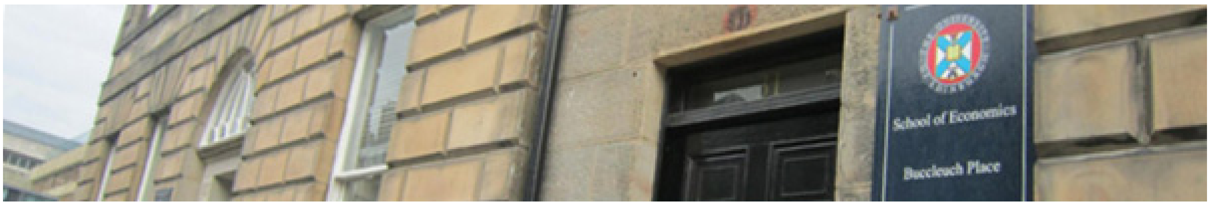

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The Costs of Mismatch

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This version November 2020

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Abstract

I calibrate a model of vertical skill mismatch to US data and demonstrate that both high-skilled workers and low-skilled workers prefer mismatch to segmentation in a decentralised environment. Using this framework, I provide estimates of the output costs of skill-mismatch with reference to a natural benchmark - a labour market in which search is fully segmented by skill. Surprisingly, I find that, despite misallocation (due to high-skilled workers undertaking low-complexity tasks), mismatch *raises* net output by around 7.5%. I show that mismatch is particularly beneficial for low-skilled workers and argue that a key mechanism underlying these net benefits is the (endogenous) response of job creation to the expanded pool of searchers under mismatch. These results call into question the view that mismatch should be seen as a form of misallocation that has deleterious effects on productivity.

1 Introduction.

Skill mismatch is pervasive and persistent. Figure (1), based on data presented in Slominczyk (2013) documents evidence of extensive, persistent over-skilled mismatch (high-skilled workers undertaking low-complexity jobs) and persistent under-skilled mismatch (low-skilled workers undertaking high-complexity activities) in US data.[1] Interest in skill-mismatch stems from concerns about equity and efficiency in a labour market in which workers can be ranked by skill and jobs by complexity. The equity issue is that particular groups of workers may be disadvantaged both during search, by the presence of differently-skilled workers searching for the same job, and as a result of search, when high skilled workers occupy vacancies for which low-skilled workers have a comparative advantage. The efficiency issue is that, because skill mismatch involves putting round pegs in square holes, it may act as a source of misallocation that may reduce aggregate productivity, although any costs of mismatch must be limited by the option to continue to search.

In this paper I quantify the costs and benefits of mismatch and analyse the sources of those costs and benefits. I provide a model of endogenous mismatch in which the sources of mismatch can be identified and calibrate the steady state of a model of skill mismatch to long-run averages in the US. I show that, in a decentralised equilibrium, workers prefer mismatch to segmentation. I quantify the output costs of skill-mismatch with reference to a labour market fully segmented by skill and use the structural model to pinpoint the origins of these costs and benefits. I find that, notwithstanding innefficiencies due to misallocation, net output is around 7.5% higher under mismatch. I show the net benefits of mismatch are primarily due to increased (endogenous) creation of low complexity jobs in the mismatch equilibrium.

Figure 1 here

[1] Slonimczyk (2013) constructs measures of skill mismatch for males and female workers inb the US economy from 1973-2002. Allowing for the gender composition of the labour force and unemployment I find that overskilled mismatch rose from around 15% to 33%, while underskilled mismatch declined from around 15% to around 10%.

Estimates of the costs of skill-mismatch inevitably involve taking a position on the structure of the labour market, and are thus model-specific. Several approaches to modelling mismatch have been introduced in the literature, which differ, inter alia, in the granularity of the treatment of worker and firm heterogeneity and the nature of the matching technology. One body of work on skill-mismatch uses the theoretical frameworks developed by Shimer and Smith (2000) and Eeckhout and Kircher (2011) to identify the nature and extent of sorting in the labour market across a continuum of skill types. A separate strand of work on mismatch builds follows Albrecht and Vroman (2002) who extend the canonical Diamond-Mortensen-Pissarides (DMP) model to a setting in which over-skilled mismatch can arise endogenously across a discrete set of job and worker types.[2,3] The latter offers a coarser treatment of heterogeneity. This may lead one to question whether it has sufficient granularity and flexibility to capture the nature of mismatch present in the data. However, by virtue of its relative simplicity and similarity to the standard DMP framework and to the framework used to study the skill-specific inequality (Acemoglu and Autor (2011)), it can draw upon a large body of results at the aggregate level to discipline the empirical analysis and may readily be applied to extend analysis of macroeconomic phenomena to include the effects of mismatch. Here I focus on the limited heterogeneity offered by discrete types of workers and jobs.

For the purposes of understanding the costs of mismatch with limited hetereogeneity, the Albrecht and Vroman framework is problematic in several respects. Firstly, it captures only the effect of over-skilled mis-

[2]Their work demonstrates a possible role for over-skilled mismatch in the analysis of the consequences of skill-biased technical change and relative supply of skills for inequality, Albrecht and Vroman (2002). Dolado, Jimenez and Jansen (2008) show that many of these effects are augmented in the presence of on the job search. Chassambouli (2011) and Khalifa (2009) highlight the role that mismatch may play in accounting for fluctuations of unemployment among particular skill groups. Davidson, Matusz and Shevchenko (2008) explore the implications for labour market mismatch of trade liberalisation and offshoring.

[3]Shi (2002) consider a 2-skill-type, 2-job-type (static) model with wage-posting and directed search. This delivers an feasible equilibria with either under-skilled mismatch (only) or segmentation. He does not calibrate his model to the data or consider the costs of mismatch.

match, whereas the data presented in Figure (1) clearly exhibit sustained under-skilled mismatch. This matters because neglecting under-skilled mismatch may bias estimates of the misallocation costs of mismatch upwards by accounting only for the lost potential output associated with over-skilled mismatch while omitting potential gains from under-skilled mismatch. Secondly, the 'ex-post' segmentation that Albrecht and Vroman propose does not mirror the type implicit in the literature on labour market inequality, see Acemoglu and Autor (2011). This confounds comparison with that literature. In particular, Albrecht and Vroman require that *both* types of workers search in the same pool for *each* type of job even when they segment. So, high-skilled individuals – who (will) refuse to mismatch with low complexity jobs - continue to search in a pool containing these jobs while firms with low-complexity jobs knowingly forgo opportunities to generate profits offered by searching in a pool containing high-skilled workers - who will reject them and delay the formation of feasible matches with low-skilled workers - instead of restricting search to low-skilled individuals.[4] Requiring workers and firms to search in a common pool in this way is tantamount to saying that it is impossible for workers to reveal their level of skill (often captured as education level). Similarly, the common search pool assumption implies that it is impossible for firms to reveal the complexity of the vacancy that they have opened – even though all parties would benefit from overcoming this informational friction and allowing ex ante, rather than ex post segmentation. A level of information friction this extensive appears implausible. Turning to methodological concerns, the Albrecht-Vroman literature can be seen as an extension of the DMP model, but all existing empirical work - see footnote 3 - fails to impose restrictions that must hold for consistency with the aggregate DMP set up. Finally, many studies simply take the mismatch equilibrium to the data, without recognising that other equilibria could arise.

To address these conceptual and methodological issues of the exist-

[4]Similarly, low skilled workers search in a pool containing high complexity vacancies which they cannot undertake, while firms with high complexity vacancies search in the common pool of workers, despite the fact that they will never hire low skilled workers.

ing literature I introduce a new model of a labour market with random search, matching frictions and discrete heterogeneity, in which (**i**) workers differ by skill and jobs differ by complexity, (**ii**) mismatch arises endogenously and may be under-skilled, over-skilled or both simulataneously (as in the data), (**iii**) segmentation, if it arises, occurs 'ex ante', when workers choose which type of job to search for, (**iv**) the model otherwise conforms to the DMP framework. I develop a model of endogenous mismatch in a heterogeneous agent DMP environment and calibrate the steady state of the mismatch equilibrium in this model to US data. Mindful of the controversies over the calibration of the aggregate DMP model, I calibrate to observable characteristics of the heterogeneous agent model and in a manner consistent with the aggregate data. I use observables to pin down unobservable/controversial parameters, such as the relative productivity of different worker types in each job type. To assess the empirical relevance of the mismatch equilibrium that arises in this framework I examine whether given the data, some or all workers would prefer an equilibrium with some segmentation (in which case, either the equilibrium with (only) under-skilled mismatch, or that with (only) over-skilled mismatch, or that with full segmentation would arise). I show that mismatch would be supported in the decentralised equilibrium.

Next, I analyse the costs of mismatch implied by the calibrated model, using a measure of aggregate output net of the costs of vacancy creation. Since segmentation (with or without matching frictions) is routinely imposed in the literature on inequality, it appears that a natural benchmark against which to compare the mismatch equilibrium is the equilibrium with ex ante segmentation. Using the parameters which arise from calibrating the model with mismatch, I show that aggregate net output is about 7.5% *higher* under mismatch than under segmentation. This result is something of a surprise. The natural preconception, at the heart of concerns over mismatch, is that mismatch reduces efficiency as high-skilled workers undertake low-complexity jobs (although this effect may be offset to the extent that low-skilled workers undertake high-complexity jobs). This preconception is false. In the calibrated model I proceed to identify the source of the benefits of mismatch by decomposing it into (i) a *compositional* effect holding employment constant (which highlights the role of misallocation) and (ii) an *aggregate activity* effect (which highlights the role of changes in employment). For each component I analyse the impact of mismatch on output and costs of vacancy creation. I find that the compositional effect accounts for the bulk of the benefits created by mismatch. Mismatch reduces the prevalence of unemployment among the low skilled, and much of the benefit of mismatch arises because of a relative decline in the fraction of high complexity vacancies, so the endogenous nature of job creation underlies the benefits of mismatch.

In the related literature, progress has been made in demonstrating the empirical relevance of mismatch, and identifying situations in which mismatch plays an important role in accounting for observed phenomena,[5] but few studies address these efficiency issues and those that do have a different focus from the quantitative standpoint that I adopt below. In the discrete skill setting, the closest study is that of Blasquez and Jansen (2008) who explore the efficiency of the equilibria generated in the Albrecht-Vroman setting. While the Hosios condition ensures that the outcomes of the decentralised economy and the social planner's problem are equivalent in the aggregate DMP setting, Blasquez and Jansen show that the Hosios condition does not hold in the Albrecht-Vroman setting. They characterise and simulate the solution to the social planner's problem, but do not work with a model that is calibrated to the data and so can offer no empirical assessment of the costs of mismatch in the decentralised environment. In the continuous skill-setting, Gautier and Tjeulings (2015) use US data to explore the costs of search frictions, Bombardini, Orefice and Toti (2015) obtain estimates of the costs of search from French matched employer-employee data using the Eeckhout-Kircher framework. Unlike these studies I use the frictional equilibrium with ex ante segmentation as a reference point; this focusses

[5] Several of contributions in both the Shimer-Smith framework and the Albrecht-Vroman framework noting the role of mismatch in trade and reallocation and economics fluctuations.

attention on the role of mismatch in generating misallocation, rather than focussing on the overal costs of search frictions. Nevertheless, these studies are useful in assessing the estimates of the cost of mismatch that I obtain.

