

# ESTIMATING CHANGES IN MARGINAL UTILITY FROM DISAGGREGATED EXPENDITURES

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ABSTRACT. Understanding how marginal utilities of expenditure evolve over time is key to the specification, the estimation, and the testing of dynamic consumer behavior. But in existing dynamic models researchers often rely on an assumption that Engel curves are linear. This allows one to specify marginal utilities as a function of nothing more than real expenditures, but is sharply at odds with strong empirical evidence against linearity, including Engel's Law. Here we show how one can use panel data on disaggregate expenditures to estimate demand systems that may feature highly non-linear Engel curves; this same estimation procedure yields summary measures of household welfare within the period which we call "neediness".

Our neediness measure is of interest in its own right, and can also be used to construct measures of inequality and poverty which match conventional measures given prevailing prices, but which also describe how inequality and poverty would be different were prices different. Beyond this, it is intimately related to the marginal utilities of expenditure that are critical in dynamic models.

We illustrate the use of these methods using data from Uganda to estimate an incomplete demand system; to estimate household neediness in different periods; and finally to use our estimates of neediness to test for full insurance in Uganda. The most serious problem with our application in Uganda has to do with the large numbers of zeros observed for some expenditure items, but these problems can be mitigated simply by using judiciously selected expenditure items with few zeros.

## 1. INTRODUCTION

The models economists use to describe dynamic consumer behavior almost invariably boil down to a description of how consumers' marginal utilities evolve over time. A central example involves the

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canonical Euler equation, which describes how consumers smooth consumption over time when they have access to credit markets (Bewley, 1977; Hall, 1978) or some more general set of asset markets (Hansen and Singleton, 1982); this same Euler equation tells us how to price assets using the mechanics of the Consumption Capital-Asset Pricing Model (Lucas, Jr., 1978; Breeden, 1979).

So, taking dynamic models of consumer behavior to the data means measuring marginal utilities. But marginal utilities are not directly observable. The usual approach to measuring these indirectly involves constructing measures of consumers' total consumption expenditures, and then plugging these total expenditures into a parametric utility function, where marginal utilities may (Hansen and Singleton, 1982; Ogaki and Atkeson, 1997) or may not Hall (1978) also depend on unknown parameters which have to be estimated. A possible justification for this approach comes the Marshallian treatment of consumer demand: Provided that consumer's intertemporal preferences are additively time-separable, then Marshallian intratemporal demand systems are functions of (all) prices within a period and (all) expenditures within that same period; further, the consumer's indirect utility can be written as a function of the same two arguments. Thus modeling demand and welfare using the Marshallian apparatus then seems to call for *measuring* all prices and total expenditures.

There are problems with this approach, both in principle and in practice. In principle, the usual practice involves plugging total expenditures into a *direct*, rather than an indirect utility function. This is defensible only if Engel curves are all linear, and we know that they are not (Engel, 1857; Houthakker, 1957). In practice the exercise of collecting the necessary data on quantities and prices of all consumption goods and services is extremely difficult, and even well-financed efforts by well-trained, ingenious economists and statisticians have yielded less than satisfactory results.

Understanding the behavior of households in low income countries through the lens of economic theory involves thinking about consumer demand. Provided that consumer's intertemporal preferences are additively time-separable, then Marshallian intratemporal demand systems are functions of (all) prices within a period and (all) expenditures within that same period. But in practice the exercise of collecting the necessary data on quantities and prices of all consumption goods and services is extremely difficult, and even well-financed efforts by well-trained, ingenious economists and statisticians have yielded less than satisfactory results.

In this paper we describe an alternative approach. Rather than constructing Marshallian demand systems and indirect utilities which depend on prices and total expenditures, and then calculating marginal utilities from the partial derivatives of the indirect utility function with respect to expenditures, we construct Frischian demand systems which are functions of prices and of a quantity we call “neediness”—this is equal to the marginal utility of expenditures for a consumer with a particular normalization of within-period utilities. We are thus able to completely avoid the task of trying to measure total expenditures on all goods and services, instead measuring disaggregate expenditures on selected goods. Avoiding aggregation allows us to also avoid the most serious of the problems described above. The key to our approach is that it takes advantage of the variation in the composition of different consumers’ consumption bundles; this is variation ignored by the Marshallian approach of simply summing all expenditures. Our approach is also practical: we simply don’t need to use data on goods or services for which data or prices are suspect; we entirely avoid the difficulties of constructing comprehensive aggregate; and it’s simply unnecessary to construct price indices to recover “real” expenditures; nominal expenditures are all that we need.

This paper proceeds by first sketching a simple model of household demand behavior, and using this model to derive a set of “Frischian” demands, using a strategy that is quite close to that taken by Attfield and Browning (1985). But where their final demand system resembles a Frischian version of the Rotterdam demand system (which they take to time series data), ours more closely resembles a Frischian version of the AID system of Deaton and Muellbauer (1980). We take this demand system to household-level data, so as to exploit differences in the composition of households’ expenditure bundles across the wealth distribution.

We next use the differential Frischian demand system we derive to develop a demand system which can be estimated using household panel data, in a specification involving first differences in logarithms of expenditures. This estimator delivers estimates of changes in each household’s neediness, along with estimates of the value of a function of shadow prices which summarizes the influence of aggregate changes on demand, and estimates of the effects of various observable household characteristics on demand.

We subsequently discuss an attractive practical feature of our demand system: it’s very simple and natural to estimate a reduced incomplete demand system. In particular, if there are some goods which seem poorly measured or which are uninformative regarding household

marginal utility, we can simply ignore these. We give two more formal criteria for making decisions about what goods ought to be included in the system being estimated.

Finally, in an appendix we illustrate our methods using data from two rounds of surveys in Uganda. We're able to obtain workable estimates of both the parameters of the demand system and of changes in household marginal utility. We're also able to show something important: namely that our estimates of household neediness are quite robust to changes in the number and type of goods being included in the estimated demand system. The chief difficulty we encounter with the Ugandan data is that many observations feature zero expenditures for many goods. We conclude with some discussion of several specific food goods that seem well suited to on-going monitoring of marginal utility in Uganda.

## 2. A FRISCH APPROACH

In the rest of this paper we'll describe an approach to measuring neediness which uses Engel-style facts about the composition of differently-situated consumers' consumption bundles; which has comparatively modest data requirements; which allows us to simply *ignore* expenditures on goods and services which are too difficult or expensive to measure well; and which completely avoids the price index problem by simply avoiding the need to construct price indices. The approach imposes fewer restrictions on the demand system than is usual; avoids the usual sausage factory from which consumption aggregates are extruded; should allow for much less expensive data collection; and directly yields measures of both household marginal utility and functions of shadow prices which can be used in subsequent analysis and model testing.

What we're calling "neediness" has a very precise theoretical interpretation: it's the rate at which a particular normalization of household utility would increase if the household's expenditures received a small increase in a given period. Provided that the household has a concave momentary utility function (the usual assumption) then neediness will decrease as resources increase. Given the normalization of utility this same quantity goes by other names, but all of them are awkward: the "marginal utility of income" (inaccurate, since a change in income will generally affect utility in several different periods); the "Lagrange multiplier on the budget constraint" (mathematically accurate, but devoid of intuition regarding the consequences for the household); the "marginal utility of expenditures" (perhaps the best of a bad lot, and a term

that Browning (1986) abbreviates to *mue*—but this is confusing for us since we use the Greek letter  $\lambda$  for this quantity).<sup>1</sup>

The question of how what we’re calling neediness is related to consumer demand and welfare was extensively considered by Ragnar Frisch (see esp. Frisch, 1959, 1964, 1978), and demand systems which depend on prices and marginal utility were apparently given the moniker “Frisch demands” by Martin Browning (Browning et al., 1985). However, previous approaches to estimating Frisch demand systems have typically imposed much more structure on the underlying consumer Engel curves than is necessary for estimating neediness.

We propose to follow the “sequential approach” advocated by (Blundell, 1998) to estimating and testing dynamic models. We take disaggregate data from one or more rounds of a household expenditure survey to estimate a Frischian demand system (demands which depend on prices and marginal utilities). Estimating such a system allows us to more or less directly recover estimates of some demand elasticities and households’ neediness in each round, which can then be used as an input to a subsequent (possibly dynamic) analysis.<sup>2</sup>

There is, of course, a vast literature on different approaches to estimating demand systems, so it’s been surprising to discover that none of these approaches seems well-suited to our problem. The first issue is simply that almost all existing approaches are aimed at estimating Marshallian demands, rather than Frischian.<sup>3</sup> Related, demand systems which are nicely behaved (e.g., linear in parameters) in a Marshallian setting are typically ill-behaved in a Frischian. This includes essentially all of the standard demand systems based on a dual approach (e.g., the AID system). Other existing demand systems can be straight-forwardly adapted to estimating Frischian demand systems, such as the Linear Expenditure System (LES), which can be derived from the primal consumer’s problem when that consumer has e.g., Cobb-Douglas utility.

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<sup>1</sup>The problem of naming this quantity has a long history; Irving Fisher was already complaining about it in 1917. Fisher himself offers the coinage “wantab” (Fisher, 1927).

<sup>2</sup>It would, of course, also be possible to estimate the demand system and the dynamic model jointly (as in, e.g., Browning et al., 1985). But since we so often are able to reject the dynamic models we estimate, joint estimation seems likely to result in a mis-specified system; here, we prefer to not impose any dynamic restrictions on the expenditure data so as to allow ourselves to remain comfortably agnostic about what the ‘right’ dynamic model ought to be.

<sup>3</sup>Notable exceptions include Browning et al. (1985); Kim (1993) and Blundell (1998)

But such systems are too restrictive, imposing a linearity in demand which is sharply at odds with observed demand behavior.

### 3. MODEL OF HOUSEHOLD BEHAVIOR

In this section we give a simple description of a “neediness” function  $\lambda$ , which maps prices and expenditures into a welfare function (higher values of the function mean that the household is in greater need), and which also serves as the central object for making predictions regarding *future* welfare.

