(h_{t'}\right)\right) \mid h_\tau\right],$$

given that after separation the firm ceases to exist. We assume that a constrained efficient contract is negotiated at t to solve:

$$f_{s_t}(U) := \max_{(w_\tau(h_\tau)\geq 0)_{\tau=t}^{\infty}} \{V_t(h_t)\} \text{ s.t. } U_t(h_t) \geq U \qquad \text{(Problem A)}$$

[12]We allow the separation rate at t to depend on s_{t-1}, in which case σ raised to a power is replaced by a product $\sigma(s_{t-1})\sigma(s_t)\ldots$, etc., below. All propositions are unaffected.

[13]This is w.l.o.g. Conditioning on the entire history from $t = 1$ would lead to the same contract provided the equilibrium is Markovian as defined below.

(where the determination of U is discussed below), and for all h_τ, $\tau > t$,

$$U_\tau(h_\tau) \geq \chi\left(y_\tau\right), \tag{2}$$

and

$$V_\tau(h_\tau) \geq 0. \tag{3}$$

The constraints (2) and (3) are the limited commitment constraints reflecting the assumption that either party can quit the relationship at any time. Note that the constraint applies *ex post* so the worker can quit after the current (date-τ) state is realized and get her outside option $\chi\left(\hat{y}_t, y_\tau\right)$, and likewise the firm can shut down immediately.

A contract is *feasible* if it satisfies (2), (3) for all histories h_τ, $\tau \geq t$.

The following characterises the evolution of wages in response to productivity changes affecting a firm.[14]

Proposition 1 *(Thomas & Worrall (1988)) For any history h_τ, the wage of an efficient contract starting at date t, $w_\tau \equiv w\left(h_\tau\right)$, is contained in a closed non-empty interval $[\underline{w}_{s_\tau}, \overline{w}_{s_\tau}]$. Moreover, $w\left(h_\tau\right)$, $\tau > t$, satisfies*

$$w_\tau = \begin{cases} \underline{w}_{s_\tau} \text{ and } U_\tau\left(h_\tau\right) = \chi\left(\hat{y}_\tau, y_\tau\right) & \text{if } w_{\tau-1} < \underline{w}_{s_\tau} \\ w_{\tau-1} & \text{if } w_{\tau-1} \in \left[\underline{w}_{s_\tau}, \overline{w}_{s_\tau}\right] \\ \overline{w}_{s_\tau} \text{ and } V_\tau\left(h_\tau\right) = 0 & \text{if } w_{\tau-1} > \overline{w}_{s_\tau} \end{cases}.$$

The following is our main theoretical characterisation. It states that higher y (hence higher revenue *and* higher outside options) is associated with increases in *both* wage-interval end-points. This implies that positive aggregate shocks may *ceteris paribus* lead to rising wages as workers' outside options are better. Correspondingly, negative shocks lead to wages being cut if a firm is against or close to its profit constraint. However the effect of higher r_j *ceteris paribus* is that only the upper end-point is affected: the top of the interval expands upwards as the profit constraint is relaxed. Hence if $(\hat{y}, y)$ does not vary, higher r_j does not lead to wage increases as the wage can be kept constant, and it translates into increased profits. Crucially, the effect is asymmetric: lower r_j will lead to wage falls for firms close to their profit constraints.[15]

[14] See an online appendix for proofs not in the main text.

[15] The i.i.d. assumption is needed to ensure future constraints are unaffected. However in our simulations similar effects arise even with very persistent shocks.