In the next section, I outline the theoretical model and characterise the equilibrium with both over-skilled and under-skilled mismatch. Section 3 outlines the calibration strategy in detail and explores key aspects of this model with mismatch as a description of the US labour market over the period 1973-2002. In Section 4, I characterise an alternative decentralised equilibrium featuring segmentation only. I show that the equilibrium featuring skill mismatch would arise in a decentralised equilibrium. In Section 5 I provide estimates of the costs of mismatch and analyse the determinants of mismatch by decomposing the effects of mismatch into changes in the composition of output (holding employment constant) and the impact of allowing aggregate activity to change. Section 6 concludes and offers suggestions for further work.

2 A Model of Endogenous Vertical-Skill Mismatch

I outline a heterogeneous agent version of a Diamond-Mortensen-Pissarides model in which workers differ by skill, jobs differ by complexity, and firms face fixed costs of vacancy creation. I focus on the simplest case, in which workers have one of two skill-levels and jobs have one of two complexities. In this environment, equilibria featuring various combinations of over-skilled mismatch, under-skilled mismatch and segmentation arise endogenously. I consider only the steady state. In this Section and the next I explore an equilibrium in which there is both underskilled and overskilled mismatch.

2.1 Setup

I consider a continuous-time model with infinitely-lived, risk-neutral agents, who discount the future at the interest rate r. Workers are characterised by the skill, $i \in \{L := low, H := high\}$. A job is characterised by its complexity, $i \in \{L := low, H := high\}$. Jobs are either vacant or filled. One unit of match-specific intermediate output, $Y_{i,j}$, is produced per unit time by an i-skilled worker matched with a vacancy for a j-complexity job. Productivity differences between matches are modelled as differences in the efficiency, $A_{i,j}$, with which match-specific intermediate outputs, $Y_{i,j}$, can be combined to produce final output: $Y = \sum_{i,j} A_{i,j} \cdot Y_{i,j}$.

When creating a vacancy, a firm chooses the complexity to maximise the value of that vacancy. The choice of complexity entails a one off cost, which can be thought of as the cost of adopting a skill-specific production technology. Without such a fixed cost, mismatch could not arise as each firm would find it profitable to adjust the complexity of its existing vacancy to suit the skill of the first worker that is encountered through search. The number of vacancies at each complexity is determined by a free entry condition, so that new vacancies with complexity $j \in \{L, H\}$ continue to be created until the value of such vacancies, $\mathcal{V}_j$ equals, K_j, the fixed cost of adopting the technology associated with the complexity-j vacancy. It is assumed that it will be more costly to adopt the technology for a high-complexity job than for a low-complexity job, that is $K_H > K_L$.

2.2 Aggregate and Skill-Specific Labour Markets

Define the total mass of workers as N and suppose that N_L are low-skilled and N_H are high-skilled. Let the fraction of low-skilled workers be $\mu_L = \mu$ then $N_L = \mu_L \cdot N = \mu \cdot N$ and $N_H = \mu_H . N = [1 - \mu] \cdot N$. Let the mass of unemployed workers as U. Suppose that there are U_L low-skilled unemployed, and U_H high-skilled unemployed. Let the fraction of unemployed workers who are low skilled be $\gamma_L = \gamma$, so $U_H = \gamma_H \cdot U = [1 - \gamma] \cdot U$, and

$$U_L = \gamma \cdot U. \tag{1}$$

Each i-skilled unemployed worker can choose whether to search for a job with complexity i or one with requirement j. On the job search is ruled out.[6] Suppose that a fraction $\lambda_{ij} \in [0, 1]$ of unemployed i-skilled

[6] This is likely to make an equilibrium with mismatch less attractive (relative to segmentation), since on-the-job-search typically increases the value to a high-skilled worker of engaging in mismatch (by providing an alternative, and more highly

workers choose to search for complexity-j jobs, then $\lambda_{HL} = 1 - \lambda_{HH}$ and $\lambda_{LH} = 1 - \lambda_{LL}$. An equilibrium in which workers segment ex ante and search for (own) skill-specific vacancies requires $\lambda_{LL} = \lambda_{HH} = 1$. If the equilibrium features under-skilled mismatch then $\lambda_{LL} < 1$; if over-skilled mismatch is present then $\lambda_{HH} < 1$. If both types of mismatch are present (as in US data) then $\lambda_{LL}, \lambda_{HH} < 1$.

The pool of workers searching for high complexity jobs is $[1 - \lambda_{LL}] \cdot U_L + \lambda_{HH} \cdot U_H$, while $\lambda_{LL} \cdot U_L + [1 - \lambda_{HH}] \cdot U_H$ workers search for low complexity jobs. If there are V_L vacancies for low complexity jobs, then define the tightness of the sub-market for low complexity jobs as $\theta_L = \frac{V_L}{\lambda_{LL} \cdot U_L + [1-\lambda_{HH}] \cdot U_H}$. Similarly tightness in the sub-market for high complexity jobs is $\theta_H = \frac{V_H}{[1-\lambda_{LL}] \cdot U_L + \lambda_{HH} \cdot U_H}$. Then aggregate tightness, θ, can be written as $\theta = \frac{V}{U} = \frac{V_L + V_H}{U} = \frac{V_L}{\lambda_{LL} \cdot U_L + [1-\lambda_{HH}] \cdot U_H} \frac{\lambda_{LL} \cdot U_L + [1-\lambda_{HH}] \cdot U_H}{U} + \frac{V_H}{[1-\lambda_{LL}] \cdot U_L + \lambda_{HH} \cdot U_H} \frac{[1-\lambda_{LL}] \cdot U_L + \lambda_{HH} \cdot U_H}{U}$. So

$$\theta = \theta_L \cdot [\lambda_{LL} \cdot \gamma + [1 - \lambda_{HH}] \cdot [1 - \gamma]] + \theta_H \cdot [[1 - \lambda_{LL}] \cdot \gamma + \lambda_{HH} \cdot [1 - \gamma]] . \tag{2}$$

Equation (2) highlights that any equilibrium, whether involving mismatch or segmentation, has implications for and must be consistent with aggregate labour market behaviour. It says that aggregate tightness, θ, is a weighted average of the tightness in each sub-market, θ_j, where the weights represent the fraction of unemployed workers searching in each sub-market, e.g. $[\lambda_{LL} \cdot \gamma + [1 - \lambda_{HH}] \cdot [1 - \gamma]]$ equals the product of the fraction of low-skilled workers who search for low-complexity jobs, λ_{LL}, and the fraction of unemployed workers who are low skilled, γ, plus the product of the fraction of high-skilled workers who search for low-complexity jobs, $1 - \lambda_{HH}$, and the fraction of unemployed workers who are high-skilled, $1 - \gamma$. If the equilibrium features segmentation then $\lambda_{LL} = \lambda_{HH} = 1$ and (2) becomes $\theta = \gamma \theta_L + [1 - \gamma] \theta_H$.

Matching for each type of vacancy, $j \in \{L, H\}$, occurs through a random matching technology, described by the matching functions $\mathcal{M}_j (U_j, V_j)$. These matching technologies are assumed to be linearly homogeneous, so that $\mathcal{M}_j (U_j, V_j) = V_j \cdot \mathcal{M}_j \left(\frac{1}{\theta_j}, 1 \right) = V_j \cdot q_j (\theta_j)$, where

remunerated, route to a high-skill-requirement position).

$\frac{\partial q_j}{\partial \theta_j} < 0$. Vacancies for complexity-j jobs, meet workers at the rate $q_j(\theta_j)$ per unit time. The rate at which unemployed workers searching for complexity j jobs meet vacancies is $\theta_j q_j(\theta_j)$ per unit time. It follows that the total number of workers exiting unemployment for low complexity jobs in the interval dt is $\theta_L \cdot q_L(\theta_L) \cdot [\lambda_{LL} \cdot U_L + [1 - \lambda_{HH}] \cdot U_H] \cdot dt$ and the figure for high complexiyt jobs is $\theta_H \cdot q_H(\theta_H) \cdot [[1 - \lambda_{LL}] \cdot U_L + \lambda_{HH} \cdot U_H] \cdot dt$. Denote the aggregate outflow from unemployment as $\theta \cdot q(\theta) \cdot U$, where θ is aggregate tightness and $q(\cdot)$ represents the standard aggregate matching function, then consistency with aggregate labour market flows requires

$$\theta \cdot q(\theta) \cdot U = \begin{array}{c} \theta_L \cdot q_L(\theta_L) \cdot [\lambda_{LL} \cdot U_L + [1 - \lambda_{HH}] \cdot U_H] + \\ \theta_H \cdot q_H(\theta_H) \cdot [[1 - \lambda_{LL}] \cdot U_L + \lambda_{HH} \cdot U_H]. \end{array}$$

Since $U_L = \gamma \cdot U$ and $U_H = [1 - \gamma] \cdot U$, this can be simplified to give the following relation between the aggregate exit rate and the exit rate for each sub-market:

$$\theta \cdot q(\theta) = \begin{array}{c} \theta_L \cdot q_L(\theta_L) \cdot [\lambda_{LL} \cdot \gamma + [1 - \lambda_{HH}] \cdot [1 - \gamma]] + \\ \theta_H \cdot q_H(\theta_H) \cdot [[1 - \lambda_{LL}] \cdot \gamma + \lambda_{HH} \cdot [1 - \gamma]] \end{array} \tag{3}$$

In an equilibrium with segmentation (3) becomes $\theta \cdot q(\theta) = \gamma \cdot \theta_L \cdot q_L(\theta_L) + [1 - \gamma] \cdot \theta_H \cdot q_H(\theta_H)$.

Besides the requirement that ties aggregate unemployment outflows to a weighted average of skill-specific outflows, a necessary feature of the steady state is that unemployment outflows and inflows must balance for separately for low-skilled workers and for high-skilled workers. To determine the mass balance equations for each skill type, I take a position on the nature of separation decisions. In general, separation rates might be skill-, job- or match-specific, and might be endogenous. I assume that separation rates are exogenous. Driven partly by a desire to reduce the number of parameters that have to be identified, I allow separation rates to be skill-specific, but not job-specific (and hence not match-specific). Let δ_L denote the separation rate for low-skilled workers and δ_H the separation rate for high-skilled workers. Empirical evidence suggests that $\delta_L > \delta_H$, Fallick and Fleischmann (2001). Then the inflow of low-skilled workers to unemployment over the time interval dt is $\delta_L [\cdot N_L - U_L] \cdot dt$. The number of low-skilled workers who leave unemployment in this interval depends both on the exit rate from each job pool, $\theta_j q_j(\theta_j)$, and on the fraction of low-skilled workers who search in each pool, λ_{Lj}. It takes the form $\lambda_{LL} \cdot U_L \cdot \theta_L \cdot q_L(\theta_L) \cdot dt + \lambda_{HL} \cdot U_L \cdot \theta_H \cdot q_H(\theta_H) \cdot dt$. Since inflows and outflows of low-skilled workers to and from unemployment must balance it follows that:

$$\lambda_{LL} \cdot U_L \cdot \theta_L \cdot q_L(\theta_L) + \lambda_{LH} \cdot U_L \cdot \theta_H \cdot q_H(\theta_H) = \delta_L \cdot [N_L - U_L].$$

$$\therefore \quad [\lambda_{LL} \cdot \theta_L \cdot q_L(\theta_L) + [1 - \lambda_{LL}] \cdot \theta_H \cdot q_H(\theta_H)] \cdot \gamma \cdot u = \delta_L \cdot [\mu - \gamma \cdot u]. \tag{4}$$

where $u = \frac{U}{N}$ is the aggregate unemployment rate. Similarly, the mass balance equation for high-skilled workers takes the form

$$[[1 - \lambda_{HH}] \cdot \theta_L \cdot q_L(\theta_L) + \lambda_{HH} \cdot \theta_H \cdot q_H(\theta_H)] \cdot [1 - \gamma] \cdot u = \delta_H \cdot [1 - \mu - [1 - \gamma] \cdot u]. \tag{5}$$

If the equilibrium features complete segmentation then equations (4) and (5) reduce to $\theta_L \cdot q_L(\theta_L) \cdot \gamma \cdot u = \delta_L \cdot [\mu - \gamma \cdot u]$ and $\theta_H \cdot q_H(\theta_H) \cdot [1 - \gamma] \cdot u = \delta_H \cdot [1 - \mu - [1 - \gamma] \cdot u]$, respectively. Consistency with aggregate labour market dynamics requires

$$\theta \cdot q(\theta) \cdot u = \delta \cdot [1 - u] \tag{6}$$

so, regardless of whether the equilibrium features mismatch or segmentation, $\delta \cdot [1 - u] = \delta_L \cdot [\mu - \gamma \cdot u] + \delta_H \cdot [1 - \mu - [1 - \gamma] \cdot u]$.