**3.1. The household’s one-period consumer problem.** To fix concepts, suppose that in a particular period  $t$  a household with some vector of characteristics  $z_t$  faces a vector of prices for goods  $p_t$  and has budgeted a quantity of the numeraire good  $x_t$  to spend on contemporaneous consumption, from which it derives utility via an increasing, concave, continuously differentiable utility function  $U$ . Within that period, the household uses this budget to purchase non-durable consumption goods and services  $c \in X \subseteq \mathbb{R}^n$ , solving the classic consumer’s problem

$$(1) \quad V(p_t, x_t) = \max_{\{c_i\}_{i=1}^n} U(c_1, \dots, c_n; z_t)$$

subject to a budget constraint

$$(2) \quad \sum_{i=1}^n p_{it} c_i \leq x_t.$$

The solution to this problem is characterized by a set of  $n$  first order conditions which take the form

$$(3) \quad U_i(c_1, \dots, c_n; z_t) = \lambda_t^* p_{it}$$

(where  $U_i$  denotes the  $i$ th partial derivative of the momentary utility function  $U$ ), along with the budget constraint (2), with which the Karush-Kuhn-Tucker multiplier  $\lambda_t^*$  is associated.

So long as  $U$  is strictly increasing the solution to this problem delivers a set of demand functions, the Marshallian indirect utility function  $V$ , and a Frischian measure of the marginal value of additional expenditures in period  $t$ ,  $\lambda_t^* = \lambda^*(p_t, x_t; z_t)$ .

It is this last object which is of central interest for our purposes. By the envelope theorem, the quantity  $\lambda_t^* = \partial V / \partial x_t$ ; it’s thus positive but decreasing in  $x_t$ , so that marginal utility decreases as the total value of per-period expenditures increase.

**3.2. The household's intertemporal problem.** Of course, we're interested in the welfare of households in a stochastic, dynamic environment. But it turns out to be simple to relate the solution of the static one-period consumer's problem to a multi-period stochastic problem; at the same time we introduce a simple form of (linear) production.

We assume the household has time-separable von Neumann-Morgenstern preferences, and that it discounts future utility using a discount factor  $\beta_t$  (allowed to vary across periods). As above, within a period  $t$ , a household is assumed to allocate funds for total expenditures in that period obtaining a total momentary utility described by the Marshallian indirect utility function  $V(p_t, x_t; z_t)$ , where  $p_t$  are time  $t$  prices,  $x_t$  are time  $t$  expenditures, and  $z_t$  are time  $t$  characteristics of the household.

The household brings a portfolio of assets with total value  $R_t b_t$  into the period, and realizes a stochastic income  $y_t$ . Given these, the household decides on investments  $b_{t+1}$  for the next period, leaving  $x_t$  for consumption expenditures during period  $t$ . More precisely, the household solves

$$\max_{\{b_{t+1+j}\}_{j=1}^{T-t}} E_t \sum_{j=0}^{T-t} \beta_j V(p_{t+j}, x_{t+j}; z_{t+j})$$

subject to the intertemporal budget constraints

$$x_{t+j} = R_{t+j} b_{t+j} + y_{t+j} - b_{t+1+j}$$

and taking the initial  $b_t$  as given.

The solution to the household's problem of allocating expenditures across time will satisfy the Euler equation

$$\frac{\partial V}{\partial x}(p_t, x_t; z_t) = \beta_j E_t R_{t+j} \frac{\partial V}{\partial x}(p_{t+j}, x_{t+j}; z_{t+j}).$$

But by definition, these partial derivatives of the indirect utility function are equal to the functions  $\lambda^*$  evaluated at the appropriate prices and expenditures, so that we have

$$(4) \quad \lambda^*(p_t, x_t; z_t) = \beta_j E_t R_{t+j} \lambda^*(p_{t+j}, x_{t+j}; z_{t+j}).$$

This expression tells us, in effect, that the household's marginal utility or marginal utility of expenditures  $\lambda_t^*$  satisfies a sort of martingale restriction, so that the current value of  $\lambda_t^*$  play a central role in predicting *future* values  $\lambda_{t+j}^*$ .

If we know the Frisch demand functions for a consumer with utility function  $U$  and observe prices and quantities demanded for some of these goods, then we can invert the demand relationship to obtain the consumer's  $\lambda_t^*$ . And notice that once we have these  $\{\lambda_t^*\}$  in hand

restrictions such as (4) are *linear* in these variables. This can simplify estimation, and perhaps also make dealing with measurement error a comparatively straight-forward procedure.

**3.3. Differentiable Demand.** We now turn our attention to the practical problem of specifying a Frischian demand relation that can be estimated using the kinds of data we have available on disaggregated expenditures. Attfield and Browning (1985) take a so-called “differentiable demand” approach to a related problem; their method yields Frischian (aggregate) demands without requiring separability. These demands will, in general, depend on all prices, yet one need only estimate demand equations for a select set of goods.

Our analysis here follows that of Attfield and Browning (1985) in outline, but where they arrive at a Rotterdam-like demand system in quantities, we obtain something importantly different in expenditures.

It’s easiest here to work with the consumer’s profit function,

$$\pi(p, r, z) = \max_c rU(c; z) - pc,$$

where  $r$  has the interpretation of being the “price” of utility. Let subscripts to the  $\pi$  function denote partial derivatives. Some immediate properties of importance: the price  $r$  is equal to the quantity  $1/\lambda^*$  from our earlier analysis; the profit function is linearly homogeneous in  $p$  and  $r$ ; by the envelope theorem  $\pi_i(p, r, z) = -c_i$  for all  $i = 1, \dots, n$  and for any  $z$ ; and (since we want to work with expenditures)  $-p_i\pi_i = x_i$ .

Using this last fact and taking the total derivative yields

$$dx_i = -\pi_i dp_i - p_i \sum_{j=1}^n \pi_{ij} dp_j - p_i \pi_{ir} dr - p_i \sum_{l=1}^{\ell} \pi_{iz_l} dz_l.$$

Now, since  $d \log x = dx/x$  for  $x > 0$ , this can be written as

$$x_i d \log x_i = -\pi_i p_i d \log p_i - p_i \sum_{j=1}^n \pi_{ij} p_j d \log p_j - p_i \pi_{ir} r d \log r - p_i \sum_{l=1}^{\ell} \pi_{iz_l} z_l.$$

Recalling that  $-\pi_i p_i = x_i$

$$(5) \quad d \log x_i = d \log p_i + \sum_{j=1}^n \frac{\pi_{ij}}{\pi_i} p_j d \log p_j + \frac{\pi_{ir}}{\pi_i} r d \log r + \sum_{l=1}^{\ell} \frac{\pi_{iz_l}}{\pi_i} z_l d \log z_l.$$

Now, let  $\theta_{ij} = -\frac{\pi_{ij}}{\pi_i} p_j$  denote the (cross-) price elasticities of demand holding  $r$  constant (Frisch, 1959, called these “want elasticities”), let  $\beta_i \delta_{il}$  denote the elasticity of demand with respect to changes in the

characteristic  $z_l$ , and let  $\beta_i = \frac{\pi_{ir}}{\pi_i} r$  denote the elasticity of demand with respect to  $r$ . Using the fact that  $1/r = \lambda^*$  we can rewrite this as

$$(6) \quad d \log x_i = d \log p_i - \sum_{j=1}^n \theta_{ij} d \log p_j - \beta_i \sum_{l=1}^{\ell} \delta_{il} d \log z_l - \beta_i d \log \lambda^*.$$

Using the linear homogeneity of the profit function, it follows that  $\beta_i = \sum_{j=1}^n \theta_{ij}$ . Also, provided only that the utility function is twice continuously differentiable, then by Young's theorem we know that  $\theta_{ij} x_i = \theta_{ji} x_j$ .

**3.4. Neediness and a normalization.** Equation (6) gives us an exact description of how expenditures will change in response to infinitesimal changes in prices for a consumer with the utility function  $U$  and characteristics  $z$ .

Now we make an assumption which is important for reasons both involving principle and practice: that the elasticities  $\Theta = [\theta_{ij}]$  (and so  $\beta = [\beta_i]$ ) and  $\delta = [\delta_{il}]$  are *constant*, and not functions of prices  $(p, r)$ . With this assumption, the matrix of parameters  $\delta$  summarizes the effects of the consumer's characteristics  $z_t$  on demand; *conditional* on these characteristics the term involving  $\lambda^*$  indicates the rate at which changes in welfare influence changes in expenditures on particular goods. Because the  $\beta_i$  are simply equal to the row sums of the matrix of elasticities  $\Theta = (\theta_{ij})$ , in this case the  $\Theta$  matrix summarizes all the pertinent information for understanding changes in demand (conditional on changes in  $z$ ); we call  $\Theta$  the matrix of "Frisch elasticities."

**Proposition 1.** *If a consumer with utility function  $U$  has log Frisch demands which are linear in log prices  $p$ , then there exists a monotonic transformation  $u = G(U)$  such that those demands are also log linear in  $r$ ; further,  $G$  is unique up to a linear transformation.*

*Proof.* For simplicity fix characteristics  $z$  so these can be ignored. Consider the consumer's profit maximization problem with utility function  $U = F(u)$ ; if log-linear in prices differential demands are

$$d \log c = \Theta [d \log p - F'(u) d \log r].$$

If  $U$  happened to be  $u$  then this expression would be log linear in  $r$ , so apply the monotonic transformation  $G = F^{-1}$ . Any other monotonic transformation  $a + bG$  for some  $a \in \mathbb{R}$  and  $b > 0$  will also produce an expression which is log-linear in  $r$ , but no other.  $\square$

This result tells us that assuming that the matrix  $\Theta$  is constant both restricts the demand system but also implicitly normalizes the utility

function which generates these demands. This may seem surprising, since the same would not be true in a Marshallian demand system: if we assumed analogously that Marshallian demands were log linear in prices and expenditures we'd severely restrict the demand system to have linear Engel curves, but the set of utility functions which generated the demand system would be the set of any monotonic transformation of *some* utility function that generated the demands. In contrast, assuming log-linear Frisch demands seems to restrict the Engel curves not at all, but to greatly restrict the set of utility functions which generate those demands.

We next assume that with constant elasticities (6) will provide us with a description of how demand changes with respect to larger changes in prices (see (Mountain, 1988) for a critical discussion of related issues of approximation in the Rotterdam demand system). Allowing also for household characteristics  $z_t$  to serve as demand shifters, we can then write a discrete-time version of (6)

$$(7) \quad \Delta \log x_{it} = \Delta \log p_{it} - \sum_{j=1}^n \theta_{ij} \Delta \log p_{jt} + \beta_i \delta_i^\top \Delta z_t - \beta_i \Delta \log \lambda_t + \Delta \xi_{it},$$

where  $\Delta \xi_{it}$  is an approximation error (For the case of the Rotterdam system Mountain (1988) argues that this approximation error must be of second or higher order; a similar argument seems likely to pertain here).