Proposition 2 *Assume that $\chi(\hat{y}, y)$ is increasing in $\hat{y}$ and y. Consider two states s, s', with $s = (\hat{y}, y, r_j)$ and $s' = (\hat{y}, y', r_j')$. (i) First assume that they differ only in y, with $y' > y$, and that $\{y_t\}$ is i.i.d. Then $\underline{w}_s < \underline{w}_{s'}$, and $\overline{w}_s < \overline{w}_{s'}$. (ii) Likewise, if they differ only in r_j, with $r_j' > r_j$, and $\{r_{jt}\}$ is i.i.d., then $\underline{w}_s = \underline{w}_{s'}$, and $\overline{w}_s < \overline{w}_{s'}$. (iii) If both $\{y_t\}$ and $\{r_{jt}\}$ are i.i.d., then, holding $\hat{y}$ fixed, $\overline{w}_s$ is increasing in yr_j, and $\underline{w}_s$ is increasing in y.*

Proof. We write χ_s for the outside option in state s, i.e., $\chi(\hat{y}, y)$. f_s is strictly decreasing, strictly concave and differentiable by standard arguments. Using standard arguments,

$$f_s(U) := \max_{w \geq 0, (U_q)_{q \in S}} \left(a_s - w + \beta\sigma E\left[f_q(U_q) \mid s\right]\right) \qquad \text{(Problem B)}$$

subject to

$$u(w) + \beta\left\{E\left[\sigma U_q + (1-\sigma)\chi_q \mid s\right]\right\} \geq U \qquad : \lambda \tag{4}$$

$$U_q \geq \chi_q \qquad : \beta\pi_{sq}\mu_q \tag{5}$$

$$f_q(U_q) \geq 0 \qquad : \beta\pi_{sq}\phi_q. \tag{6}$$

(i) We have

$$f_{s'}(U) = f_s(U) + r_j\hat{y}(y' - y), \tag{7}$$

since if $w, (U_q)_{q \in S}$ attains the maximum in Problem B, it also attains the maximum in state s' as the constraint set in Problem B is the same (the distribution of q conditional on s is unchanged given y_t is i.i.d. and $r_j = r_j'$). This holds for all U such that the constraint set is non-empty, so we can differentiate (7) w.r.t. U to get

$$f_{s'}'(U) = f_s'(U). \tag{8}$$

From (7):

$$f_{s'}\left(\overline{U}_s\right) = f_s\left(\overline{U}_s\right) + r_j\hat{y}(y' - y) > 0,$$

using $f_s\left(\overline{U}_s\right) = 0$, so that $f_{s'}\left(\overline{U}_s\right) > 0$ and thus $\overline{U}_{s'} > \overline{U}_s$ (by $f' < 0$ and $f_{s'}\left(\overline{U}_{s'}\right) = 0$). Consequently we have

$$f_{s'}'\left(\overline{U}_{s'}\right) < f_{s'}'\left(\overline{U}_s\right) = f_s'\left(\overline{U}_s\right),$$

by the strict concavity of f and by (8). Hence $\overline{w}_s < \overline{w}_{s'}$ from $f_q'\left(\overline{U}_q\right) = -1/u'\left(\overline{w}_q\right)$ (using the first-order conditions from the recursive problem). Similarly, by $\chi(\hat{y}, y) < \chi(\hat{y}, y')$, so $\chi_{s'} > \chi_s$,

$$f_{s'}'(\chi_{s'}) < f_{s'}'(\chi_s) = f_s'(\chi_s),$$

so that $\underline{w}_s < \underline{w}_{s'}$ from $f'_q(\chi_q) = -1/u'(\underline{w}_q)$. (ii) Following similar reasoning (7) holds in this case, given that again the distribution of q is unchanged, so

$$f_{s'}(U) = f_s(U) + \hat{y}y\left(r'_j - r_j\right), \tag{9}$$

and we get $f'_{s'}\left(\overline{U}_{s'}\right) < f'_s\left(\overline{U}_s\right)$, so $\overline{w}_s < \overline{w}_{s'}$, but now $\chi_{s'} = \chi_s$,so $f'_{s'}(\chi_{s'}) = f'_s(\chi_s)$ and thus $\underline{w}_s = \underline{w}_{s'}$. (iii) (7) holds again, and following the reasoning above, $\overline{w}_s < \overline{w}_{s'}$ if $y'r'_j - yr_j > 0$ and $\underline{w}_s < \underline{w}_{s'}$ if (and only if) $\chi(y) < \chi(y')$. ■