To model job creation and mismatch I need to determine the fraction of individuals in the pool of workers searching for j-complexity jobs, who are of skill i. Denote this by $\phi_{i,j}$, where $\phi_{i,j} = \frac{\lambda_{ij} \cdot U_i}{\lambda_{ij} \cdot U_i + \lambda_{i'j} \cdot U_{i'}}$, and $i' \neq i$. So, for example, the fraction of workers in the pool of individuals searching for low-complexity jobs who are low-skilled, is

$$\phi_{LL} = \frac{\lambda_{LL} \cdot U_L}{\lambda_{LL} \cdot U_L + \lambda_{HL} \cdot U_H} = \frac{\gamma \cdot \lambda_{LL}}{\gamma \cdot \lambda_{LL} + [1 - \gamma] \cdot [1 - \lambda_{HH}]}, \tag{7}$$

while the fraction who are high-skilled is $\phi_{HL} = 1 - \phi_{LL}$. Similarly, the fraction of workers in the pool of individuals searching for high-complexity jobs who are high-skilled, is

$$\phi_{HH} = \frac{\lambda_{HH} \cdot U_H}{\lambda_{LH} \cdot U_L + \lambda_{HH} \cdot U_H} = \frac{[1-\gamma] \cdot \lambda_{HH}}{\gamma \cdot [1-\lambda_{LL}] + [1-\gamma] \cdot \lambda_{HH}}, \tag{8}$$

and the fraction who are low-skilled is $\phi_{LH} = 1 - \phi_{HH}$. In an equilibrium with segmentation $\lambda_{LL} = \lambda_{HH} = 1$, so $\phi_{LL} = \phi_{HH} = 1$ and $\phi_{LH} = \phi_{HL} = 0$.

Finally, I determine the extent of each type of mismatch and the skill premium. Suppose that i-skilled workers in j-skill- requirement jobs earn the wage w_{ij}. The wage premium of skilled workers over unskilled workers will reflect the average wage paid to skilled workers divided by the average wage paid to unskilled workers. In general, the average wage paid to a particular skill type of workers may be a rather complex object, since the composition of jobs may vary over time due to differences in unemployment exit rates by skill and job, separation rates across workers, jobs and matches, and out-of-steady state dynamics. However, by focussing on the steady state, and assuming that exit rates from employment are skill-specific, but independent of the complexity of jobs, the skill-composition of employment reflects only exit rates from unemployment to jobs of each complexity. Then the fraction of low-skilled workers, and high-skilled workers respectively who are *not* mismatched, is

$$s_{LL} = \frac{\lambda_{LL} \cdot \theta_L \cdot q_L(\theta_L)}{[1-\lambda_{LL}] \cdot \theta_H \cdot q_H(\theta_H) + \lambda_{LL} \cdot \theta_L \cdot q_L(\theta_L)} \tag{9}$$

$$s_{HH} = \frac{\lambda_{HH} \cdot \theta_H \cdot q_H(\theta_H)}{[1-\lambda_{HH}] \cdot \theta_L \cdot q_L(\theta_L) + \lambda_{HH} \cdot \theta_H \cdot q_H(\theta_H)} \tag{10}$$

Under-skilled mismatch is then $s_{LH} = 1 - s_{HH}$, while over-skilled mismatch is $s_{HL} = 1 - s_{LL}$, and the wage premium, χ, under mismatch is

$$\chi = \frac{s_{HH}.w_{HH} + s_{HL} \cdot w_{HL}}{s_{LL} \cdot w_{LL} + s_{LH} \cdot w_{LH}} = \frac{s_{HH} \cdot w_{HH} + [1-s_{HH}] \cdot w_{HL}}{s_{LL} \cdot w_{LL} + [1-s_{LL}] \cdot w_{LH}} \tag{11}$$

If there is complete segmentation then $s_{LL} = s_{HH} = 1$ and the wage premium $\chi = \frac{w_{HH}}{w_{LL}}$.

2.3 Production

The output of final good per worker, $y = \frac{Y}{N}$ is produced by combining match-specific outputs according to the production function[7]

$$y = A_{HH} \cdot y_{HH} + A_{LH} \cdot y_{LH} + A_{HL} \cdot y_{HL} + A_{LL} \cdot y_{LL}, \tag{12}$$

where $A_{i,j}$, $i, j \in \{L, H\}$ represent the productivity in final output of a good produced by an i-skilled worker in a j-complexity job and $y_{i,j} = s_{ij} \cdot [\mu_j - \gamma_j \cdot u]$ represents both units of output of a match between an i-skilled worker and complexity j job and the fraction of the workforce employed in $i - j$ matches. The profit maximisation problem for price-taking final goods producers is: $\max_{y_{ij};i,j\in\{L,H\}} y - P_{HH} \cdot y_{HH} + P_{LH} \cdot y_{LH} + P_{HL} \cdot y_{HL} + P_{LL} \cdot y_{LL}$, where the price, P, of final goods is normalised to 1. So, for any worker-i firm-j match $i, j \in \{L, H\}$ which engages in production I have

$$A_{ij} = P_{ij}. \tag{13}$$

In an equilibrium with complete segmentation $y = A_{HH} \cdot y_{HH} + A_{LL} \cdot y_{LL}$

2.4 Value Functions and Wages

In this Section I discuss the value functions associated with employment and unemployment and with filled jobs and unfilled vacancies. I also determine wages.

2.4.1 Filled Jobs and Employment

The value function for a complexity-j job filled by an i-skilled worker reflects the sum of the operating profit flow over the interval dt, $[P_{i,j} - w_{i,j}] \cdot dt$ (revenue, $P_{i,j}$, less wages, $w_{i,j}$) and the expected capital loss arising from exogenous separation, which occurs at rate $\delta_i dt$ and leaves the firm with an open vacancy of complexity j whose valiue is $\mathcal{V}_j$:

$$\mathcal{J}_{i,j} = \frac{1}{1 + r\ dt} \cdot [[P_{ij} - w_{i,j}] \cdot dt + \delta_i \cdot \mathcal{V}_j \cdot dt + [1 - \delta_i \cdot dt] \cdot \mathcal{J}_{i,j}]$$

[7] This production function assumes that the output of high-skill requirement and low-skill requirement positions are perfect substitutes in the production of final goods.

The flow value, $r \cdot \mathcal{J}_{i,j}$, is summarised in equation (14),

$$r \cdot \mathcal{J}_{i,j} = P_{ij} - w_{i,j} + \delta_i \cdot [\mathcal{V}_j - \mathcal{J}_{i,j}] \,. \tag{14}$$

For a level i-skilled worker, the value of employment, $\mathcal{E}_{i,j}$, in a job requiring skill j is the discounted value of the after tax wage payment and the capital loss in the event of job loss:

$$\mathcal{E}_{i,j} = \frac{1}{1 + r\ dt} \cdot [[1 - \tau_i] \cdot w_{i,j} \cdot dt + \mathcal{U}_{i,j} \cdot \delta_i \cdot dt + [1 - \delta_i \cdot dt] \cdot \mathcal{E}_{i,j}] \,.$$

where τ_i is the skill-specific tax rate and $\mathcal{U}_{i,j}$ is the value of unemployment for a worker of type i in the pool of workers searching for a type j job. This simplifies to equation (15).

$$r \cdot \mathcal{E}_{i,j} = [1 - \tau_i] \cdot w_{i,j} + \delta_i \cdot [\mathcal{U}_{ij} - \mathcal{E}_{i,j}] \,. \tag{15}$$

2.4.2 Wages

Once formed the surplus resulting from a match between an i-skilled worker and a job with complexity j job is $S_{i,j} \equiv \mathcal{E}_{i,j} + \mathcal{J}_{i,j} - \mathcal{U}_{i,j} - \mathcal{V}_j$. The wage in each match is determined by Nash Bargaining. The wage $w_{i,j}$ for a worker with skill level i in a job with complexity j, is given by condition (16), which apportions to the worker a share β_{ij} of the surplus $S_{i,j}$. If the worker's share / bargaining power is neither job-, nor worker-, nor match-specific, then $\beta_{ij} = \beta$ and

$$\mathcal{E}_{i,j} - \mathcal{U}_{i,j} = \beta \cdot [\mathcal{E}_{i,j} + \mathcal{J}_{i,j} - \mathcal{U}_{i,j} - \mathcal{V}_j] \tag{16}$$

Using (14) and (15) in (16) the after tax wage, $[1 - \tau_i] \cdot w_{ij}$, for an i-skilled worker in a j-complexity job is

$$[1 - \tau_i] \cdot w_{ij} = \beta \cdot [P_{ij} - r \cdot \mathcal{V}_j] + [1 - \beta] \cdot r \cdot \mathcal{U}_{ij} \tag{17}$$

Using these results the capital gain for an i-skilled worker moving from unemployment to employment at a job with complexity j, $\mathcal{E}_{i,j} - \mathcal{U}_{i,j}$, and the capital gain for a firm which fills a complexity-j vacancy with an i-skilled worker, $\mathcal{J}_{i,j} - \mathcal{V}_j$ are:

$$\mathcal{E}_{i,j} - \mathcal{U}_{i,j} = \frac{\beta \cdot [[1-\tau_i] \cdot [P_{ij} - r \cdot \mathcal{V}_j] - r \cdot \mathcal{U}_{ij}]}{r + \delta_i} \text{ and}$$

$$\mathcal{J}_{i,j} - \mathcal{V}_j = [1-\beta] \cdot \frac{P_{ij} - r\mathcal{V}_j - \frac{r \cdot \mathcal{U}_{ij}}{[1-\tau_i]}}{r + \delta_i}.$$