One further normalization is useful. Observe that in a population of  $N$  households each will typically face a different “price”  $r$ . Normalize *all* prices by assuming that in some initial period the geometric mean of  $r$  across households is equal to one. With these two normalizations, we're finally able to define exactly what we mean by “neediness.”

**Definition 1.** Let a household  $j$  with characteristics  $z$  have a momentary utility function  $u$  such that its Frisch demands are log linear in prices  $(p, r_j)$  and characteristics  $z$ . Then the household's neediness is  $\lambda_j \prod_{l=1}^{\ell} z_l$ , where  $\lambda_j = 1/r_j$  is its *conditional* neediness.

#### 4. ESTIMATION USING A PANEL

Suppose we have data on expenditures at two or more different periods (but lack data on prices). We want to use these data to estimate the parameters of (7). However, that equation describes only the demand system for a single household. Adapting it, let  $j = 1, \dots, N$  index different households, and assume that household characteristics for the  $j$ th household at time  $t$  include both observable characteristics

$z_t^j$  and time-varying unobservable characteristics  $\epsilon_{it}^j$ . Then we can write our structural estimating equation as

$$(8) \quad \Delta \log x_{it}^j = \left( \Delta \log p_{it} - \sum_{k=1}^n \theta_{ij} \Delta \log p_{kt} \right) + \beta_i \delta_i^\top \Delta z_t^j - \beta_i \Delta \log \lambda_t^j + \Delta \xi_{it}^j + \beta_i \Delta \epsilon_{it}^j.$$

We assume that prices are unknown to the econometrician, but that all households face the same prices. Expressed as a reduced form, we write

$$(9) \quad y_{it}^j = a_{it} + b_i^\top (\Delta z_t^j - \overline{\Delta z_t^j}) + c_i w_t^j + e_{it}^j,$$

where

$$\begin{aligned} y_{it}^j &= \Delta \log x_{it}^j \\ a_{it} &= \left[ \Delta \log p_{it} - \sum_{k=1}^n \theta_{ij} \Delta \log p_{kt} \right] - \beta_i \overline{\Delta \log \lambda_t} + \beta_i \overline{\Delta \epsilon_{it}} + \overline{\Delta \xi_{it}} \\ b_i &= \beta_i \delta_i \\ e_{it}^j &= \beta_i (\Delta \epsilon_{it}^j - \overline{\Delta \epsilon_{it}}) + (\Delta \xi_{it}^j - \overline{\Delta \xi_{it}}) \\ c_i w_t^j &= -\beta_i (\Delta \log \lambda_t^j - \overline{\Delta \log \lambda_t}). \end{aligned}$$

We obtain the reduced form parameters  $(a_{it}, b_i)$  simply by using least squares to estimate (9), treating the  $a_{it}$  as a set of good-time effects.

**4.1. Identification of the Parameters of Interest.** What parameters and unobserved quantities can we identify? There are three different groups of objects which are likely to be of interest: the  $n^2$  parameters of the demand system  $\Theta$ ; the conditional neediness  $\{\lambda_t^j\}$ ; and the  $nT$  prices  $\{p_{it}\}$ . We consider each group in turn.

**4.1.1. Marginal Utilities.** The residuals are then equal to  $c_i w_t^j + e_{it}^j$ . The first term of this sum is what we're interested in. Arrange the residuals as an  $n \times NT$  matrix  $\mathbf{Y}$ . The first term in the equation captures the role that variation in  $\lambda$  plays in explaining variation in expenditures. Because it's computed as the outer product of two vectors, this first term is at most of rank one. The second term captures the further role that changes in household observables play in changes in demand; if there are  $\ell$  such observables, then this second term is of at most rank  $\bar{\ell} = \min(\ell, n - 1)$ .

We proceed by considering the singular value decomposition of  $\mathbf{Y} = \mathbf{U}\Sigma\mathbf{V}^\top$ , where  $\mathbf{U}$  and  $\mathbf{V}$  are unitary matrices, and where  $\Sigma$  is a diagonal matrix of the singular values of  $\mathbf{Y}$ , ordered from the largest to the smallest. Then the rank one matrix that depends on

$\lambda$  is  $\mathbf{g}\mathbf{w}^\top = \sigma_1 \mathbf{u}_1 \mathbf{v}_1^\top$ , while the second matrix (of at most rank  $\bar{\ell}$ ) is  $\mathbf{d}\mathbf{Z}^\top = \sum_{k=2}^{\bar{\ell}} \sigma_k \mathbf{u}_k \mathbf{v}_k^\top$ , where  $\sigma_k$  denotes the  $k$ th singular value of  $\mathbf{Y}$ , and where the subscripts on  $\mathbf{u}$  and  $\mathbf{v}$  indicate the column of the corresponding matrices  $\mathbf{U}$  and  $\mathbf{V}$ . The sum of these matrices is the optimal  $1 + \bar{\ell}$  rank approximation to  $\mathbf{Y}$ , in the sense that by the Eckart-Young theorem this is the solution to the problem of minimizing the Frobenius distance between  $\mathbf{Y}$  and the approximation; accordingly, this is also the least-squares solution (Golub and Reinsch, 1970).

The singular value decomposition thus identifies the structural parameters  $\beta_i$  and changes in log marginal utility up to a factor  $\phi$ , so that we obtain estimates of  $\phi\beta_i$  and of  $(\Delta \log \lambda_t^j - \overline{\Delta \log \lambda_t})/\phi$ . We adopt a final normalization which chooses  $\phi$  so that it's equal to the reciprocal of the mean of the estimated  $\beta_i$  across goods in the estimated demand system.

4.1.2. *Additional Parameters of the Demand System.* To completely identify the demand system, we'd like to estimate the  $n^2$  Frisch elasticities  $\Theta_{ij}$  (which in turn pin down the  $n$  elasticities  $\beta_i$ ); and the  $n\ell$  effects of household observables on demands  $\delta_i$ .

In the estimation procedure described above, we first obtain a set of estimated good-time effects  $\{a_{it}\}$ ; this is the only place in which the full set of coefficients in  $\Theta$  appear, so the question of identifying the Frisch matrix of elasticities is the question of being able to compute  $\Theta$  from the matrix equation

$$\mathbf{A} = (\Delta \log \mathbf{p})(\mathbf{I} - \Theta) + \mathbf{E}$$

where  $\mathbf{A}$  and  $\Delta \log p$  are  $n \times T$  matrices, and where  $\mathbf{E} = \beta_i \overline{\Delta \log \lambda_t} + \beta_i \overline{\Delta \epsilon_{it}} + \overline{\Delta \xi_{it}}$  is a matrix of residuals averaged over households.

We've already established that the elasticities  $\beta_i$  can be separately identified up to an unknown scalar. This imposes  $n$  adding-up restrictions on the matrix  $\Theta$ , leaving  $n(n - 1)$  degrees of freedom. If we do not observe prices, little more can be said about  $\Theta$ . If we observe  $n$  prices in each period, then we must have  $T \geq n(n - 1)$  if we are to estimate  $\Theta$  (Larson, 1966, using, e.g., the methods of).

In the application of this paper this requirement is *not* satisfied, and so we cannot estimate without additional restrictions. This means that we will not be in a position to discuss the intra-temporal substitution elasticities between different goods.

4.1.3. *Prices.* If we have independent information on the matrix  $\Theta$ , it may be possible to draw inferences regarding changes in prices, using the same relationship between the latent good-time effects  $A$  and prices

described above. If  $\mathbf{I} - \Theta$  is invertible, then we have

$$\Delta \log \mathbf{p} = (\mathbf{A} - \mathbf{E})(\mathbf{I} - \Theta)^{-1}.$$

If we know less about  $\Theta$  but are willing to assume some additional structure we may also be able to make progress. For example, if  $\Theta$  is known (or assumed) to be diagonal, then we can use estimates of the  $\beta_i$  to construct an estimated  $\hat{\Theta}$ , and then proceed as above.

## 5. SELECTING PARTICULAR GOODS

One of the attractive features of Frisch demand systems is that it's very simple to estimate *incomplete* demand systems, featuring just a few goods, something which isn't easy or straightforward in a Marshallian demand context unless one invokes some very strong assumptions regarding separability.

The upshot of this is that if we're interested in measuring marginal utility, we don't have to know about expenditures for all goods; instead, we can simply choose a *few* goods. The cost of using a smaller set of goods is simply that one ignores a possible additional useful source of information on marginal utility. However, this cost may be quite small for some goods. First, if the elasticity parameter  $\beta_i$  is small, then variation in expenditures simply isn't closely related to variation in marginal utility. Second, the demand equations we estimate may be a poor fit to the expenditure data, whether because of measurement error, a high importance of unobservable household characteristics in determining demand for a particular good, large approximation error, or simply because the specification of demand is particularly poor for some goods. In any of these cases the fit of the estimated demand system will tend to be poor, and variation in expenditures may depend more on the error term in the regression than on variation in marginal utility.

The preceding considerations suggest two simple criteria for determining what goods to use. The first merely involves examining the magnitude of the estimated elasticities  $\beta_i$ , and preferring goods with larger (absolute) elasticities. The second involves simply considering a measure of equation fit, such as the simple  $R^2$  statistic.

## 6. CONCLUSION

In this paper we've outlined some of the key methodological ingredients needed in a recipe to measure what we've termed a household's marginal utility of contemporaneous expenditures in a manner which is theoretically coherent, which lends itself to straightforward statistical

inference and hypothesis testing, and which is very parsimonious in its data requirements.

Our goal is to devise procedures to easily *measure* and *monitor* households' marginal utility over both different environments and across time. To this end, we've described an approach which involves estimating an incomplete demand system of a new sort which features a highly flexible relationship between total expenditures and demand. The method worked out in this paper involves using a panel of data on household expenditures on different goods and/or services, and requires at least two periods of data. It seems likely that related methods could be developed to permit the analysis of repeated cross-sections of data, but this is left for future research.