With CRRA preferences it is straightforward to see that for each (y, r), wage intervals will be proportional to $\hat{y}$:

Proposition 3 *Write $f_s(U) = f(U; \hat{y}, y, r)$. Suppose that $u(w) = w^{1-\alpha}/(1-\alpha)$, $\alpha \neq 1$, and $\chi(\hat{y}, y) = \tilde{\chi}(y)\,\hat{y}^{(1-\alpha)}$ for some increasing function $\tilde{\chi}(y)$. Then $f(U; \hat{y}, y, r) = \hat{y}f\left(\hat{y}^{-(1-\alpha)}U; 1, y, r\right)$, and $\overline{w}_{(\hat{y},y,r)} = \hat{y}\overline{w}_{(1,y,r)}$, $\underline{w}_{(\hat{y},y,r)} = \hat{y}\underline{w}_{(1,y,r)}$.*

This implies that increases in $\hat{y}$ have similar qualitative effects to changes in y in that both end-points of wage intervals increase/decrease with increases/decreases in $\hat{y}$.

3.1 Endogenizing outside options

In our simulations we will analyse the CRRA case and look for an equilibrium that conforms with Proposition 3. To determine $\tilde{\chi}(y)$ we assume that there is free entry of firms, with initial idiosyncratic productivity fixed at $r^* \in \mathcal{R}$ for all entrants, so the initial state for a firm entering at time t is $s_t = (\hat{y}_t, y_t, r^*)$. Because of competition between new entrants, we assume that a separated worker can, immediately,[16] extract full surplus from an entrant firm, receiving a utility $\overline{U}_{s_t}$.[17]

[16] Our interest is with wages in ongoing matches so for simplicity we abstract from unemployment. If the outside option involved entering unemployment and searching for a new match, this would both reduce the value of outside options, and modify the impact of aggregate shocks on them. This is unlikely to change the qualitative predictions of our model which depend on wage movements in ongoing contracts, although initial values of wages within those contracts may be affected if surplus is more evenly divided, and more states may become viable. Rudanko (2009) shows that limited commitment contracts combined with (directed) search and aggregate shocks do little to amplify unemployment volatility, so that we would not expect the effect of the aggregate state on outside options to be magnified much.

[17] We will w.l.o.g. assume r^* is the worst idiosyncratic state: this guarantees that all states are viable in the sense that there is positive match surplus. If it is not, then any inferior state would not be viable and would lead to endogenous job destruction as a worker would be better off quitting and starting a new job; however such states would be subsumed into $1-\sigma$, and the above analysis applies mutatis mutandis.

A Markov equilibrium is a function $\tilde{\chi}(y)$ and for each y a contract $(w_\tau(h_\tau) \geq 0)_{\tau=1}^{\infty}$ where $h_1 = (1, y, r^*)$, such that this contract solves Problem A above where $s_t = h_1$, and where $\chi(\hat{y}, y) = \tilde{\chi}(y)\,\hat{y}^{(1-\alpha)}$. While wages within a match will in general be history dependent, in a Markov equilibrium in which χ depends only on $(\hat{y}_t, y_t)$ new entrant firms face the same future for any given $(\hat{y}_t, y_t)$ and so will agree a contract depending only on $(\hat{y}_t, y_t)$.

3.2 Simulations

In the simulations, time is treated as quarterly. This allows for the inclusion of low persistence shocks in the idiosyncratic process – shocks that only last a quarter – in addition to longer shocks which may last many years.