2.4.3 The Value of Unemployment and the Search Decision

Here I discuss the value of unemployment and the unemployed worker's decision over which type of job to search for. For an unemployed i-skilled worker searching for a complexity-j job, the value of unemployment, $\mathcal{U}_{ij}$, is the sum of the flow benefits of unemployment, b, and the expected capital gain from the possibility of forming an employment relationship. This second term depends on the skill-level of the worker, i, the complexity, j, of the job for which he/she searches and the arrival rate, $\phi_{ij} \cdot \theta_j \cdot q_j(\theta_j)$, of complexity-$j$ jobs (which in turn depends on the tightness, θ_j, of the sub-market for complexity-j jobs and, ϕ_{ij}, the fraction of workers earching for jobs of complexity-j who possess skill level i). The value of unemployment takes the form:

$$\mathcal{U}_{ij} = \frac{1}{1 + r\ dt} \cdot \left[b \cdot dt + \mathcal{E}_{ij} \cdot \phi_{ij} \cdot \theta_j \cdot q_j(\theta_j) \cdot dt + \mathcal{U}_{jj} \cdot \left[1 - \phi_{ij} \cdot \theta_j \cdot q_j(\theta_j) \cdot dt\right]\right]$$

Using the earlier results for $\mathcal{E}_{ij} - \mathcal{U}_{ij}$ the value of unemployment for particular skill and complexity combinations are:

$$r \cdot \mathcal{U}_{LL} = \frac{[r+\delta_L] \cdot b + [1-\tau_L] \cdot \beta \cdot \phi_{LL} \cdot \theta_L \cdot q_L(\theta_L) \cdot [P_{LL} - r \cdot \mathcal{V}_L]}{r + \delta_L + \beta \cdot \phi_{LL} \cdot \theta_L \cdot q_L(\theta_L)},$$

$$r \cdot \mathcal{U}_{LH} = \frac{[r+\delta_L] \cdot b + [1-\tau_L] \cdot \beta \cdot [1-\phi_{HH}] \cdot \theta_H \cdot q_H(\theta_H) \cdot [P_{LH} - r \cdot \mathcal{V}_H]}{r + \delta_L + \beta [1-\phi_{HH}] \theta_H q_H(\theta_H)},$$

$$r \cdot \mathcal{U}_{HH} = \frac{[r+\delta_H] \cdot b + [1-\tau_H] \beta \cdot \phi_{HH} \cdot \theta_H \cdot q_H(\theta_H) \cdot [P_{HH} - r \cdot \mathcal{V}_H]}{r + \delta_H + \beta \cdot \phi_{HH} \cdot \theta_H \cdot q_H(\theta_H)},$$

$$r \cdot \mathcal{U}_{HL} = \frac{[r+\delta_H] \cdot b + [1-\tau_H] \cdot \beta \cdot [1-\phi_{LL}] \cdot \theta_L \cdot q_L(\theta_L) \cdot [P_{HL} - r \cdot \mathcal{V}_L]}{r + \delta_H + \beta \cdot [1-\phi_{LL}] \cdot \theta_L \cdot q_L(\theta_L)}.$$

Each equilibrium implies a particular relationships between $\mathcal{U}_{LL}$ and $\mathcal{U}_{LH}$ and between $\mathcal{U}_{HH}$ and $\mathcal{U}_{HL}$.[8] For example, in an equilibrium with under-skilled mismatch, low-skilled workers must be indifferent between searching in each sub-market, which corresponds to the condition:

$$\mathcal{U}_{LL} = \mathcal{U}_{LH} = \mathcal{U}_{L} > b \tag{18}$$

In an equilibrium with over-skilled mismatch, high-skilled workers must be indifferent between searching in each sub-market, which corresponds to the condition:

$$\mathcal{U}_{HL} = \mathcal{U}_{HH} = \mathcal{U}_{H} > b \tag{19}$$

In an equilibrium with both over-skilled mismatch and under-skilled mismatch, both (18) and (19) hold. In an equilibrium with complete segmentation low-skilled workers must prefer to search for low-complexity jobs rather than mismatch, so $\mathcal{U}_{LL} = \mathcal{U}_{L} > \mathcal{U}_{LH} = b$ and high-skilled workers must prefer to search for high-complexity jobs rather than mismatch, so $\mathcal{U}_{HH} = \mathcal{U}_{H} > \mathcal{U}_{HL} = b$.

2.4.4 Vacancy Valuation and Vacancy Creation Decisions

The steady state value of a j-skill- requirement vacancy, $\mathcal{V}_j$ satisfies the following condition

$$r\mathcal{V}_j = q_j(\theta_j)\left[\phi_{i,j}\left[\mathcal{J}_{i,j} - \mathcal{V}_j\right] + \phi_{i',j}\left[\mathcal{J}_{i',j} - \mathcal{V}_j\right]\right]$$

This states that the flow return to a vacancy for a j-complexity job is the expected capital gain from forming a match. This is the weighted value of filling the vacancy, where the weights reflect the arrival rates of i-skilled workers, $\phi_{ij} \cdot q_j(\theta_j)$, and i'-skilled workers, $\phi_{i'j} \cdot q_j(\theta_j)$, respectively. An encounter with a i-skilled worker results in a match which produces a capital gain $J_{ij} - \mathcal{V}_j$, whilst an encounter with a i'-skilled worker results in a match which produces a capital gain $\mathcal{J}_{i',j} - \mathcal{V}_j$.

[8] There are 9 combinations, but here, I focus on the equilibria with both over-skilled and under-skilled mismatch and that with complete segmentation. The former have the greatest relevance from an empirical viewpoint and the latter is the most relevant benchmark from the perspective of the literature.

Notice that I omit the standard flow cost of posting a vacancy (often denoted c).[9] There is limited evidence on the form of these costs, but it is not possible to distinguish between flow costs, c_j, and a one-off cost of vacancy creation, K_j on the basis of the steady state alone. So, since fixed costs of vacancy creation are crucial to the feasibility of a mismatch equilibrium, I focus on K_j alone and suppress flow costs, c_j. Using the expression for $\mathcal{J}_{i,j} - \mathcal{V}_j$ above, it is straightforward to show that for an equilibrium with both under-skilled mismatch and over-skilled mismatch, asses values for low-complexity and high-complexity vacancies satisfy (20) and (21) respectively:

$$r \cdot \mathcal{V}_L = [1-\beta] \cdot q_L(\theta_L) \cdot \begin{bmatrix} \phi_{LL} \cdot \left[\frac{[1-\tau_L]\cdot[P_{LL}-r\cdot\mathcal{V}_L]-b}{[1-\tau_L]\cdot[r+\delta_L+\beta\cdot\phi_{LL}\theta_L\cdot q_L(\theta_L)]}\right] + \\ [1-\phi_{LL}] \cdot \left[\frac{[1-\tau_H]\cdot[P_{HL}-r\cdot\mathcal{V}_L]-b}{[1-\tau_H]\cdot[r+\delta_H+\beta\cdot[1-\phi_{LL}]\cdot\theta_L\cdot q_L(\theta_L)]}\right] \end{bmatrix}, \tag{20}$$

$$r \cdot \mathcal{V}_H = [1-\beta] \cdot q_H(\theta_H) \cdot \begin{bmatrix} [1-\phi_{HH}] \cdot \left[\frac{[1-\tau_L][P_{LH}-r\cdot\mathcal{V}_H]-b}{[1-\tau_L][r+\delta_L+\beta\cdot[1-\phi_{HH}]\cdot\theta_H\cdot q_H(\theta_H)]}\right] \\ +\phi_{HH} \cdot \left[\frac{[1-\tau_H]\cdot[P_{HH}-r\cdot\mathcal{V}_H]-b}{[1-\tau_H]\cdot[r+\delta_H+\beta\cdot\phi_{HH}\cdot\theta_H\cdot q_H(\theta_H)]}\right] \end{bmatrix}. \tag{21}$$

For an equilibrium which features complete segmentation

$$r \cdot \mathcal{V}_L = [1-\beta] \cdot q_L(\theta_L) \cdot \left[\frac{[1-\tau_L]\cdot[P_{LL}-r\cdot\mathcal{V}_L]-b}{[1-\tau_L]\cdot[r+\delta_L+\beta\cdot\theta_L\cdot q_L(\theta_L)]}\right] \tag{22}$$

$$r \cdot \mathcal{V}_H = [1-\beta] \cdot q_H(\theta_H) \left[\frac{[1-\tau_H]\cdot[P_{HH}-r\cdot\mathcal{V}_H]-b}{[1-\tau_H][r+\delta_H+\beta\cdot\theta_H\cdot q_H(\theta_H)]}\right]. \tag{23}$$

In any equilibrium, these asset value of vacancies satisfy the free entry conditions

$$\mathcal{V}_j = K_j, j \in \{L, H\}. \tag{24}$$

Notice that in (20), (21), (22) and (23) in equilibrium the complexity-specific fixed cost of creating a vacancy, K_j, enters each condition twice in its flow equivalent form, $r \cdot K_j$. This structure is formally equivalent to

[9] In the DMP framework an absence of heterogeneity between workers (and jobs) makes the distinction between K_j and c_j irrelevant when considering the steady state. One-off fixed costs of vacancy creation can matter out of steady state. For example, Fujita and Ramey (2007), in a contribution to the controversy over the cyclical properties of the DMP model, calibrate fixed costs of vacancy creation to match the persistence properties of vacancies in a DMP environment, while retaining the traditional flow cost of vacancy creation.

allowing *both* a flow cost $c_j = r \cdot K_j$ of posting a vacancy while it remains unfilled *and* a flow cost of production, $\kappa_j = rK_j$ for each j-complexity match engaged in production.

2.5 Government

The government raises taxes τ_i, on labour income w_{ij}, $i, j \in \{L, H\}$to finance unemployment benefits, b, and runs a balanced budget at all dates:

$$b \cdot u = \sum_{i,j \in \{L,H\}} \tau_i \cdot w_{ij}. \tag{25}$$

3 Calibrated Mismatch Equilibrium

In this Section I outline and implement a strategy to calibrate the steady state of the mismatch equilibrium outlined in Section 2 to match US data and discuss its principal features. I focus on the equilibrium with both over-skilled and under-skilled mismatch because the evidence, e.g. Slomnczyk, supports this. I explore whether (and if so how) other feasible equilibria (such as segmentation) can be ruled out in the next Section.

The model in Section 2 is a heterogeneous agent extension of the canonical DMP model. I adopt a strategy informed by empirical work of Shimer (2005), Hagedorn and Manowksii (2008) and Pissarides (2009) on the canonical DMP model, which (i) respects the relationship between aggregates and skill-specific quanitities and (ii) that makes use of information about observable economic aggregates. The sample period for the calibration is determined by the availability of data on mismatch; Slominczyk's evidence of ongoing over-skilled and under-skilled mismatch in the US is for the period 1973-2002, is the longest consistent series on mismatch, so I calibrate the model to this data. This calibration is used in subsequent sections to demonstrate that the calibrated mismatch equilibrium arises in the decentralised economy and evaluate the costs of mismatch. I calibrate to observable targets at the aggregate and skill-specific level, using 17 equilibrium conditions and accounting identities (1)-(13), (18)-(21) (under the free-entry condition (24)) and (25) to pin down values of the unobserved parameters. The parameters are summarised in Table 1 and the calibrated objects and the equations used to calibrate them are summarised in Table 2.

Tables 1 and 2

I begin by identifying a range of parameters from labour market flows identities.

I first determine some aggregate objects that would be familiar in analysis of the aggregate DMP model. I assume that the length of a period is 1 month. I set the steady state unemployment rate $u = U/N$ as the sample average unemployment rate, $u = 0.052$. Using Shimer's labour flows data, the aggregate separation rate for 1973-2002 is $\delta = 0.037$. Then, using equation (6) the aggregate exit rate is $\theta \cdot q(\theta) = \frac{1-u}{u} \cdot \delta = 0.675$. Following Pissarides (2009), I adopt aggregate labour market tightness: $\theta = 0.72$.