In an application developed in an appendix to this paper we illustrate the use of our methods using two rounds of data from Uganda. We focus on food expenditures in this dataset; initially estimating a system of 29 demands, we find that a much smaller system of goods suffices to estimate household marginal utility. In particular, a scheme of monitoring marginal utility by carefully tracking expenditures on just 21 food items would do just as well as the current collection of 61 food items, and could do much better if care was taken to reduce the proportion of households reporting zero expenditures for these items. Of particular importance are just a handful of food goods with high elasticities of marginal utility: fresh cassava, fresh sweet potatoes, bread, rice, fresh milk, tea, beef, sugar, cooking oil, onions, and tomatoes.

The presence of zeros in households' reported expenditures is the chief difficulty in our empirical exercise. These zeros cause problems for two reasons. First, the construction of the differential demand system (6) assumes that expenditures are positive, and simply isn't valid for settings in which expenditures are sometimes zero. Addressing this problem in the theory may be possible, by imposing non-negativity constraints on the demands for consumers. How this will change the estimating equation is unknown. Second, some of the recorded zeros may be the result of a particular kind of measurement error, perhaps related to the fatigue or inattention of enumerators or respondents. If it's possible to change data collection practices (perhaps by focusing on fewer goods) so as to reduce this possible source of error, then perhaps it would be possible to avoid the theoretical difficulties posed by zero expenditures simply by focusing on goods for which few households report zeros.

All this said, while the existence of a large numbers of reported zeros clearly leads to problems of bias in our estimated elasticities, in practice our estimates of changes in households' marginal utility seems

not to be very sensitive to zeros. After fairly extensive experimentation with including or excluding different goods from the estimation, we find that estimated changes in household marginal utility are quite highly correlated across these different experiments. Indeed, it seems very likely that estimates of changes in marginal utility will be much less sensitive to this sort of reduction in the set of expenditures being considered than changes in poverty (based on the construction of corresponding expenditure ‘sub-aggregates’) would be.

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## APPENDIX A. APPLICATION IN UGANDA

To illustrate some of the methods and issues discussed above, we use data from two rounds of surveys conducted in Uganda (in 2005–06 and 2009–10).<sup>4</sup> We first give a descriptive account of some of the data on expenditures from these surveys, and then supply data on estimates of household neediness and the neediness elasticities ( $\beta_i$ ). We finally supply some discussion of the effects of using only selected goods in our estimation procedure.

**A.1. Data on Expenditures & Results from Estimation.** Excluding durables, taxes, fees & transfers, there are 99 categories of expenditure in the data, of which 61 are different food items or categories, and 38 are nondurables or services. We consider some different collections of goods, first describing all the goods, then turning our attention to a group of what we call “slightly aggregated” foods.

*A.1.1. All Expenditures.* Figure 1 paints a picture of aggregate expenditure shares across these categories, listing mean and aggregate expenditure shares for all goods which had an aggregate expenditure share greater than one percent in 2005. A glance reveals that shares of aggregates is fairly stable across the two rounds, with an increase in the share of housing from 8% to 10% the most notable change. It should be noted, however, that stability of shares over time is not a prediction of theory—changes in incomes or relative prices could easily be responsible for changes in shares.

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<sup>4</sup>These datasets are available at <http://go.worldbank.org/M05MSKCQSO>, with documentation available at <http://go.worldbank.org/S233P3YC30>.

## ESTIMATING CHANGES IN MARGINAL UTILITY FROM DISAGGREGATED EXPENDITURES

TABLE 1. Shares of Aggregate Expenditures in Uganda (2005 and 2009), for all goods with aggregate shares greater than 1% in 2005.

Expenditure Item	Agg. Shares 2005	Agg. Shares 2009	Mean 2005	Mean 2009
Imputed rent of owned house	0.078	0.100	0.060	0.067
Matoke (Bunch)	0.050	0.054	0.044	0.050
Sweet potatoes (Fresh)	0.042	0.045	0.051	0.056
Maize (flour)	0.040	0.038	0.052	0.049
Medicines etc	0.038	0.041	0.034	0.038
Water	0.033	0.030	0.044	0.033
Food (restaurant)	0.033	0.033	0.030	0.031
Beef	0.033	0.033	0.027	0.028
Sugar	0.031	0.029	0.030	0.029
Beans (dry)	0.031	0.033	0.040	0.044
Taxi fares	0.027	0.023	0.020	0.016
Firewood	0.027	0.030	0.040	0.042
Hospital/ clinic charges	0.026	0.020	0.021	0.018
Cassava (Fresh)	0.024	0.026	0.029	0.034
Fresh Milk	0.022	0.024	0.018	0.021
Air time & services fee for owned fixed/mobile phones	0.022	0.035	0.011	0.023
Cassava (Dry/ Flour)	0.020	0.024	0.028	0.032
Rent of rented house	0.019	0.017	0.017	0.015
Rice	0.015	0.015	0.012	0.013
Fresh Fish	0.013	0.012	0.012	0.012
Paraffin (Kerosene)	0.013	0.011	0.016	0.015
Cooking oil	0.012	0.011	0.015	0.013
Washing soap	0.012	0.012	0.016	0.015
Charcoal	0.012	0.014	0.010	0.011
Barber and Beauty Shops	0.011	0.011	0.008	0.008
Tomatoes	0.011	0.010	0.012	0.011
Petrol, diesel etc	0.011	0.015	0.003	0.006

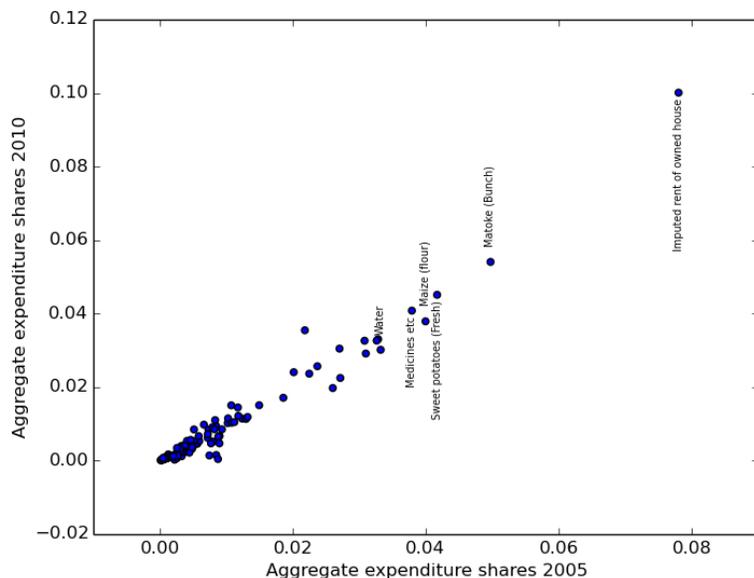


FIGURE 1. Aggregate expenditure shares in 2005 and 2009, using all expenditure items.

Table 1 describes the share of aggregate expenditures on different consumption items in each of the two rounds of the survey, for goods with an expenditures share greater than 1% in 2005. The second pair columns instead provides the average expenditure shares; that is, averaging shares across households, instead of summing across households and then computing shares. These two different measures of shares give an interesting indication of what shares are more important for wealthy or poor households, since wealthy households are over-represented in the calculation of aggregate shares relative to poor households. This general point is made perhaps more effectively by Figure 2. In this figure goods are ordered by the log of the *ratio* of the mean share to the aggregate share in 2005. Accordingly, this statistic for goods with an outsized share of wealthy households' total expenditures take values less than zero, while goods that take a larger share in poor households total expenditures take values greater than one. Some of the goods at extremes are labeled: it appears that wealthier households tend to spend proportionally more on servants and motor fuels, for example, while poorer households spend proportionally more on foods (consistent with Engel's law) such as sorghum and maize.

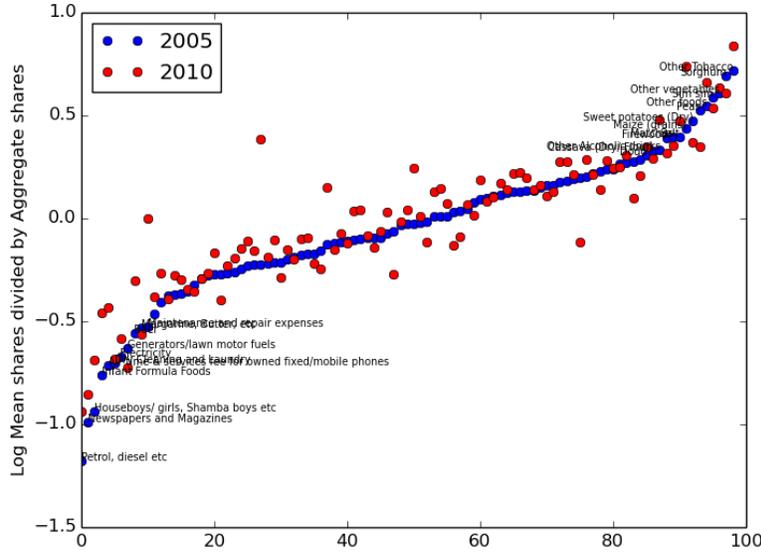


FIGURE 2. Ratio of mean shares to aggregate shares. Items on the left form a disproportionately large share of the budget of the rich; items on the right a disproportionately large budget share of the needy.

Table 2: Estimated parameters of the demand system for all food items. The first two columns report the values of  $\hat{\delta}_i$  associated with household characteristics. The third column reports estimates of the elasticity of marginal utility, up to a common factor  $\phi$ . The fourth column reports the proportion of variance in the residual term accounted for by variation in  $\Delta \log \lambda_t^j$ ; the final column indicates the proportion of observations with zero recorded for the expenditure category.

Expenditure category	HHSIZE	Rural	$\phi\beta_i$	$R^2$	% Zeros
Other foods	0.05	-1.30	-0.22	0.00	0.92
Imputed rent of free house	2.14	-23.52	-0.06	0.00	0.94
Food (restaurant)	1.77	17.26	-0.04	0.00	0.83
Beer (restaurant)	0.68	0.01	-0.04	0.00	0.99
Dry Cleaning and Laundry	0.81	0.05	-0.02	0.00	0.99
Sweet potatoes (Dry)	-0.38	-33.56	-0.02	0.00	0.98

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Table 2: Estimated parameters of the demand system for all food items. The first two columns report the values of  $\hat{\delta}_i$  associated with household characteristics. The third column reports estimates of the elasticity of marginal utility, up to a common factor  $\phi$ . The fourth column reports the proportion of variance in the residual term accounted for by variation in  $\Delta \log \lambda_t^j$ ; the final column indicates the proportion of observations with zero recorded for the expenditure category.