The Productivity Processes; We consider an eight-state model where y takes on two possible values; $y \in \{y^l, y^h\}$ and is persistent with $\Pr[y_t = y_{t-1}] = q^{agg}$, and r consists of two independent two-point shocks, one is iid $r^i \in \{r^l, r^h\}$ and the other is persistent $r^p \in \{r^l, r^h\}$ with $\Pr[r_t^p = r_{t-1}^P] = q^{id}$, and $r = r^i \times r^p$. We have constrained the jumps in the temporary and persistent processes to be equal for reasons of parsimony. The above productivity processes are determined by five parameters; the gap between the good and bad states ($\log(y^h) - \log(y^l)$ and $\log(r^h) - \log(r^l)$), the size of the movements in the aggregate stochastic trend ξ, the switch rate of the idiosyncratic process q^{id} and the switch rate of the stationary aggregate component. We set these five parameters to exactly match the following five annual moments of the German data; i) and ii) the standard deviation of the change in log GDP per worker and its first-order autocorrelation iii) and iv) the standard deviation of the change in the log of our idiosyncratic productivity measure and its first order autocorrelation[18], and v) the standard deviation of the change in HP filtered log real GDP per worker (the stationary component of GDP growth).

Economic Parameters: We assume that jobs can only be lost when firms are in the worst state i.e. $r = \underline{r}^i \times \underline{r}^p$. In this case we may tie down σ to a value that ensures average tenure equals that found in the German data. β was fixed at $0.97^{\frac{1}{4}}$ (we show the results were not sensitive to reasonable deviations from this). Workers taking the outside option $\chi(y)$ are assumed to instantly find a new job at a firm that has $\underline{r}^i = \underline{r}^p = r^l$. The importance of this frictionless assumption is checked with a robustness check that allows for a small utility cost of switching job. We assume utility is CRRA with coefficient α, set

[18] Following the arguments in section 2.5 we set σ to .35 to allow for measurement error.

to 0.5 (as was the case with β we show that our results are not sensitive to this choice).

Parameter	Value	Informative Moments	Model	Data
$\log(y^h) - \log(y^l)$	0.0346	Standard Deviation of Δy	0.0156	0.016
$\log(r^h) - \log(r^l)$	0.8323	Standard Deviation of Δr	0.35	0.35
q^{ag}	0.8	Autocorrelation of ΔGDP	-0.05	-0.05
q^{id}	0.98	Autocorrelation of Δr	-0.3009	-0.3
ξ	0.0189	Standard Deviation of Δ(GDP)	0.0213	0.0210
σ	0.82	Average Tenure of Worker (Years)	10.55	10.5

Table 2: Internally Calibrated Parameters

We simulate the model to generate "quarterly data" and then aggregate these to annual frequency. We then re-estimate (1) (without controls) with the said parameterisation, as well as a variety of robustness checks. The results are presented in Table 3.

Model	δ	γ^+	γ^-
Baseline ($\alpha = 0.5$)	0.264	4.31×10^{-4}	0.0118
$\alpha = 0.2$	0.264	4.30×10^{-4}	0.0118
$\alpha = 1.5$	0.263	4.13×10^{-4}	0.0117
Baseline with 1% annual discounting	0.255	4.13×10^{-4}	0.0112
Baseline with 5% annual discounting	0.271	4.86×10^{-4}	0.0123
Baseline with small mobility cost	0.186	-1.09×10^{-3}	0.0096
Baseline with 3-year differences	0.321	-2.34×10^{-3}	0.0197

Table 3: Regression Results from Model Simulation. Results rounded to three significant figures. In the Small Mobility Cost case, a fixed utility penalty is imposed on the worker upon switching jobs equivalent to approximately one week's worth of wages.

In general, the model matches the empirical results well; γ^+ is close to zero, γ^- lies somewhere between the IV and OLS estimates and δ is only slightly above the value estimated from the data. Like the data (both OLS and IV estimates), the model displays larger γ and δ coefficients when three-year differences are used instead of one. The proportional rise in γ^- is about the same as in the data but for δ the rise is only one half of that found in the data. Although much smaller than δ, γ^- is quantitatively as important in terms of its contribution to wage variance. This is not true of γ^+ whose contribution to wage volatility is negligible. The inclusion of a mobility cost creates a space between the initial wage upon starting a job, and the lower bound which would make the worker indifferent between staying in the job and quitting. This reduces the size of δ and γ^{-}, but the broader qualitative implications are the same.