From equation (1), the number of low-skilled unemployed workers is $U_L = \gamma U$. It follows that $\frac{U_L}{N_L} \cdot \frac{N_L}{N} = \gamma \cdot \frac{U}{N}$ so $u_L \cdot \mu = \gamma \cdot u$, which ties together skill-specific and aggregate unemployment rates. Using US data, the sample average fraction of unskilled workers in the workforce (measured as those with (only) secondary school qualifications) over the period 1973-2002 is $\mu = 0.528$ and $u_L = 0.068$ is the unemployment rate amongst low-skilled workers, so the fraction of the unemployed who are low-skilled is pinned down as $\gamma = \mu \cdot \frac{u_L}{u} = 0.692$.

The total of monthly skill-specific separations must equal the measure of aggregate separations. Combining information on aggregate separations from equations (3) and (6) with the mass balance equations low-skilled workers,(4), and for high-skilled workers, (5), gives the following expression:

$$\delta_L = \frac{\delta \cdot [1-u]}{[[\mu - \gamma \cdot u] + \delta_{H/L} \cdot [1-\mu] - [1-\gamma] \cdot u]}. \tag{26}$$

This pins down the separation rate for low-skilled workers, δ_L in terms of the aggregate separation rate, δ, the unemployment rate, u, the fraction of low-skilled in the workforce, μ, the fraction of low-skilled amongst the unemployed, γ, and the relative separation rate for high-skilled workers to that of low skilled workers, which I denote as, $\delta_{H/L}$, where

$$\delta_H = \delta_{HL} \cdot \delta_L. \tag{27}$$

This approach takes the ratio of skill-specific separation rates as given and allows the levels to be pinned down in a manner consistent with the rest of the calibration. I use data on skill-specific separation rates from Fallick and Fleishmann (2001) for the US for the period 1994-2000 to pin down the relative separation rates for high-skilled and low-skilled workers $\delta_{H/L} = 0.574$. Then using (26) the monthly separation rate for low-skilled workers is $\delta_L = 0.041$, which exceeds the aggregate separation rate, $\delta = 0.036$. It follows from (27) that the monthly separation rate for high-skilled workers, $\delta_H = 0.023$, is lower than aggregate.

Next, I calibrate skill-specific exit rates, $\theta_j \cdot q_j(\theta_j)$, and the fraction of j-skilled workers searching for j-complexity jobs, λ_{jj} $j \in \{H, L\}$. To do this I use a system of four equations - the expression for the fraction of non-mismatch for each skill type $j \in \{H, L\}$, (9) and (10), the mass balance equations for low-skilled workers, (4), and for high-skilled workers, (5) - combined with estimates from Slominczyk (2013) of (non-mismatch) for low-skilled workers, $s_{LL} = 0.899$, and high-skilled workers, $s_{HH} = 0.742$ and the values of δ_i and γ determined above. Using this approach a fraction $\lambda_{LL} = 0.922$ of low skilled workers search for low-complexity positions while a smaller faction $\lambda_{HH} = 0.684$ of high-skilled workers search for high-skilled jobs. The exit rate from unemployment for workers searching for low-complexity jobs is $\theta_L \cdot q_L(\theta_L) = 0.546$, around 7.5% lower than the aggregate rate, while the exit rate from unemployment for workers searching for high-complexity jobs is $\theta_H \cdot q_H(\theta_H) = 0.726$ - about 23% higher than the aggregate rate. The difference between λ_{LL} and λ_{HH} indicates the greater willingness to search for mismatched positions amongst high-skilled workers. This in turn reflects the higher proportion of mismatch amongst high-skilled workers and the lower exit rate from unemployment amongst those searching for low-complexity jobs.

The profitability of complexity-j vacancies depends both on labour market tightness, θ_j, and the fraction, ϕ_{jj}, of workers searching for complexity-j jobs who are of skill j. In particular, vacancies for high complexity jobs will be more profitable the higher is ϕ_{HH} while vacancies for low complexity jobs will be more profitable the lower is ϕ_{LL}. For high-skilled workers in the high-complexity sub-market this term can be computed using (8) as $\phi_{HH} = 0.796$. Similarly, for low-skilled workers in the low-complexity sub-market $\phi_{LL} = 0.868$ using (7). Differences in the values of ϕ_{LL} and ϕ_{HH} reflect the greater prevalence of low-skilled workers in the US labour force and the pool of unemployed over the sample period, this outweighs the effects of greater prevalence of over-skilled mismatch rather than under-skilled mismatch in US data.

Notice that I have characterised a variety of skill- and complexity-specific quantities (exit rates from unemployment, fraction of workers of skill j searching for i-complexity jobs, fraction of workers searching for complexity-j jobs who are of skill i) of the heterogeneous labour market without any formal reference to the superstructure provided by the model of equilibrium unemployment. The critical issue is how to calibrate the remaining 11 objects: the flow benefits of unemployment, b, the fixed cost of posting a vacancy for a low complexity position, K_L, the fixed cost of creating a vacancy for a high complexity position, K_H and the price of a unit of match-specific output $P_{i,j}$, for $i, j \in \{H, L\}$, which will be proportional to match-specific productivities, as in equation (13), tightness for low complexity and high complexity jobs θ_j, $j \in \{H, L\}$ and the income tax rates, τ_i, required to ensure a balanced budget.

While 13 objects remain to be determined, there are only 7 as-yet-unused equations characterising the mismatch equilibrium. These equations include the wage premium (11); the arbitrage conditions that ensure mismatch - specifically, (18), that low-skilled workers are indifferent between searching for low-complexity jobs and high-complexity jobs and (19) which characterises the equivalent indifference condition for high-skilled workers; the free entry conditions for low-complexity positions, (20), and for high-complexity positions, (21), the condition linking aggregate and complexity-specific tightness, (2), and the government budget constraint (25).

In the 7 remaining equations, skill-specific tightness θ_j appears directly in the aggregate tightness condition, (2), and in the free entry conditions (20) and (21) where it also appears in the the term $q_j(\theta_j)$ nd in the exit rates $\theta_j \cdot q_j(\theta_j)$. Now the exit rates are known as have been determined above, but the match efficiencies and hence θ_j and $q_j(\theta_j)$ are unknown. However, I use the result that $q_j(\theta_j) = \frac{\theta_j \cdot q_j(\theta_j)}{\theta_j}$, to write the free entry conditions in terms of the known exit rates $\theta_L \cdot q_L(\theta_L)$ and $\theta_H \cdot q_H(\theta_H)$ and the two unknowns, θ_L and θ_H. It is then straightforward to use (2) to substitute for θ_H in (21) in terms of γ and θ (both known) and the unknown complexity-specific tightness θ_L.

To make progress, rather than allow a large number of free parameters, which would undermine the calibration procedure, I normalise some parameters relating to match-specific output prices (productivities), vacancy creation costs and skill-specific income tax rates, so as to reduce the number of unknowns to match the number of available equations. **(i)** I normalise the relative price of output from low-skilled workers in low-complexity jobs to 1, $P_{LL} = 1$. Through (13) this pins down $P_{LL} = A_{LL}$. **(ii)** I set $P_{HL} = P_{LH}$, so that the productivity of low-skilled workers in low-complexity jobs equals that of high-skilled workers in low-complexity jobs. These normalisations mean that only 2 prices P_{HL} and P_{HH} need to be identified rather than 4.[10] **(iii)** Next I assume that $K_H = P_{HH} \cdot K_L$, so that it is only necessary to solve for K_L. **(iv)** Turning to tax rates I set $\tau_L = 0$, and choose τ_H to ensure that the governemnt budget constraint is satisfied.[11]

I set the wage premium (the average wage of high-skilled workers to the average wage of low-skilled workers) at 1.68, consistent with US data, see Acemoglu and Autor (2012). Finally, I set real interest rate $r = 0.004$, consistent with an annual interest rate of 4%. I adopt the

[10] I anticipate $P_{HH} > P_{HL} > P_{LL} \equiv 1$ and reject any putative equilibrium for which this condition is violated.

[11] I focus on this progressive tax scheme because of its simplicity, because some form of tax-progressivity seems plausible and because the wages of low-skilled workers in the calibrated model are close to the calibrated value of b, so that a flat rate of income tax across all workers has a large effect on the employment of low-skilled workers. I also explore an alternative scheme with flat income tax rate for all workers on income in excess of unemployment benefits, b. Results are similar.

standard approach in the literature and set $\beta = 0.5$.[12]

For $\beta = 0.5$, set a tax rate τ_H, and solve a system of 5 equations - the wage premium (11), the arbitrage conditions for each worker type, (18) and (19), the free entry conditions (20) and (21) (written in terms of $\theta_H \cdot q_H(\theta_H)$, γ, θ and θ_L) and the governmemt budget constraint - to determine the six objects: $\{b, K_L, \theta_L, P_{HL}, P_{HH}\}$. I repeat this pocedure, searching over feasible values of τ_H to ensure that the government budget constraint holds.

I find that the model generates a value of $b = 0.164$, that is around one sixth of the value of low skilled output in low complexity jobs. High skilled workers in high complexity jobs are 56% more productive as low skilled workers in low complexity jobs: $P_{HH} = 1.564$. While $P_{HL} = 1.430$ so high-skilled workers in low-complexity jobs are around 43% more productive than low-skilled workers in similar positions. Equally, low skilled workers in high complexity jobs are around 10% less productive than high skilled worker in similar positions. The calibrated fixed cost of vacancy creation is $K_L = 161.275$ the monthly flow equivalent, $r \cdot V_L = r \cdot K_L = 0.645$, which is not dissimilar in magnitude to the per-period flow cost of vacancy posting identified in aggregate studies. The calibrated fixed cost of high-complexity vacancy creation is $K_H = 252.248$, so that the monthly flow equivalent is $r \cdot V_H = r \cdot K_H = 1.008$. Labour market tightness for low complexity jobs, $\theta_L = 0.796$ exceeds that for low high complexity jobs, $\theta_H = 0.503$. The government's budget is balanced for a tax rate $\tau_H = 0.0324$.