Expenditure category	HHSize	Rural	$\phi\beta_i$	$R^2$	% Zeros
Others (Rent of rented house/fuel/power)	7.40	234.88	-0.01	0.00	0.98
Infant Formula Foods	-7.83	180.65	-0.00	0.00	1.00
Other Tobacco	6.85	-47.64	0.01	0.00	0.91
Other Meat	1.51	-0.42	0.02	0.00	0.98
Other juice	-2.61	-13.27	0.05	0.00	0.96
Matoke (Others)	1.32	1.48	0.05	0.00	0.98
Stamps, envelopes, etc.	-0.01	13.14	0.05	0.00	0.99
Generators/lawn motor fuels	0.54	-6.54	0.06	0.00	0.99
Others (Transport and communication)	0.60	10.24	0.06	0.00	0.98
Traditional Doctors fees/ medicines	0.51	0.37	0.08	0.00	0.98
Consultation Fees	0.47	0.52	0.09	0.00	0.96
Ground nuts (in shell)	0.78	-5.26	0.13	0.00	0.97
Soda (restaurant)	-1.12	0.47	0.13	0.00	0.96
Rent of rented house	-0.80	-47.92	0.13	0.00	0.86
Other drinks	-0.10	-4.10	0.14	0.00	0.96
Bus fares	0.48	-8.40	0.17	0.00	0.95
Houseboys/ girls, Shamba boys etc	-0.32	3.34	0.19	0.01	0.96
Matoke (Heap)	-0.05	-22.52	0.20	0.00	0.92
Others (Non-durable and personal goods)	0.11	6.54	0.20	0.00	0.95
Maize (grains)	0.22	3.01	0.22	0.00	0.96
Margarine, Butter, etc	-0.14	2.93	0.24	0.01	0.96
Petrol, diesel etc	0.55	-3.91	0.25	0.01	0.95
Maintenance and repair expenses	0.24	-7.30	0.26	0.00	0.92
Handbags, travel bags etc	0.15	-2.24	0.26	0.01	0.96
Ghee	0.16	-2.57	0.26	0.01	0.95
Cigarettes	0.26	-4.30	0.30	0.01	0.91
Beer	-0.01	6.51	0.33	0.01	0.95
Electricity	0.42	-2.16	0.33	0.01	0.88
Others (Health and Medical Care)	0.34	3.02	0.36	0.01	0.91
Newspapers and Magazines	-0.07	-3.58	0.37	0.02	0.94

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Table 2: Estimated parameters of the demand system for all food items. The first two columns report the values of  $\hat{\delta}_i$  associated with household characteristics. The third column reports estimates of the elasticity of marginal utility, up to a common factor  $\phi$ . The fourth column reports the proportion of variance in the residual term accounted for by variation in  $\Delta \log \lambda_t^j$ ; the final column indicates the proportion of observations with zero recorded for the expenditure category.

Expenditure category	HHSize	Rural	$\phi\beta_i$	$R^2$	% Zeros
Sports, theaters, etc	0.42	3.47	0.37	0.01	0.96
Sorghum	0.16	5.55	0.38	0.01	0.86
Coffee	0.37	2.71	0.43	0.01	0.93
Matoke (Cluster)	0.19	3.24	0.46	0.01	0.95
Peas	0.29	-2.89	0.47	0.01	0.91
Beans (fresh)	-0.09	1.17	0.51	0.01	0.84
Goat Meat	-0.11	1.44	0.51	0.02	0.94
Pork	-0.12	-0.04	0.52	0.02	0.94
Other Alcoholic drinks	0.50	2.71	0.52	0.01	0.83
Other vegetables	0.18	8.23	0.55	0.01	0.66
Mangos	0.50	3.61	0.56	0.01	0.88
Maize (cobs)	0.14	2.24	0.57	0.01	0.86
Hospital/ clinic charges	0.29	0.13	0.60	0.01	0.79
Ground nuts (shelled)	-0.06	0.26	0.60	0.01	0.89
Sim sim	0.25	3.13	0.65	0.02	0.89
Oranges	-0.07	1.72	0.66	0.02	0.92
Tires, tubes, spares, etc	0.34	1.96	0.71	0.02	0.84
Dodo	0.23	1.54	0.87	0.01	0.67
Cassava (Dry/ Flour)	0.12	0.70	0.90	0.03	0.73
Chicken	0.13	-0.71	0.94	0.04	0.92
Expenditure on phones not owned	0.02	4.82	0.96	0.03	0.84
Washing soap	0.15	0.81	1.00	0.09	0.04
Water	0.03	-0.01	1.02	0.02	0.22
Imputed rent of owned house	0.38	5.30	1.04	0.04	0.23
Irish Potatoes	0.11	-1.31	1.05	0.05	0.88
Millet	0.15	2.64	1.07	0.04	0.84
Matches	0.16	0.52	1.07	0.11	0.05
Medicines etc	0.80	-1.62	1.12	0.02	0.49
Paraffin (Kerosene)	0.18	1.52	1.17	0.06	0.14
Firewood	0.34	3.55	1.20	0.05	0.28

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Table 2: Estimated parameters of the demand system for all food items. The first two columns report the values of  $\hat{\delta}_i$  associated with household characteristics. The third column reports estimates of the elasticity of marginal utility, up to a common factor  $\phi$ . The fourth column reports the proportion of variance in the residual term accounted for by variation in  $\Delta \log \lambda_t^j$ ; the final column indicates the proportion of observations with zero recorded for the expenditure category.

Expenditure category	HHSize	Rural	$\phi\beta_i$	$R^2$	% Zeros
Soda	-0.02	-3.35	1.24	0.06	0.90
Eggs	0.02	-0.63	1.29	0.06	0.87
Airtime & fees for owned phones	0.18	-1.72	1.33	0.04	0.65
Fresh Fish	0.09	1.21	1.36	0.05	0.80
Charcoal	0.02	-2.62	1.45	0.10	0.75
Passion Fruits	0.07	0.93	1.46	0.09	0.88
Dry/ Smoked fish	0.23	-2.51	1.47	0.04	0.75
Matoke (Bunch)	0.21	-0.36	1.50	0.04	0.66
Other Fruits	0.09	0.43	1.56	0.06	0.82
Sweet Bananas	0.04	0.29	1.58	0.08	0.85
Batteries (Dry cells)	0.10	-0.34	1.61	0.05	0.47
Salt	0.22	0.20	1.70	0.19	0.08
Ground nuts (pounded)	0.15	1.16	1.70	0.06	0.70
Boda boda fares	0.11	0.27	1.72	0.06	0.73
Cabbages	0.12	0.32	1.85	0.09	0.81
Maize (flour)	0.25	-3.35	1.98	0.06	0.46
Tooth paste	0.01	-0.95	2.04	0.11	0.52
Cassava (Fresh)	0.17	3.60	2.05	0.07	0.57
Bathing soap	0.04	-1.38	2.13	0.11	0.68
Bread	-0.01	-1.51	2.20	0.14	0.78
Beans (dry)	0.31	1.01	2.23	0.08	0.37
Taxi fares	0.00	0.38	2.24	0.10	0.69
Rice	0.13	0.33	2.25	0.12	0.74
Sweet potatoes (Fresh)	0.23	2.08	2.42	0.10	0.46
Cosmetics	0.15	0.44	2.44	0.11	0.32
Fresh Milk	0.07	0.25	2.60	0.13	0.65
Barber and Beauty Shops	0.11	1.05	2.79	0.14	0.55
Tea	0.09	0.61	3.03	0.22	0.39
Beef	0.07	-0.62	3.28	0.19	0.67
Cooking oil	0.17	0.08	3.84	0.30	0.42

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Table 2: Estimated parameters of the demand system for all food items. The first two columns report the values of  $\hat{\delta}_i$  associated with household characteristics. The third column reports estimates of the elasticity of marginal utility, up to a common factor  $\phi$ . The fourth column reports the proportion of variance in the residual term accounted for by variation in  $\Delta \log \lambda_t^j$ ; the final column indicates the proportion of observations with zero recorded for the expenditure category.

Expenditure category	HHSize	Rural	$\phi\beta_i$	$R^2$	% Zeros
Tomatoes	0.10	-0.75	3.85	0.32	0.33
Onions	0.15	-0.19	3.88	0.33	0.31
Sugar	0.11	0.09	3.90	0.31	0.36

Table 2 reports estimates and some diagnostics from the estimation of the complete system of 99 food and nondurable goods and services, ordered by the size of the critical estimated marginal utility elasticities (up to a unknown scale parameter)  $\phi\beta_i$ . The table also reports estimated coefficients associated with the household characteristics of household size (number of people in the household) and a dummy variable indicating whether the household is rural. Table 2 also reports a statistic labeled  $R^2$  which reports the proportion of variation in the residual term accounted for by variation in changes in log marginal utility.

As discussed above, the goods and services most useful goods for measuring marginal utility are those with larger estimated elasticities and with smaller measurement error. The  $R^2$  statistics here provide a measure of the variance of measurement error (smaller  $R^2$  statistics indicate more measurement error). The goods in Table 2 with the largest estimated elasticities also tend to have larger  $R^2$  statistics; they also happen to be food items. We next explore this association in some detail by focusing *just* on food items.