4 Concluding comments

The paper studies the pass-through of firm productivity changes to wages. A simple and parsimonious limited commitment model of wage contracting is able to reproduce well the salient features of pass through of aggregate and firm-idiosyncratic productivity to wages that we find in German data.

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Appendices

A Computational Approach

While the model can be written recursively in terms of utility promised to the worker, and future (state-contingent) utilities promised, it is much simpler from a computational standpoint to treat the wage as a state variable. Since the structure of the policy function is already known, i.e., the wage stays the same over time or it coincides with one of the state-contingent lower or upper bounds. For this reason, it is useful to solve the following dynamic program, which reformulates the problem:

$$F^*(w,S) = y(S) - w + \beta\sigma_S \mathbb{E}_{g,S'|S}\Big(g\hat{F}(\frac{w}{g},S')\Big), \tag{10}$$

where :

$$\hat{F}(w,S) = \begin{cases} F^*(w,S), & \text{if} \quad F^*(w,S) \geq 0, \quad \text{and} \quad U^*(w,S) \geq \chi^*(S) - c \\ 0, & \text{if} \quad F^*(w,S) < 0, \quad \text{and} \quad U^*(w,S) \geq \chi^*(S) - c \\ F^{extraction}(S), & \text{if} \quad F^*(w,S) \geq 0, \quad \text{and} \quad U^*(w,S) < \chi^*(S) - c \\ 0, & \text{if} \quad F^*(w,S) < 0, \quad \text{and} \quad U^*(w,S) < \chi^*(S) - c \end{cases}$$

A mobility cost, c, is included for completeness, although this is shut down in the baseline. Here, $F^*(w,S)$ denotes the firm's expected discounted profits conditional on paying wage w and being in state S. $y(S)-w$ is therefore flow profit. The continuation value must be discounted by β, weighed by the probability that the match survives, σ_S. g refers to the growth in the trend process between the current and next period, which can take on values $e^{\frac{1}{2}\xi}$ and $e^{-\frac{1}{2}\xi}$ with equal probability. Since growth is permanent, it effectively re-scales the firm value, hence the need to scale by g for comparability to the current value. Similarly, from the perspective of next period, the current wage will have to be normalised, hence the entry $\frac{w}{g}$. $\hat{F}()$ is convenient notation as it allows for various possibilities. If sticking with the same wage is such that neither worker nor firm wish to leave, then $\hat{F}$ is the same as F^*. However, if one, or both agents wish to leave by trying to keep the wage the same, then this is like hitting a wage bound. Either the worker wishes to leave, in which case the worker will be given the wage that makes them indifferent, and the firm will get the "extraction" value, $F^{extraction}$, or the firm wishes to leave which ensures that $\hat{F}$ will be set to zero. The extraction value is defined as the maximum amount of profit the firm is able to make if they had full market power over the wage i.e.

$$F^{extraction}(S) = \max_w \Big\{F^*(w,S)\Big\}, \quad \text{subject to} \quad U^*(w,S) \geq \chi^*(S) - c. \tag{11}$$

If, somehow both agents wish the match to end (which does not occur in the equilibrium but could occur during iterations of the value functions) then the firm value is allocated zero.