The exit-rate expressions $\theta_L \cdot q_L(\theta_L)$ and $\theta_H \cdot q_H(\theta_H)$ permit different

[12] Note that the following objects related to labour market flows, $\theta_j q_j(\theta_j)$, δ_i, λ_{ij}, ϕ_{ij}, γ are unaffected by the choice of β. The value of worker bargaining power has been a source of recent controversy in the analysis of labour market fluctuations using the DMP matching model, see Shimer (2005), Hagedorn and Manowskii (2008). If the focus of the paper was the cyclical behaviour of the labour market, then it would be straightforward, in principle, to assign this parameter to target some dynamic property of the model, as in the DMP literature. Also, the division of vacancy posting costs between fixed K_j and flow-fixed c_j could be chosen to match a dynamic target, as in Ramey and Fujita (2007). However, my concern here is with whether the model, and particularly the equilibrium exhibiting mismatch is even capable of explaining the steady state. This is, in a sense, logically prior to any question about the cyclical properties of the model, which typically concerns itself with deviations around such a steady state.

functional forms for complexity specific matching functions and these may differ from the aggregate matching function. The latter is, consistent with the literature, assumed to have a Cobb-Douglas form $\theta \cdot q(\theta) = m_0 \cdot \theta^{1-a}$, where m_0 represents the efficiency of the aggregate matching process. I maintain the Cobb-Douglas structure and identify separate complexity-specific efficiency parameters, m_L and m_H using (28) and (29). These satisfy both the exit rate identity (3) and the aggregate tightness identity (2) simultaneously. Setting $\alpha = 0.5$ (standard from the literature), the match efficiency parameters are: $m_L = 0.696$ and $m_H = 1.164$.

$$\theta_L \cdot q_L(\theta_L) = m_L \cdot \theta_L^{1-\alpha} \tag{28}$$

$$\theta_H \cdot q_H(\theta_H) = m_H \cdot \theta_H^{1-\alpha} \tag{29}$$

From these results, we can determine several other noteworthy features of the calibrated mismatch equilibrium, such as the units of match-specific output per worker, y_{ij}, for $i, j \in \{H, L\}$, aggregate output per worker, y and the share of output produced by different groups or firms and workers. Since only a small proportion of workers are mismatched, the bulk of output per worker comes from low skilled workers in low complexity jobs, $y_{LL} = s_{LL} \cdot [\mu - \gamma \cdot u] = 0.442$, and high skilled workers in high complexity jobs: $y_{HH} = s_{HH} \cdot [1 - \mu - [1 - \gamma] \cdot u] = 0.338$. Around a quarter of high skilled workers mismatch to low complexity jobs, $y_{HL} = [1 - s_{HH}] \cdot [1 - \mu - [1 - \gamma] \cdot u] = 0.118$, while only one tenth of low skilled workers secure high complexity jobs, $y_{LH} = [1 - s_{LL}] \cdot [\mu - \gamma \cdot u] = 0.050$. Note that $\sum\limits_{i,j \in \{L,H\}} y_{ij} = 1 - u$ which is the fraction of the workforce in employment. As a share of the workforce, 0.560 are engaged in the production of low complexity goods and 0.388 are employed to produce high complexity goods - the remainder are unemployed. Taking account of the relative prices, $P_{i,j}$ (= productivities, A_{ij}) of specific matches and the production function for final goods, (12), the value of final output per worker in the mismatch equilibrium is $y = 1.211$, of which just under half, 49.5%, is attributable to high complexity goods production, and the remaining 50.5% comes from low complexity goods production. Viewed from the perspective of relative contribution to final output the situation is rather different: around 57.5% of final output is produced by high skilled workers.

4 Which Equilibrium Arises in the Decentralised Economy?

In the model of Section 2 mismatch, if it arises, does so endogenously: workers *choose* to mismatch over segmentation. Indeed, the endogenous nature of mismatch is, arguably, an interesting feature of the theoretical model. By contrast the analysis of the previous Section simply imposes mismatch, (through conditions (19) and (18)) without regard to alternative equilibria. If the model is to provide a robust empirical explanation of the data, then I must show that in the decentralised environment workers of each skill level prefer mismatch to segmentation so that the mismatch equilibrium holds.

Fortunately, the logic that led to these arbitrage conditions under the mismatch equilibrium generate related conditions that can characterise alternative equilibria: unemployed i-skilled workers choose to search in a segmented fashion or to search in a way that permits mismatch on the basis of whether the capitalised value of segmented search, $\mathcal{U}_{ii}$, is greater than or equal to the capitalised value of mismatched search, $\mathcal{U}_{ij}$. As indicated in Table 3, segmented search by high skilled and low skilled workers is characterised by $\mathcal{U}_{HH} > \mathcal{U}_{HL}$, and $\mathcal{U}_{LL} > \mathcal{U}_{LH}$.Over-skilled mismatch (only) arises if high skilled workers are prepared to mismatch while low skilled workers are not, so that $\mathcal{U}_{HH} = \mathcal{U}_{HL}$ and $\mathcal{U}_{LL} > \mathcal{U}_{LH}$. Under-skilled mismatch (only) will occur if low skilled workers are prepared to mismatch while high skilled workers are not, so that $\mathcal{U}_{HH} > \mathcal{U}_{HL}$ and $\mathcal{U}_{LL} > \mathcal{U}_{LH}$.[13]

Table 3

These conditions provide a means to operationalise a theory-consistent empirical evaluation of the mismatch equilibrium. Specifically, the ques-

[13] In Appendix B, I document 5 other equilibria which arise if I allow $\mathcal{U}_{ii} \gtreqless \mathcal{U}_{ij}, i, j \in \{H, L\}$ $i \neq j$. I do not explore these equilibria further, because they would require at least one skill-group to engage only in negative assortative matching, which would seem to misrepresent the data.

tion that an i-skilled worker asks is 'Will I be better off undertaking segmented search or mismatched search?' To answer this question the worker must compare the value of unemployment from segmented search, $\mathcal{U}_{ii}$ and that from mismatched search, $\mathcal{U}_{i,j}$, conditioning on various patterns of search activity amongst differently skilled individuals, at a common set of parameter values. Implicitly, workers compute the value of unemployment under one equilibrium with the value of unemployment under an alternative equilibrium, where the set of alternative equilibria depends on the worker's skill and the behaviour of workers of different skills. Since the data exhibit both under-skilled and over-skilled mismatch, it is natural to take the parameter values (of features such as productivities, A_{ij}, costs of vacancy creation, K_j, separation rates, δ_i and so on), as those identified under the calibration of the mismatch equilibrium in Section 3. So I can mimic the workers' search decision by taking the parameters from the mismatch equilibrium and computing the equilibrium for each of the alternative equilibria (segmentation, over-skilled mismatch, under-skilled mismatch) using the parameter values for the calibrated mismatch equilibrium and the conditions for the alternative equilibrium under consideration. These conditions are stated in Appendix A. Then I compare the capitalised value of unemployment obtained in each case. The parameters inherited from Section 3 are equilibrium-invariant general parameters, $\{r, \alpha\ , \mu, \beta\}$ and the mismatch-equilibrium-specific parameters $\{\delta_L, \delta_H, m_L, m_H, A_{LL}, A_{LH,}A_{HL}, A_{HH}, b, K_L, K_H\}$. Note that A_{LH} and A_{HL} are only used in constructing the under-skilled mismatch equilibrium and the over-skilled mismatch equilibrium repectively.

For notational clarity, I denote the capitalised value of unemployment under these alternative equilibria with a superscript, $\mathcal{U}^a_{ii}$, $a \in \{s, o, u\}$ where s corresponds to the equilibrium in which both groups of workers segment, o, corresponds to the equilibrium in which high skilled workers mismatch, but low skilled workers segment, and u represents the equilibrium in which low skilled workers mismatch and high skilled workers segment. Now in computing each equilibrium, $a \in \{s, o, u\}$ the relevant conditions in Table (3) must hold, so when the alternative equilibrium involves mismatch by i-skilled workers, $\mathcal{U}^a_{ii} = \mathcal{U}^a_{ij}$, and I can evaluate the mismatch equilibrium against the other putative equilibria by comparing $\mathcal{U}_{ii}$ with $\mathcal{U}^a_{ii}$, $a \in \{s, o, u\}$. Using this approach, the calibrated mismatch equilibrium discussed in Section 3 can be said to exist if neither high skilled workers, nor low skilled workers would strictly prefer to segment. In particular, mismatch arises provided that the value of unemployment for high skilled workers obtained in the mismatch equilibrium satisfies $r\mathcal{U}_{HH} \geq \{r\mathcal{U}^s_{HH}, r\mathcal{U}^u_{HH}\}$ and the value of unemployment for low skilled workers satisfies $r\mathcal{U}_{LL} \geq \{r\mathcal{U}^s_{LL}, r\mathcal{U}^o_{LL}\}$.[14] Notice that what matters to high skilled workers is whether the mismatch equilibrium is preferred to the set of alternative equilibria in which high skilled workers segment. They do not consider whether $r\mathcal{U}_{HH} \geq r\mathcal{U}^o_{HH}$; that is they do not have to decide between the mismatch equilibrium and the over-skilled mismatch (only) equilibrium (since they mismatch in both cases). Rather, conditional on high skilled workers choosing to mismatch, it is low skilled workers' decision on whether or not to segment that is critical in determining whether mismatch or overskilled mismatch (only) will arise. In an entirely analagous way, low skilled workers do not need to decide between the mismatch equilibrium and the under-skilled mismatch (only) equilibrium.

Table (4) displays the results of this exercise. The results for high skilled workers decisions indicate that regardless of whether low skilled workers choose to segment or to mismatch, high skilled workers will be strictly better off by pursuing opportunities for mismatch, since $r\mathcal{U}_{HH} > \{r\mathcal{U}^s_{HH}, r\mathcal{U}^u_{HH}\}$. At the same time, regardless of whether skilled workers were to choose to segment or to mismatch, low skilled workers would be strictly better off if they mismatch, since $r\mathcal{U}_{LL} > \{r\mathcal{U}^s_{LL}, r\mathcal{U}^o_{LL}\}$. So neither group of workers prefers to segment, therefore the mismatch equi-

[14]Notice that in these putative equilibria there, by definition, no need to match the outcomes observed in the data for equilibrium objects such as aggregate and skill-specific unemployment, extent of mismatch etc. Indeed, if I were to proceed simply to calibrate each putative equilibrium to match properties in the data, I would have to assign *new* values for parameters such as match-specific productivities, vacancy creation costs and so on, and then a comparison of the the capitalised value of unemployment across equilibria would be confounded by different underlying parameter values, and would not mirror the search decision facing workers.

librium can be claimed to exist and provides a theoretically consistent empirical framework.[15]

Table 4

5 Is Mismatch Costly?

In this Section I outline how to calculate the costs of mismatch, develop estimates of the costs of mismatch in the calibrated model, explore how the costs of mismatch have evolved over time and discuss the causes of these changes in the costs of mismatch.

5.1 Measuring the Costs of Mismatch

An evaluation of the costs of mismatch requires both (i) a reference point against which to compare the decentralised mismatch equilibrium and (ii) a measure of the efficiency of an equilibrium. Much of the macro-labour literature on skill-specific inequality adopts a framework that imposes segmentation (with or without matching frictions) and precludes mismatch, see Acemoglu and Autor (2011) and Cahuc, Carcillo and Zylberberg (2014). So for the model outlined in Section 2 a natural and literature-relevant reference point is the decentralised equilibrium in which both groups of workers choose to segment. In the canonical decentralised Diamond-Mortensen-Pissarides model of search with matching frictions, a convenient, standard flow measure is real net output per capita, $\bar{\omega} = y + b \cdot u - c \cdot V/N$, defined as the sum of final output per capita, $y = Y/N$, the per capita value of benefits to the unemployed, $b \cdot u = b \cdot U/N$, less the costs, $c \cdot V/N$, of vacancy creation (where I have used c, the flow cost of vacancy posting common to the literature). However, if unemployment benefits are funded by (income) taxes, then a more appropriate measure of net output per capita is $\omega = y - c \cdot V/N$ otherwise unemployment benefits are double counted. Here, I generalise the net output measure to a heterogeneous agent setting. Denoting the

[15] Note that, under the mismatch equilibrium parameter values, high skilled workers would prefer to be in an equilibrium with over-skilled mismatch (only), but in a decentralised environment they cannot compel low skilled workers to segment or credibly commit to compensate low skilled workers for doing so (to bring about the equilibrium that they would prefer).

net output per capita measures under mismatch and under segmentation respectively as ω and ω^s, the costs of mismatch can be calculated as the percentage difference in net output arising under mismatch versus that arising under segmentation:[16]

$$\xi = -100 \cdot \frac{\omega - \omega^s}{\omega^s}. \tag{30}$$

Notice that ξ will take on positive values only if per capita net output is lower under mismatch than under segmentation, which indicates that mismatch is costly. In the remainder of this subsection I define and quantify net output measures under mismatch and under segmentation and use them to provide a quantitative assessment of the costs of mismatch.