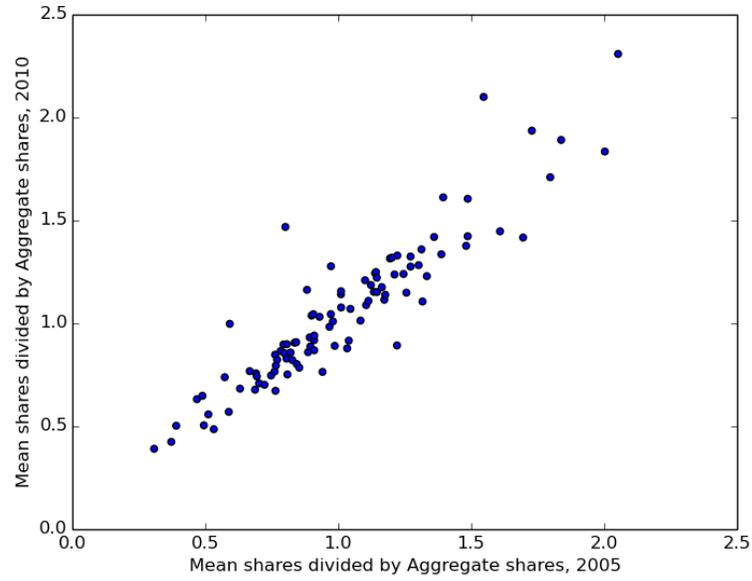


FIGURE 3. Changes in ratio of mean shares to aggregate shares across survey rounds, using all expenditure categories.

A.1.2. *All Food Categories.*

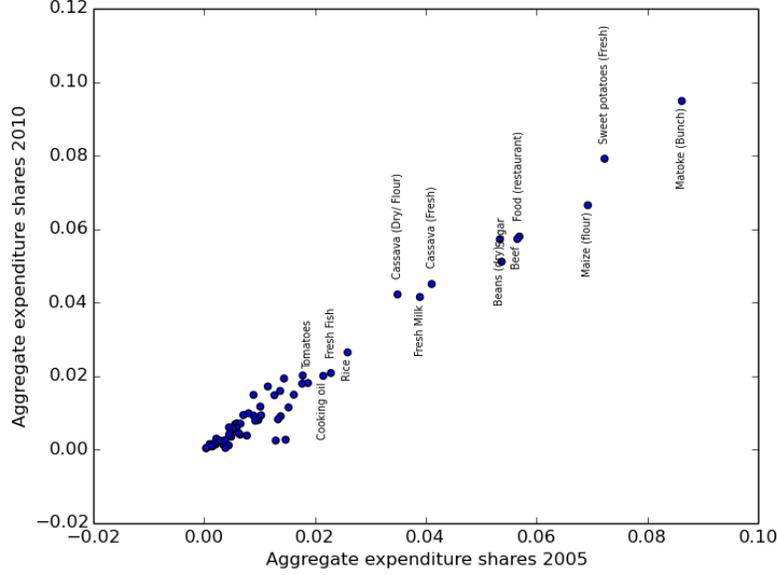


FIGURE 4. Aggregate expenditure shares in 2005 and 2009, using all food items.

Table 3 reports statistics obtained from the estimation of a demand system consisting of just the 61 different food items. As in Table 2 expenditure categories are ordered according to the estimated marginal utility elasticity  $\beta_i$ . The ordering of foods in Table 3 is extremely close to the ordering observed in Table 2.

Table 3: Estimated parameters of the demand system for all food items. The first two columns report the values of  $\hat{\delta}_i$  associated with household characteristics. The third column reports estimates of the elasticity of marginal utility, up to a common factor  $\phi$ . The fourth column reports the proportion of variance in the residual term accounted for by variation in  $\Delta \log \lambda_t^j$ ; the final column indicates the proportion of observations with zero recorded for the expenditure category.

Food category	HHSize	Rural	$\phi\beta_i$	$R^2$	% Zeros
Other foods	0.07	-0.74	-0.30	0.01	0.92
Food (restaurant)	0.31	-0.01	-0.24	0.00	0.83
Beer (restaurant)	0.63	0.10	-0.03	0.00	0.99

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Table 3: Estimated parameters of the demand system for all food items. The first two columns report the values of  $\hat{\delta}_i$  associated with household characteristics. The third column reports estimates of the elasticity of marginal utility, up to a common factor  $\phi$ . The fourth column reports the proportion of variance in the residual term accounted for by variation in  $\Delta \log \lambda_t^j$ ; the final column indicates the proportion of observations with zero recorded for the expenditure category.

Food category	HHSize	Rural	$\phi\beta_i$	$R^2$	% Zeros
Infant Formula Foods	-4.14	98.29	-0.01	0.00	1.00
Sweet potatoes (Dry)	0.44	-135.09	-0.00	0.00	0.98
Other juice	-53.77	-328.61	0.00	0.00	0.96
Matoke (Others)	3.46	3.34	0.02	0.00	0.98
Other Tobacco	2.22	-18.50	0.03	0.00	0.91
Other Meat	0.77	-0.15	0.04	0.00	0.98
Soda (restaurant)	-2.28	11.03	0.06	0.00	0.96
Other drinks	-0.30	-4.73	0.11	0.00	0.96
Ground nuts (in shell)	0.86	-4.89	0.13	0.00	0.97
Matoke (Heap)	0.06	-24.08	0.18	0.00	0.92
Cigarettes	0.54	-6.44	0.18	0.00	0.91
Maize (grains)	0.10	3.09	0.20	0.01	0.96
Ghee	0.19	-2.97	0.21	0.01	0.95
Margarine, Butter, etc	-0.20	2.29	0.28	0.01	0.96
Beer	0.01	6.73	0.30	0.01	0.95
Coffee	0.43	3.18	0.35	0.01	0.93
Other Alcoholic drinks	0.73	3.74	0.36	0.01	0.83
Sorghum	0.22	5.39	0.37	0.01	0.86
Peas	0.36	-2.94	0.43	0.01	0.91
Goat Meat	-0.09	1.51	0.45	0.02	0.94
Matoke (Cluster)	0.08	3.06	0.45	0.02	0.95
Mangos	0.53	2.67	0.47	0.01	0.88
Pork	-0.10	-1.47	0.48	0.02	0.94
Beans (fresh)	-0.06	1.07	0.52	0.01	0.84
Maize (cobs)	0.12	2.28	0.52	0.01	0.86
Sim sim	0.25	3.56	0.54	0.02	0.89
Other vegetables	0.26	6.61	0.55	0.01	0.66
Ground nuts (shelled)	-0.07	0.25	0.57	0.02	0.89
Oranges	-0.05	0.63	0.64	0.03	0.92
Cassava (Dry/ Flour)	0.19	0.81	0.75	0.03	0.73

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Table 3: Estimated parameters of the demand system for all food items. The first two columns report the values of  $\hat{\delta}_i$  associated with household characteristics. The third column reports estimates of the elasticity of marginal utility, up to a common factor  $\phi$ . The fourth column reports the proportion of variance in the residual term accounted for by variation in  $\Delta \log \lambda_t^j$ ; the final column indicates the proportion of observations with zero recorded for the expenditure category.

Food category	HHSIZE	Rural	$\phi\beta_i$	$R^2$	% Zeros
Dodo	0.23	1.56	0.82	0.02	0.67
Chicken	0.16	-1.50	0.88	0.05	0.92
Millet	0.18	2.88	0.92	0.04	0.84
Irish Potatoes	0.18	-1.32	0.96	0.05	0.88
Soda	0.03	-4.74	1.08	0.07	0.89
Eggs	0.04	-0.63	1.19	0.07	0.87
Other Fruits	0.09	0.51	1.26	0.05	0.82
Fresh Fish	0.11	1.23	1.27	0.06	0.80
Passion Fruits	0.07	0.99	1.30	0.09	0.88
Dry/ Smoked fish	0.20	-2.63	1.31	0.05	0.75
Sweet Bananas	0.06	0.33	1.36	0.08	0.85
Matoke (Bunch)	0.28	-0.81	1.39	0.05	0.66
Ground nuts (pounded)	0.12	1.71	1.43	0.06	0.70
Salt	0.24	0.20	1.52	0.21	0.08
Cabbages	0.11	0.35	1.64	0.09	0.81
Maize (flour)	0.27	-3.26	1.89	0.08	0.45
Beans (dry)	0.35	0.81	1.93	0.08	0.37
Cassava (Fresh)	0.18	3.11	2.01	0.09	0.57
Sweet potatoes (Fresh)	0.28	2.34	2.03	0.10	0.46
Bread	0.01	-1.24	2.07	0.16	0.78
Rice	0.17	0.04	2.10	0.15	0.74
Fresh Milk	0.07	0.52	2.33	0.15	0.65
Tea	0.10	0.66	2.71	0.24	0.39
Beef	0.06	-0.82	3.14	0.23	0.67
Sugar	0.13	0.11	3.34	0.32	0.36
Cooking oil	0.18	0.09	3.43	0.34	0.42
Onions	0.17	-0.20	3.49	0.37	0.31
Tomatoes	0.09	-0.76	3.56	0.37	0.33

The parameter estimates reported in Table 3 mostly seem reasonable enough, though one should bear in mind that the critical  $\beta_i$  parameters

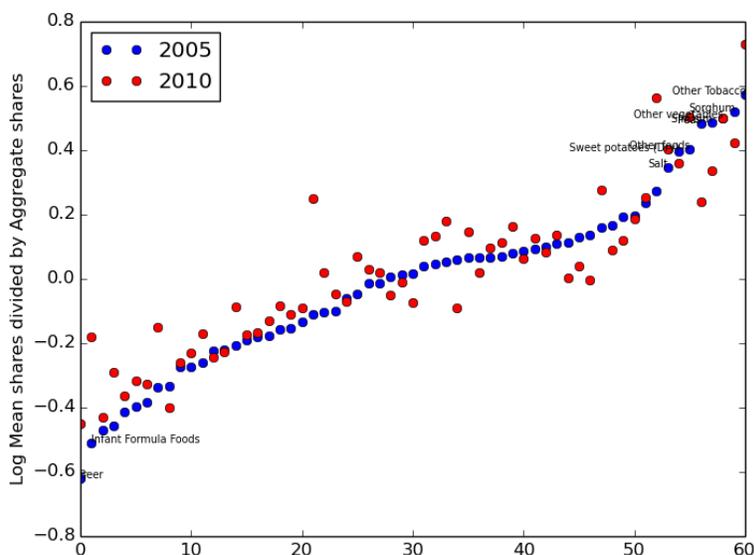


FIGURE 5. Ratio of mean shares to aggregate shares, using all food items. Items on the left form a disproportionately large share of the budget of the rich; items on the right a disproportionately large budget share of the needy.

are only identified up to a scale parameter (so that one can't really say that some goods are "luxuries" while others are "necessities", for example). However, one can rank the  $\beta_i$ , and most of the goods that have the smallest elasticities also have a very high proportion of "zero" recorded for expenditures. Moving from the lowest elasticity up (the rows are ordered by the estimated  $\phi\beta_i$ ), with a single exception ("Other vegetables") one has to move more than halfway down the table before reaching a good (Cassava dry/flour) for which more than 20% of the observations report positive expenditures. What seems to be happening for at least some of these goods is that our estimator is trying to fit a large proportion of zeros, and these zeros occur across a large part of the marginal utility distribution, leading to very low estimates of  $\beta_i$  for these goods.