These make reference to the worker's utility under wage w and state S which must be defined:

$$U^*(w,S) = \frac{w^{1-\alpha}}{1-\alpha} + \beta \mathbb{E}_{g,S'|S}\Big(g^{1-\alpha}(\sigma_S \hat{U}(\frac{w}{g}, S') + (1-\sigma_S)(\chi^*(S') - c))\Big), \tag{12}$$

where :

$$\hat{U}(w,S) = \begin{cases} U^*(w,S), & \text{if} \quad F^*(w,S) \geq 0, \quad \text{and} \quad U^*(w,S) \geq \chi(S) - c \\ U^{extraction}(S), & \text{if} \quad F^*(w,S) < 0, \quad \text{and} \quad U^*(w,S) \geq \chi(S) - c \\ \chi(S) - c, & \text{if} \quad F^*(w,S) \geq 0, \quad \text{and} \quad U^*(w,S) < \chi(S) - c \\ \chi(S) - c, & \text{if} \quad F^*(w,S) < 0, \quad \text{and} \quad U^*(w,S) < \chi(S) - c. \end{cases}$$

The intuition for the worker's value function $U^*(w,S)$ is similar to that of the firm's. The worker gains flow utility from the wage, and they have value $\chi(S) - c$ if they lose the job. Notice that when accounting for growth, the utility is scaled by $g^{1-\alpha}$, not g, to account for the worker's utility function. $\hat{U}$ is then defined in a similar manner to $\hat{F}$; if both wish to stay under keeping the wage the same then $\hat{U} = U^*$, if the firm wants to leave but the worker does not then the worker gets the extraction value, and if the worker wants to leave then irrelevant of the firm's wishes, the worker gets their outside value $\chi(S) - c$. The worker's extraction value and outside options are defined as:

$$U^{extraction}(S) = \max_w \Big\{U^*(w,S)\Big\}, \quad \text{subject to} \quad F^*(w,S) \geq 0,$$

$$\chi(S) = \max_{w} \left\{ U^*(w, S^{id:low}) \right\}, \quad \text{subject to} \quad F^*(w, L(S)) \geq 0. \tag{13}$$

The worker extraction value is symmetric to that of the firms. The definition of $\chi(S)$ uses the rule that upon leaving a job, the worker matches to a low-idiosyncratic productivity firm (hence the crude notation $S^{low:id}$). So far, this constitutes a dynamic program which is straightforward to solve numerically. This is done by defining a logarithmically spaced wage grid set with a minimum of -0.2 and maximum of 1, with spacing 10^{-4} apart (meaning each gap is $\sim$ one hundredth of a percent). Over S there are 12 possible combinations. Value function iteration is repeated until mean root squared error over all the elements of U^* and F^* is less than 10^{-4}. When this solution is reached, now the wage bounds associated with each state can be found by solving:

$$w_{LB}(S) = \underset{w}{\text{argmin}}\{w\}, \quad \text{subject to} \quad U^*(w, S) \geq \chi^*(S) - c,$$
$$w_{UB}(S) = \underset{w}{\text{argmax}}\{w\}, \quad \text{subject to} \quad F^*(w, S) \geq 0.$$

With these bounds, the simulation can now be run. This is done for 120,000 periods. This then generates a quarterly time series which has the aggregate and idiosyncratic productivity, as well as the wage of the worker. To compare this to annual data, the data is bunched and aggregated over groups of four quarters, which then is used to do the regressions. An example of the evolution of the wage bounds and the wage is given in figures 1 and 2.