With vacancies of differing complexity and fixed costs of vacancy creation, the steady state flow value of net output per capita under mismatch can be written as

$$\omega = y - c_L \cdot \frac{V_L}{N} - \kappa_L \cdot [y_{LL} + y_{HL}] - c_H \cdot \frac{V_H}{N} - \kappa_H \cdot [y_{LH} + y_{HH}]. \tag{31}$$

In (31), $y = A_{HH} \cdot y_{HH} + A_{LH} \cdot y_{LH} + A_{HL} \cdot y_{HL} + A_{LL} \cdot y_{LL}$, as in (12) and the remaining terms capture the costs of vacancies. [17] Although vacancy creation must incur fixed costs if mismatch is to exist, it is helpful to *interpret* the impact of vacancy posting costs in flow equivalent terms. The flow measures exploit the formal equivalence, in conditions (21) and (20), of fixed vacancy creation costs to a set up in which firms pay a flow cost of vacancy posting, $c_j = r \cdot K_j$ for each unfilled vacancy *and* a flow cost of production, $\kappa_j = r \cdot K_j$ for each complexity j match engaged in production. The (flow equivalent) expenditure on posting low complex-

[16] It is also straightforward to calculate net output per capita in the standard way that incorporates unemployment benefits as $\bar{\xi} = -100 \cdot \frac{\bar{\omega} - \bar{\omega}^s}{\bar{\omega}^s}$.

[17] Under the free entry condition, (24), a complexity-j vacancy satisfies $r \cdot K_j =$

$$[1-\beta] \cdot q_j(\theta_j) \cdot \left[\begin{array}{c} [1-\phi_{jj}] \cdot \left[\frac{[1-\tau_{j'}][P_{j'j} - r \cdot K_j] - b}{[1-\tau_{j'}][r + \delta_{j'} + \beta \cdot [1-\phi_{jj}] \cdot \theta_j \cdot q_j(\theta_j)]} \right] \\ +\phi_{jj} \cdot \left[\frac{[1-\tau_j] \cdot [P_{jj} - r \cdot K_j] - b}{[1-\tau_j] \cdot [r + \delta_j + \beta \cdot \phi_{jj} \cdot \theta_j \cdot q_j(\theta_j)]} \right] \end{array} \right], \; j, j' \in \{H, L\}, \, j \neq j'.$$

Here $r \cdot K_j$ appears both on the left hand side, in the manner of a flow cost of vacancy posting, c_j, and (twice) on the right hand side where it takes the form of a complexity-specific production cost, κ_j .

ity vacancies is $c_L \cdot V_L/N = r \cdot K_L \cdot \theta_L \cdot [\lambda_{LL} \cdot \gamma + [1 - \lambda_{HH}] \cdot [1 - \gamma]] \cdot u$. In this expression θ_L is tightness in the sub-market for low complexity jobs and $[\lambda_{LL} \cdot \gamma + [1 - \lambda_{HH}] \cdot [1 - \gamma]] \cdot u$ is the measure of unemployed workers of each skill level searching for low complexity jobs, so $V_L/N = \theta_L \cdot [\lambda_{LL} \cdot \gamma + [1 - \lambda_{HH}] \cdot [1 - \gamma]] \cdot u$ represents the number of vacancies for low complexity jobs created per period in steady state and $r \cdot K_L$ represents the real (flow equivalent) cost of posting a vacancy. The flow costs of production associated with low complexity output is $\kappa_L \cdot [y_{LL} + y_{HL}] = r \cdot K_L \cdot [s_{LL} \cdot [\mu - \gamma \cdot u] + [1 - s_{HH}] \cdot [1 - \mu - [1 - \gamma] \cdot u]]$, where $y_{LL} = s_{LL} \cdot [\mu - \gamma \cdot u]$ is the number of matches between low skilled workers and low complexity jobs in the mismatch equilibrum and $y_{HL} = [1 - s_{HH}] \cdot [1 - \mu - [1 - \gamma] \cdot u]$ is the number of matches between high skilled workers and low complexity jobs. Similarly, $c_H \cdot V_H/N = r \cdot K_H \cdot \theta_H \cdot [[1 - \lambda_{LL}] \cdot \gamma + \lambda_{HH} \cdot [1 - \gamma]] \cdot u$ and $\kappa_H \cdot [y_{LH} + y_{HH}] = r \cdot K_H \cdot [[1 - s_{LL}] \cdot [\mu - \gamma \cdot u] + s_{HH} \cdot [1 - \mu - [1 - \gamma] \cdot u]]$.

Next, I provide a quantitative assessment of each component of ω in equation (31). Using the calibrated values outlined in Section 3, final output per capita is $y = 1.211$, with high complexity jobs accounting for around 49.5% of output per capita, but only around 38% of employment. The equivalent flow cost of all vacancies held open for low complexity positions is $c_L \cdot V_L/N = 0.020$. This is less than 2% of final output. To glean greater insight consider the components of these costs; $c_L = r \cdot K_L = 0.645$ and $V_L/N = \theta_L \cdot [\lambda_{LL} \cdot \gamma + [1 - \lambda_{HH}] \cdot [1 - \gamma]] \cdot u = 0.795 \cdot 0.735 \cdot 0.052 = 0.030$. Similarly, for high-complexity positions, $c_H \cdot V_H/N = 0.007$, which is under 1% of final output. Here $c_H = r \cdot K_H = 1.008$ and $V_H/N = \theta_H \cdot [[1 - \lambda_{LL}] \cdot \gamma + \lambda_{HH} \cdot [1 - \gamma]] \cdot u = 0.503 \cdot 0.264 \cdot 0.052 = 0.007$. The flow costs of production for low complexity jobs can be expressed as the product of the costs of production for low complexity jobs and the number of matches producing low complexity goods: $\kappa_L \cdot [y_{LL} + y_{HL}] = r \cdot K_L \cdot [s_{LL} \cdot [\mu - \gamma \cdot u] + [1 - s_{LL}] \cdot [1 - \mu - [1 - \gamma] \cdot u]] = 0.645 \cdot [0.442 + 0.118] = 0.361$; these production costs are more substantial at around 30% of final output. The flow costs of production for high complexity jobs are slightly larger at $\kappa_H \cdot [y_{LH} + y_{HH}] = 1.008 \cdot [0.050 + 0.338] = 0.391$ around 32% of final output. Taking account of all these components, ω, the net output under mismatch is 0.432. Note for future reference that the flow cost of posting vacancies is an order of magnitude smaller than the production costs element.

Next I discuss the measure of net output per capita under segmentation, ω^s, at the parameter values of the claibrated mismatch equilibrium. This allows the costs of mismatch to be evaluated by comparing equilibrium balues under mismatch and segmentation without altering other features of the economy. This ceteris paribus comparison should hold constant these deep parameters, so as to explore the consequences of the decision to mismatch on equilibrium objects. Bearing these issues in mind I take deep parameters identified by the mismatch calibration as given and solve the segmentation equilibrium conditions, (outlined in Appendix A) to determine equilibrium values of unemployment, u^s, fraction of the unemployed who are low-skilled, γ^s, tightness for high-complexity jobs, θ^s_H and low-complexity jobs, θ^s_L. To ensure that the segmentation equilibrium also corresponds to a sustainable steady state I choose a segmentation-specific tax rate, τ^s_H, to ensure a balanced budget. Pinning down equilibrium values for $\{u^s, \gamma^s, \tau^s_H, \theta^s_H, \theta^s_L\}$ allows match-specific output per worker, y^s_{ii} and aggregate output per worker $y^s = A_{HH} \cdot y^s_{HH} + y^s_{LL}$ to be computed. Key variables for the mismatch and segmentation equilibria are summarised in Table (5). The equivalent net output measure is:

$$\omega^s = y^s - c_L \cdot \frac{V^s_L}{N} - \kappa_L \cdot y^s_{LL} - c_H \cdot \frac{V^s_H}{N} - \kappa_H \cdot y^s_{HH}. \qquad (32)$$

The final four terms represent (fixed) costs of creating new vacancies in a fow equivalent form. These are $c_j = r{\cdot}K_j$, for all open j-skill-requirement vacancies and $\kappa_j = r \cdot K_j$ for all matches producing j-skill requirement goods are unaltered from the mismatch calibration. Then the (flow equivalent) cost of posting low-skill-requirement vacancies is $c_L{\cdot}V^s_L/N = r \cdot K_L \cdot \theta^s_L \cdot \gamma^s \cdot u^s$. The flow costs of production of low-skill-requirement output can be written as $\kappa_L \cdot y^s_{LL} = r \cdot K_L \cdot [\mu - \gamma^s \cdot u^s]$. Similarly, for high-skill-requirement positions the analogous terms are $c_H \cdot V^s_H/N = r{\cdot}K_H{\cdot}\theta^s_H{\cdot}[1-\gamma^s]{\cdot}u^s$ and $\kappa_H{\cdot}y^s_{HH} = r{\cdot}K_H{\cdot}[1-\mu-[1-\gamma^s] \cdot u^s]$. For the segmentation equilibrium evaluated at the parameter values calibrated under mismatch, output is $y^s = 1.174$, around 3% lower than that under mismatch. The total costs of vacancies in the segmentation equilibrium is 0.773, larger than in the mismatch equilibrium. So net output under segmentation is 0.401. Also, in the segmentation equilibrium, a larger share (almost 60%) of the costs of vacancies is associated with high complexity positions.

Table 5

Putting these results together the costs of mismatch are: $\xi = -100 \cdot \frac{\omega - \omega^s}{\omega^s} = -7.668\%$ of net output under segmentation.[18] Since ξ is negative, it follows that mismatch generates benefits rather than costs. This is an intriguing result since, to the extent that one views mismatch as a form of misallocation of workers to inappropriate tasks, it would be natural to expect mismatch to be costly. To develop some understanding of this result I examine the manner in which the components of net output differ in the mismatch equilribium and the segmentation equilibrium.

5.2 Understanding the Costs and Benefits of Mismatch

The most obvious difference between the segmentation equilibrium and the mismatch equilibrium is the composition of match-specific outputs. Through its impact upon this composition, mismatch directly affects the level of final output per worker and vacancy creation costs. Focussing on compositional effects effects highlights the scope for mismatch to act as a source of misallocation. However, compositional effects reflect only part of the difference between mismatch and segmentation. The other key feature of mismatch is the impact that it has on the overall level of activity, through its effect on expected profitability of low complexity jobs, and hence on complexity-specific tightness, vacancy creation, job-finding rates and employment. Changes in the level of activity also impact on final output per worker and both flow equivalent elements of

[18] If the value of unemployment is included, then net output under mismatch is $\bar{\omega} = 0.440$, while under segmentation it is $\bar{\omega}^s = 0.414$, and the net costs of mismatch would be $\bar{\xi} = -6.21\%$ of net output.

fixed production costs. In this Section I develop a decomposition which allows me to express the costs of mismatch in terms of the compositional effect and the aggregate activity effect and provide a quantitative assessment of the role of each element.