Once one gets down the table to goods that have a proportion of zeros less than one half the relative elasticities seem to be fairly plausible: it's easy to believe that demands for tomatoes, onions, cooking oil, sugar

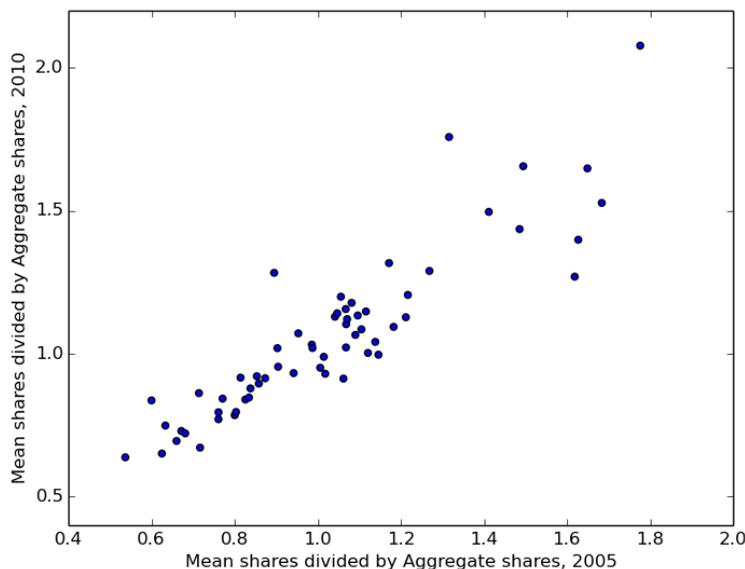


FIGURE 6. Changes in ratio of mean shares to aggregate shares across survey rounds, using all food items.

and beef are more elastic than demands for maize flour, dry beans, and fresh cassava, for example.

Recall that the goods which are most useful for drawing inferences regarding marginal utility are those which either have large (relative) values of  $\beta_i$ , or small errors (large  $R^2$  statistics). The fact that we earlier offered two distinct criteria for choosing goods created the possibility that these criteria might conflict. However, Table 3 indicates a very high degree of correlation between these two criteria, suggesting that the conflict may not arise often in practice.

Figure 7 provides a histogram of estimated changes in log marginal utility. These estimates have a mean of zero by construction using this estimator—time varying differences common to all households are swept out with the good-time effects of the estimating equation (9). Further, the scale is only determined by an arbitrary normalization of the  $\beta_i$ , so the information in this histogram all pertains to the shape of the distribution. In this case the distribution appears to be reasonably symmetric, though slightly skew right (the third central moment is positive) and with slightly fatter tails than the normal distribution (the fourth central moment is approximately 55, while if the distribution were normal we'd expect this moment to be approximately 40).

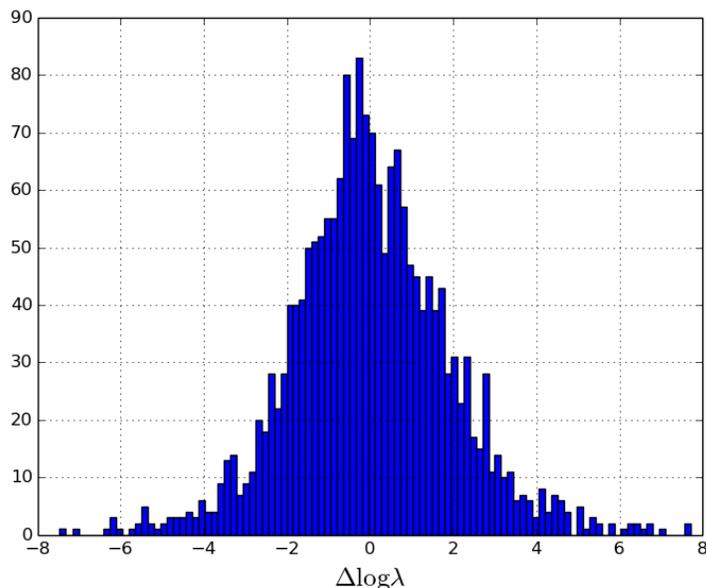


FIGURE 7. Histogram of  $\Delta \log \lambda_t^j$  using all food items.

These deviations seem fairly minor, however; taking the distribution of changes in marginal utility to be log-normal would not seem to do too much violence to the data.

One might wonder what would happen if we simply dropped the goods with a high proportion of zeros. In this case dropping any good with a proportion of zeros greater than eighty per cent reduces the number of goods in the demand system from 61 to 21. The (Pearson) correlation between the estimated  $\Delta \log \lambda_t^j$  in the 61 good and the 21 good system is 0.98, and the ordering of the goods by elasticity is almost unchanged (bread and milk are slightly less relatively elastic in the 21 good system). Dropping an additional 10 goods, leaving the 11 most elastic, yields a correlation of 0.94 between changes in log marginal utility estimated using the 61 and 11 good systems.

A.1.3. *Food in Slightly Aggregated Groups.* Is there a better way to deal with the problem of the most detailed expenditure categories having a high proportion of zeros? Though there are 61 different food expenditure categories (including two different categories for tobacco) many of these are categories that seem that they must be very close substitutes. For instance, the four different forms in which Matoke is acquired (bunches, clusters, heaps, and others) are all elicited as

separate expenditure items. Other staple items are also elicited in detail, with two different expenditure items each for sweet potatoes and cassava (fresh and dry), three for maize and ground nut, and so on. Expenditures on five different kinds of fresh fruit and five different kinds of fresh vegetables are also collected.

This level of detail in expenditure isn't a problem in principle, but in practice many of the detailed categories feature zero expenditures for many households. Supposing these to be "true" zeros (rather than measurement error), we'd interpret these as corner solutions for the households where they occur, but the requirement that expenditures be non-negative isn't something that's taken into account in the derivation of our demand system. Adding this to our modeling exercise is undoubtedly the correct way to proceed, but is sufficiently difficult that we try to reduce the impact of this by aggregating goods that seem likely to be close substitutes.

Simply adding up expenditures which, according to their descriptions, seem as though they might be closely related yields a system of 29 goods; we'll refer to this as our "slightly aggregated food" system. Expenditure shares from this system are reported in Table ??, paralleling the complete set of foods described in Table ?. Similar figures and tables also parallel those for the complete set of foods.

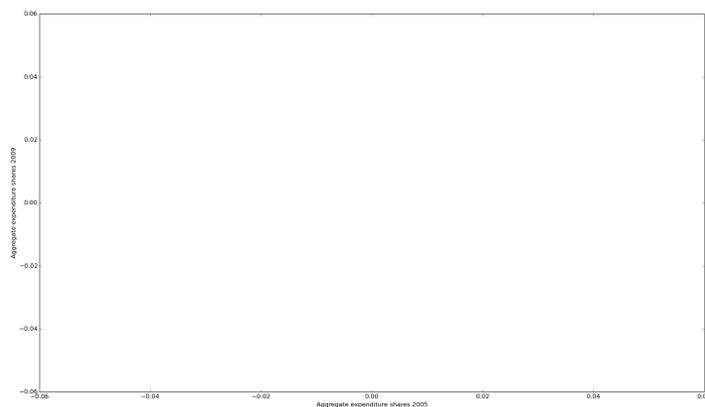


FIGURE 8. Aggregate expenditure shares in 2005 and 2009, using slightly aggregated food groups.

Figure 8 reveals aggregate shares that are reasonably stable over time, though Figure 9 indicates that the ratio of mean to aggregate shares shows some greater variability. The most prominent examples are the goods with the largest such ratio in 2005: salt, "other foods", peas, sim sim (a sweetener), and sorghum, all goods that feature more prominently in the budgets of poorer rather than wealthier households.

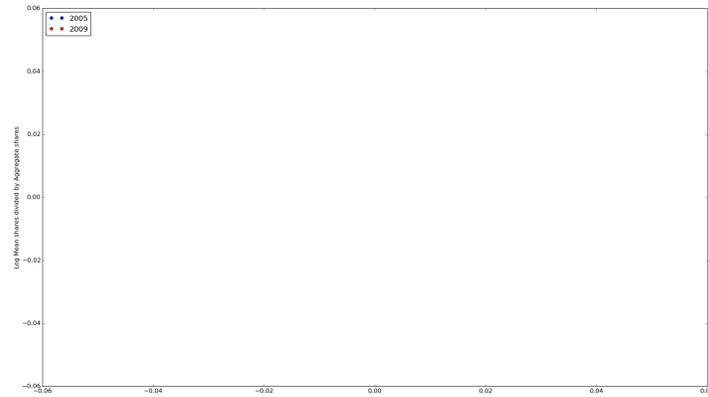


FIGURE 9. Ratio of mean shares to aggregate shares, using slightly aggregated food groups. Items on the left form a disproportionately large share of the budget of the rich; items on the right a disproportionately large budget share of the needy.

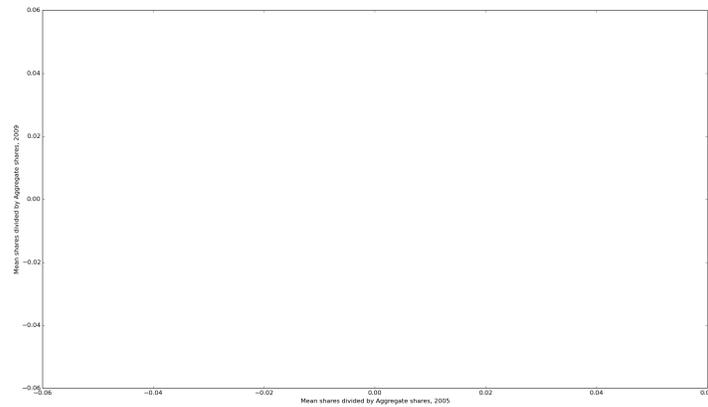


FIGURE 10. Changes in the ratio of mean shares to aggregate shares across survey rounds, using slightly aggregated food groups.