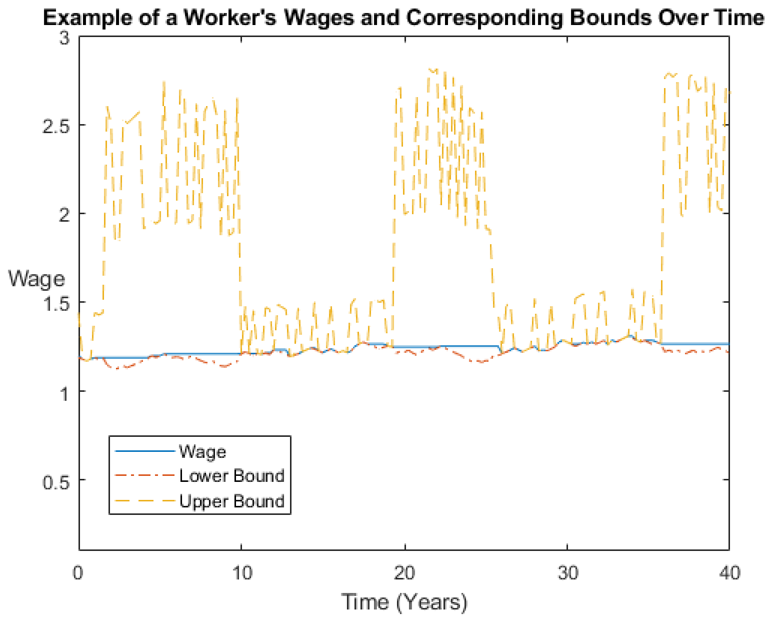

Figure 1: 40 year example of simulation.

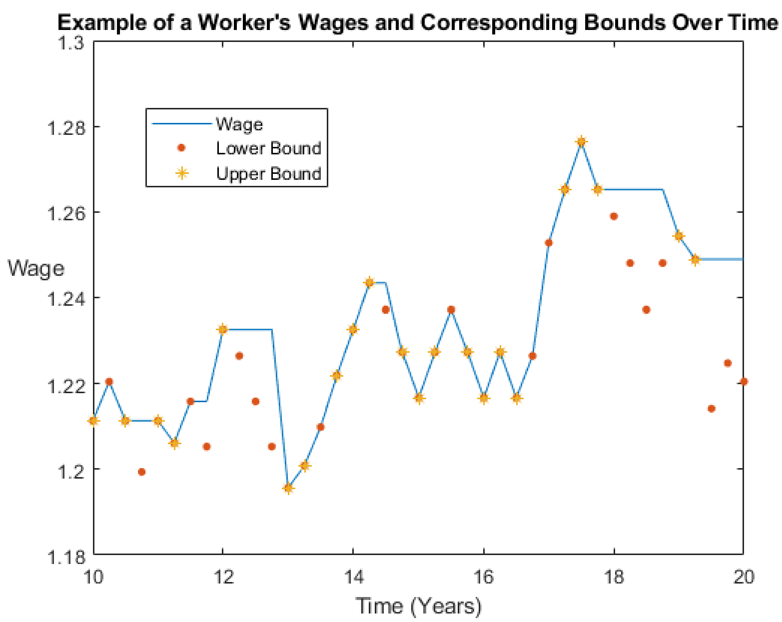

Figure 2: This shows the same series as A but zoomed in to the 10 - 20 year section and a rescaled y-axis. Notice sometimes the lower bound pushes the wage up and the upper bound (which often disappears off the top of the range) pushes the wage down.

B Proofs

Proof of Proposition 1.

Proof. Consider Problem B in the text. Note that $f_s(U)$ is potentially defined outside of the interval where $U \geq \chi_s$ and $f_s(U) \geq 0$ as the participation constraints (5) and (6) only apply in the future, but we define $\overline{U}_s$ by $f_s\left(\overline{U}_s\right) = 0$ to be the highest U the firm can offer in s.

First-order conditions are

$$-1 + \lambda u'(w) = 0, \tag{14}$$

and

$$\beta\sigma\pi_{sq}f_q'(U_q) + \lambda\beta\sigma\pi_{sq} + \beta\pi_{sq}\mu_q + \beta\pi_{sq}\phi_q f_q'(U_q) = 0$$

or rearranging

$$f_q'(U_q)\left(1+\phi_q\right) + \lambda + \mu_q = 0 \tag{15}$$

together with the envelope condition

$$f_s'(U) = -\lambda. \tag{16}$$

From (14), (15) and (16) (in states s and q),

$$1/u'(w_q) = \left(1/u'(w) + \mu_q\right)/\left(1+\phi_q\right),$$

where w_q is the wage next period in state q. It follows that if $w_q > w$ then $\mu_q > 0$, so that $U_q = \chi_q$, and w_q is at the solution to Problem B for $s = q$ and $U = \chi_q$, which we denote by $\underline{w}_q$, and from (14) and (16)

$$f_q'(\chi_q) = -1/u'\left(\underline{w}_q\right). \tag{17}$$

Also, $w_q < \underline{w}_q$ implies $f_q'(U_q) < f_q'(\chi_q)$ from (14), (16) and (17), and so $U_q < \chi_q$, violating (5). So $w_q \geq \underline{w}_q$. Hence if $w < \underline{w}_q$, then $w_q > w$ and we showed this implies $w_q = \underline{w}_q$.