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Table 4: Estimated parameters of the demand system for “slightly aggregated” food items. The first two columns report the values of  $\hat{\delta}_i$  associated with household characteristics. The third column reports estimates of the elasticity of marginal utility, up to a common factor  $\phi$ . The fourth column reports the proportion of variance in the residual term accounted for by variation in  $\Delta \log \lambda_t^j$ ; the final column indicates the proportion of observations with zero recorded for the expenditure category.

Food category	HHSize	Rural	$\phi\beta_i$	$R^2$	% Zeros
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Table 4: Estimated parameters of the demand system for “slightly aggregated” food items. The first two columns report the values of  $\hat{\delta}_i$  associated with household characteristics. The third column reports estimates of the elasticity of marginal utility, up to a common factor  $\phi$ . The fourth column reports the proportion of variance in the residual term accounted for by variation in  $\Delta \log \lambda_t^j$ ; the final column indicates the proportion of observations with zero recorded for the expenditure category.

Food category	HHSize	Rural	$\phi\beta_i$	$R^2$	% Zeros
Other foods	0.11	-1.53	-0.15	0.00	0.92
Restaurant meals	1.11	-0.10	-0.11	0.00	0.83
Infant formula	-17.86	543.66	-0.00	0.00	1.00
Tobacco	1.26	-10.27	0.12	0.00	0.83
Coffee	0.63	6.27	0.17	0.01	0.93
Peas	0.63	-5.24	0.24	0.01	0.91
Sorghum	0.40	7.28	0.28	0.01	0.86
Sim sim	0.46	6.14	0.31	0.02	0.89
Millet	0.44	5.03	0.53	0.03	0.84
Irish Potatoes	0.21	-2.05	0.62	0.06	0.88
Eggs	0.05	-0.97	0.77	0.07	0.87
Salt	0.43	0.36	0.86	0.18	0.08
Beans	0.49	0.72	1.12	0.11	0.25
Drinks	0.15	-0.96	1.13	0.08	0.67
Cassava	0.27	3.79	1.20	0.10	0.42
Sweet potatoes	0.42	4.45	1.21	0.09	0.45
Vegetables	0.38	-0.14	1.23	0.24	0.10
Bread	0.06	-2.01	1.27	0.16	0.78
Matoke	0.34	-3.14	1.27	0.13	0.53

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Table 4: Estimated parameters of the demand system for “slightly aggregated” food items. The first two columns report the values of  $\hat{\delta}_i$  associated with household characteristics. The third column reports estimates of the elasticity of marginal utility, up to a common factor  $\phi$ . The fourth column reports the proportion of variance in the residual term accounted for by variation in  $\Delta \log \lambda_t^j$ ; the final column indicates the proportion of observations with zero recorded for the expenditure category.

Food category	HHSize	Rural	$\phi\beta_i$	$R^2$	% Zeros
Rice	0.31	0.07	1.30	0.15	0.74
Fish	0.22	-0.05	1.36	0.13	0.60
Maize	0.28	-3.51	1.40	0.12	0.38
Ground nut	0.22	1.40	1.41	0.12	0.58
Fresh milk	0.19	0.84	1.47	0.16	0.65
Tea	0.17	1.12	1.57	0.23	0.39
Sugar	0.25	0.19	1.97	0.31	0.36
Fruits	0.22	1.53	2.00	0.23	0.58
Oils	0.27	-0.13	2.03	0.31	0.37
Meat	0.09	-1.09	2.41	0.33	0.57

This aggregation helps with the problem of “zeros” to a considerable extent; now one only has to go to the eleventh good reported in Table 4 to find one with proportions of zeros less than 80% (salt). And the ordering of goods by elasticity seems quite consistent with the ordering using the 61-good system of all food expenditures; as before, meat, oils, fruit, sugar, and tea are among the most elastic goods. Less optimistically, it’s still the case that even with this aggregation there are a great number of zeros—more than half of all households report no expenditures on any kind of meat, for example. Knowing whether this is accurate or evidence of measurement error requires evidence outside of the dataset, but it seems possible that some of these zeros may be due to respondent (or enumerator) fatigue. Proposing to collect data on future expenditures only on the eighteen slightly aggregated categories (salt through meat) would involve inquiring about 47 distinct expenditure categories but with accompanying instructions which reduced the proportion of recorded zeros might be wise (and further reductions in the number of categories are surely possible if the proportion of zeros can be reduced).

However, perhaps the most critical question is whether this aggregation harms our estimates of household marginal utility. The answer illustrates our last point. If we eliminate those expenditure categories for which more than eighty percent of observations are zero, we're left with 18 of 29 categories; the correlation between estimates of changes in marginal utility for this 18 good category with the 29 good category is 0.9964, so there's essentially no loss here. Again, asking about this relatively limited set of expenditure categories while eliciting fewer zeros might be a worthwhile trade-off.

**A.2. Distribution of Changes in Marginal Utility.** Figure 11 illustrates the relationship between our estimated changes in (minus) log marginal utility and changes in aggregate expenditures. Though there's obviously a strong positive relationship, it's also apparent that the relationship is considerably less than perfect.

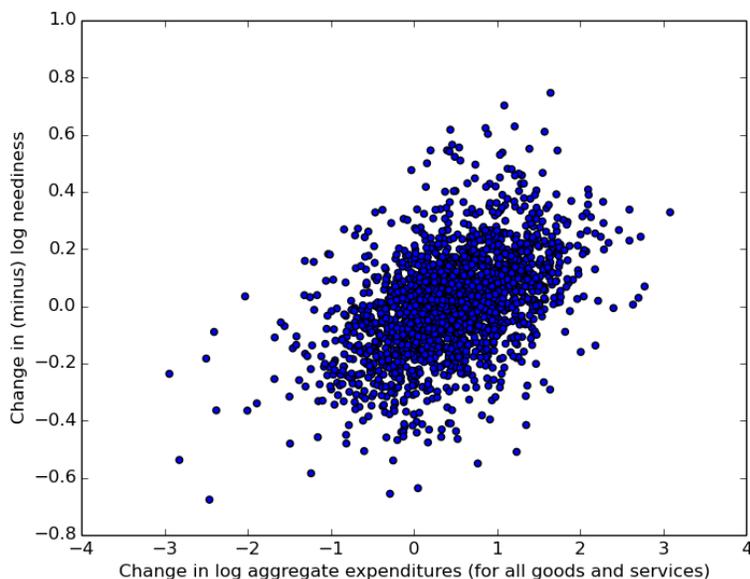


FIGURE 11. Relationship between changes in (the logarithm of) a consumption aggregate and changes in estimated marginal utility, using estimates from slightly aggregated foods.

In this appendix we have discussed a variety of different approaches to estimating (changes in log) marginal utility. Table 5 reports on the relationship between these different approaches, by reporting correlations between them. The first column (and row) use as a measure

TABLE 5. Correlations between estimates of changes in marginal utility using different demand systems. Pearson correlation coefficients are below the diagonal; Spearman correlation coefficients above. Diagonal elements are estimates of the proportion of total residual variation accounted for by variation in marginal utility. “Agg. Exp.” is aggregate expenditures (across all food and nondurable goods and services); remaining columns are estimated  $\Delta \log \lambda_t^j$ . “All” uses the 99 good demand system of all food and nondurables; “All Food” uses all 61 food expenditure categories; “S.A. Food” is the ‘slightly aggregated’ demand system of 29 categories. Numbers in parentheses indicate that categories are only included if the proportion of zeros is less than that number.

	Agg. Exp.	All	All Food	All Food (0.8)	All Food (0.5)	S.A. Food	S.A. Food (0.8)
Agg. Exp.	—	-0.490	-0.481	-0.441	-0.334	-0.508	-0.504
All	-0.489	0.074	0.937	0.906	0.804	0.868	0.862
All Food	-0.471	0.945	0.095	0.971	0.852	0.921	0.915
All Food (0.8)	-0.432	0.921	0.978	0.150	0.901	0.870	0.876
All Food (0.5)	-0.343	0.837	0.883	0.925	0.280	0.718	0.725
S.A. Food	-0.500	0.897	0.942	0.904	0.774	0.142	0.995
S.A. Food (0.8)	-0.497	0.893	0.938	0.909	0.782	0.996	0.174

changes in log total expenditures; this is the “consumption aggregate” approach typically used for measuring poverty at the World Bank. The remaining columns (and rows) use measures of  $\Delta \log \lambda_t^j$  estimated using the different demand systems described above. “All” is the 99 good system of foods, non-durables and services; “All Food” is the 61 good system of food (and tobacco) expenditures; “S.A. Food” is the “slightly aggregated” system of 29 different food expenditure categories. Where a number appears in parentheses the demand system has been reduced by eliminating any goods for which the proportion of zeros exceeds the parenthetical number; for example, “All Food (0.8)” excludes the food expenditure categories for which fewer than 20% of observations have a positive value.

The table has three parts. Statistics below the diagonal are Pearson correlation coefficients, while statistics above the diagonal are Spearman rank correlation coefficients. These two different measures of correlation are generally in fairly close agreement, reflecting the roughly log-normal distribution of estimated  $\Delta \log \lambda_t^j$  noted in our discussion

of Figure 7. Along the diagonal is a measure of the overall ability of the demand system to fit the data: the statistics reported here use the singular values obtained when we computed the  $\beta_i$  and  $\Delta \log \lambda_i^j$ , and are the ratio of the square of the first singular value to the sum of the squares of all the singular values. This ratio can be interpreted as the proportion of total residual variance accounted for by variation in household marginal utility.

Increases in log aggregate expenditures are negatively correlated with changes in log marginal utility, as we'd expect, though the correlation is far from perfect (echoing the lesson of Figure 11), taking values of at most (minus) 0.50. It's not at all clear that it *should* be perfect—even in the complete absence of measurement error marginal utility is generally a highly nonlinear (though monotonic) function of total expenditures. Neither is it clear whether aggregate expenditures or estimates of marginal utility are more affected by measurement error. Measures of aggregate expenditures will be more sensitive to the problem of many expenditure categories having zero expenditures for a large proportion of observations than will our marginal utility estimates, since our estimation approach tends to assign low weights (in the form of the estimated  $\beta_i$ ) to goods with large proportions of zeros.