Likewise if $w_q < w$, $U_q = \overline{U}_q$ and w_q is at the corresponding solution to Problem B, denoted by $\overline{w}_q$ where

$$f_q'\left(\overline{U}_q\right) = -1/u'\left(\overline{w}_q\right). \tag{18}$$

By a symmetric argument with the previous case, $w > \overline{w}_q$ implies $w_q = \overline{w}_q$.

If $w \in \left[\underline{w}_q, \overline{w}_q\right]$ then $w_q = w$ as otherwise, if $w_q > w$ then from above $\mu_q > 0$ so $w_q = \underline{w}_q$, a contradiction, and symmetrically if $w_q < w$. ■

Proof of Proposition 3

Proof. Consider first a solution $(\hat{w}_\tau(h_\tau))_{\tau=t}^{\infty}$ to Problem A with $s_t = (\hat{y} = 1, y, r)$. Now consider Problem A with $s_t = (\hat{y} \neq 1, y, r)$, and the contract $\tilde{w}_\tau(h_\tau) = \hat{y}\hat{w}_\tau(h'_\tau)$ where h'_τ is h_τ with each $\hat{y}_{t'}$, $t' \geq t$, replaced by $\hat{y}\hat{y}_{t'}$. It follows from the definition of a geometric random walk and the assumption $\chi(\hat{y}, y) = \tilde{\chi}(y)\,\hat{y}^{(1-\alpha)}$ that $\tilde{U}_\tau(h_\tau) = \hat{y}^{(1-\alpha)}\hat{U}_\tau(h_\tau)$ and $\tilde{V}_\tau(h_\tau) = y\hat{V}_\tau(h_\tau)$ (using obvious notation). Thus $(\tilde{w}_\tau(h_\tau))_{\tau \geq t}$ satisfies (5) and (6) and delivers values $\hat{y}^{(1-\alpha)}\hat{U}_\tau(h_\tau)$ and $\hat{y}\hat{V}_\tau(h_\tau)$. No other feasible contract, say $(w'_\tau(h_\tau))_{\tau=t}^{\infty}$, Pareto-dominates this with profits strictly higher; otherwise using the same logic there would be a contract $(\hat{y}^{-1}w'_\tau(h_\tau))_{\tau=t}^{\infty}$ in the original problem that dominated $(\hat{w}_\tau(h_\tau))_{\tau=t}^{\infty}$, a contradiction. Thus $f\left(\hat{y}^{(1-\alpha)}U; y, r\right) = \hat{y}f(U; 1, r)$. Next, differentiating this at $\overline{U}_{(\hat{y},y,r)} = \hat{y}^{(1-\alpha)}\overline{U}_{(1,y,r)}$ we get $f'\left(\overline{U}_{(\hat{y},y,r)}; \hat{y}, y, r\right) = \hat{y}^{\alpha} f'\left(\hat{y}^{-(1-\alpha)}\overline{U}_{(1,y,r)}; 1, y, r\right) = -1/u'\left(\overline{w}_{(\hat{y},y,r)}\right)$ (using (18)) $= -\overline{w}^{\alpha}_{(\hat{y},y,r)}$, so $\overline{w}_{(\hat{y},y,r)} = \hat{y}\overline{w}_{(1,y,r)}$. Likewise $\underline{w}_{(\hat{y},y,r)} = \hat{y}\underline{w}_{(1,y,r)}$. ■