To explore these effects I first define by $\hat{\omega}$ the level of net output that arises if (i) mismatch is suppressed (so $\hat{\lambda}_{LL} = \hat{\lambda}_{HH} = 1$), while (ii) aggregate unemployment, υ, and skill -specific unemployment, γ, and complexity specific tightness θ_j, $j \in \{H, L\}$ are held at the values as in the mismatch equilibrium. Then, since $\hat{\lambda}_{LL} = \hat{\lambda}_{HH} = 1$, the pool of workers searching for high skilled jobs is simply the measure of high skilled unemployed workers in the mismatch equilibrium, $[1-\gamma] \cdot u$, it follows that vacancies for high complexity jobs must be $\hat{V}_H/N = \theta_H \cdot [1-\gamma] \cdot u$. Then the cost of posting vacancies for high complexity jobs is $c_H \cdot \hat{V}_H/N = r \cdot K_H \cdot \theta_H \cdot [1-\gamma] \cdot u$. Similarly, the cost of posting vacancies for low complexity jobs is $c_L \cdot \hat{V}_L/N = r \cdot K_L \cdot \theta_L \cdot \gamma \cdot u$. So

$$\hat{\omega} = \hat{y} - c_L \frac{\hat{V}_L}{N} - \kappa_L \cdot [y_{LL} + y_{LH}] - c_H \cdot \frac{\hat{V}_H}{N} - \kappa_H \cdot [y_{HH} + y_{HH}],$$

where $\hat{y} = A_{HH} \cdot [y_{HH} + y_{HL}] + A_{LL} \cdot [y_{LH} + y_{LL}]$. Then the (compositional) effect, which assesses that part of the costs of mismatch which arises if one holds aggregate (and skill-specific) employment constant (at the level of the calibrated mismatch equilibrium) and notionally reallocates workers from an environment with segmentation, is defined as $\omega - \hat{\omega}$ and (ii) the (aggregate activity) effect, which measures what happens if one maintains that $\hat{\lambda}_{LL} = \hat{\lambda}_{HH} = 1$ (as under perfect segmentation) and notionally changes the overall (and skill-specific) unemployment rates from the values under the segmentation equilibrium (evaluated at the parameter values identified in Section 3) to the values under the mismatch equilibrium, is given by $\hat{\omega} - \omega^s$. . This decomposition can be summarised in (33). If there is no change in aggregate (or skill-specific) unemployment as a result of mismatch, then the second term in (33) is zero and the output cost of mismatch will reflect the compositional effects associated with a reallocation of workers alone. However, any change in aggregate unemployment or in the skill-composition of the unemployed as a result of mismatch will impact directly the output costs of mismatch.

$$\omega - \omega^s = \{\omega - \hat{\omega}\} + \{\hat{\omega} - \omega^s\} \tag{33}$$

$$= \begin{array}{c} \{\Delta \text{ Net-output holding employment fixed}\} + \\ \{\Delta \text{ Net-output allowing employment variation}\} \end{array} \tag{34}$$

Now

$$\omega - \hat{\omega} = \begin{array}{c} [[A_{LH} - 1] - [\kappa_H - \kappa_L]]\, y_{LH} - [[A_{HH} - A_{HL}] - [\kappa_H - \kappa_L]]\, y_{HL} \\ -c_L \cdot \theta_L \cdot [[\lambda_{LL} - 1] \cdot \gamma + [1 - \lambda_{HH}] \cdot [1 - \gamma]] \cdot u \\ -c_H \cdot \theta_H \cdot [[1 - \lambda_{LL}] \cdot \gamma + [\lambda_{HH} - 1] \cdot [1 - \gamma]] \cdot u \end{array}$$

and

$$\hat{\omega} - \omega^s = \begin{array}{c} +[A_{HH} - \kappa_H] \cdot [y_{HH} + y_{HL} - y^s_{HH}] + [1 - \kappa_L] \cdot [y_{LH} + y_{LL} - y^s_{LL}] \\ -c_L \cdot [\theta_L \cdot \gamma \cdot u - \theta^s_L \cdot \gamma^s \cdot u^s] - c_H \cdot [\theta_H \cdot [1 - \gamma] \cdot u - \theta^s_H \cdot [1 - \gamma^s] \cdot u^s] \end{array}$$

Finally, in (35) I compute $\omega - \omega^s = \omega - \hat{\omega} + \hat{\omega} - \omega^s$ and substitute for match-specific output $\{y_{ij}, y^s_{ii}\}$ for $i, j \in \{H, L\}$ to show how costs of mismatch relate to the amount of mismatch, $[1 - s_{ii}]$, the relative supply of skills, μ, unemployment under mismatch, u, and segmentation, u^s, and the composition of unemployment under mismatch, γ, and under segmentation, γ^s. Here the first four lines capture the compositional effects and the last two lines capture the aggregate activity effect. This final decomposition shows that the extent of mismatch $[1 - s_{ii}]$ and the relative supply of skills, μ, have a direct impact associated with the effect of eliminating mismatch while holding employment constant, but no direct effect once employment variation is permitted.

$$\omega - \omega^s = \begin{Bmatrix} [[A_{LH}-1]-[\kappa_H-\kappa_L]]\,[1-s_{LL}]\cdot[\mu-\gamma\cdot u]- \\ [[A_{HH}-A_{HL}]-[\kappa_H-\kappa_L]]\,[1-s_{HH}]\cdot[1-\mu-[1-\gamma]\cdot u] \\ -c_L\cdot\theta_L\cdot[[\lambda_{LL}-1]\cdot\gamma+[1-\lambda_{HH}]\cdot[1-\gamma]]\cdot u \\ -c_H\cdot[\theta_H\cdot[[1-\lambda_{LL}]\cdot\gamma+[\lambda_{HH}-1]\cdot[1-\gamma]]\cdot u] \end{Bmatrix} + \begin{Bmatrix} [A_{HH}-\kappa_H]\cdot[[1-\gamma]\cdot u-[1-\gamma^s]\cdot u^s]+[1-\kappa_L]\cdot[\gamma\cdot u-\gamma^s\cdot u^s] \\ -c_L\cdot[\theta_L\cdot\gamma\cdot u-\theta_L^s\cdot\gamma^s\cdot u^s]-c_H\cdot[\theta_H\cdot[1-\gamma]\cdot u-\theta_H^s\cdot[1-\gamma^s]\cdot u^s] \end{Bmatrix} \tag{35}$$

Turning to quantitative analysis of this decomposition, I begin by focussing on the compositional effects, holding employment fixed. Notice that since both under-skilled mismatch and over-skilled mismatch are permitted (in the mismatch equilibrium), the reallocation required, holding employment constant, as a result of moving from segmentation to mismatch need not even lower (gross) output per capita (raise the costs of mismatch). This depends on the extent of the productivity gains / losses implied by each type of mismatch (under-skilled and over-skilled) and upon the extent of each type of mismatch.[19] That is it depends on whether $[A_{LH}-1]\cdot y_{LH}-[A_{HH}-A_{HL}]\cdot y_{HL}\gtrless 0$ see line 1 of (35). If the costs of mismatch were due only to compositional changes, then the effect on output would depend on the productivity of the workers who are mismatched in the mismatch equilibrium but cease to be mismatched in the segmentation equilibrium. Although over-skilled mismatch is more than 2 times as prevalent as under-skilled mismatch in the mismatch equilibrium: ($y_{HL}=0.118$, and, $y_{LH}=0.050$) it turns out that under-skilled mismatch as a larger impact on the costs of mismatch. This is because mismatch for high-skilled workers would raise the costs of mismatch (lower final output) by $[A_{HH}-A_{HL}]\cdot y_{HL}=0.134\cdot 0.118=0.016$, whereas under-skilled mismatch by low skilled workers would raise final output (lower the costs of mismatch) by $[A_{LH}-1]\cdot y_{LH}=0.021$. So the compositional effects of

[19] By contrast, the the literature which follows from Albrecht and Vroman's work suppreses under-skilled mismatch by assumption and (combined with the absence of complexity-specific fixed costs of vacancy creation) forces the compositional effect associated with mismatch to reduce output.

eliminating mismatch would raise output (lower costs of mismatch) by 0.005. However, worker reallocation also affects production costs - see line 2 of (35). For high skilled workers the compositional effect of mismatch reduces production costs (raises net output) and reduces the costs of mismatch by $[\kappa_H - \kappa_L] \cdot y_{HL} = [1.008 - 0.645] \cdot 0.118 = 0.043$, and for low skilled workers the reallocation implicit in the composition effect raises production costs and the costs of mismatch by $[\kappa_H - \kappa_L] \cdot y_{LH} = 0.018$. So production costs would fall by 0.025 if mismatch replaced segmentation, while holding employment constant. The impact of compositional changes on vacancy posting costs is captured in lines 3 and 4 of (35). For low complexity vacancies the compositional effect of mismatch increases the number of low complexity vacancies and hence the costs of vacancy posting by $c_L \cdot \theta_L \cdot [[\lambda_{LL} - 1] \cdot \gamma + [1 - \lambda_{HH}] \cdot [1 - \gamma]] \cdot u = 0.645 \cdot [0.735 - 0.692] \cdot 0.052 = 0.001$. This tends to reduce net output and raise the costs of mismatch. By contrast the number of high complexity vacancies declines under mismatch which reduces the flow cost of vacancies by the amount $c_H \cdot [\theta_H \cdot [[1 - \lambda_{LL}] \cdot \gamma + [\lambda_{HH} - 1] \cdot [1 - \gamma]] \cdot u] = 1.008 \cdot [0.264 - 0.308] \cdot 0.052 = -0.001$ which tends to lower the costs of mismatch and more than offsets the effect for low complexity vacancies. The results are summarised in Table (6).

Table 6

Putting these insights together, if one were to hold employment constant, the net output would be higher under mismatch than under segmentation by $0.021 - 0.016 - [0.018 - 0.043 + 0.001 - 0.001] = 0.030$. That is the costs of mismatch would be higher and the net benefits of mismatch would be around 7.56% of net output using (30). Moreover, the majority of the net benefits come from the net change in costs of vacancies - while changes in the flow costs of posting high and low complexity vacancies cancels out, the flow production costs are muchlarger and do not balanceout, since the reductin costs due to high skilled workers in low complexity jobs outweighs the rise in production costs for the much smaller number of low-skilled workers who occupy high complexity jobs. So, under the parameter values identified in Section 3, around half of the benefits of mismatch arise because of compositional effects holding employment constant, predominantly associated with the allocation of high-skilled workers. By ruling out mismatch, the segmentation equilibrium generates an increase in flow-production costs of high complexity matches that outweighs the increase in final output that arises from employing high skilled workers in more productive matches.

The calculations in the previous paragraph ignore the aggregate activity effect of mismatch. This is revealed most clearly by the unemployment rate in the segmentation equilibrium, $u^s = 0.083$, which is 1.58 times that under mismatch, $u = 0.052$. This is equivalent to saying that by allowing mismatch, the number of matches increases by around 3.5%. What are the consequences of this aggregate activity effect for the costs of mismatch? In fact, the change in the overall level of activity also features a compositional element, namely the differential impact on the skill-composition of the unemployment pool: allowing mismatch disproportionately improves the employment prospects of low skilled workers. This is embedded in the fact that the fraction of the unemployed who are low skilled is higher in the segmentation equilibrium than in the mismatch equilibrium, $\gamma^s = 0.786 > 0.692 = \gamma$, so there are $\gamma^s \cdot u^s = 0.065$ low skilled unemployed under segmentation almost two times the number under mismatch $\gamma \cdot u = 0.036$, while the number of high skilled unemployed barely changes $[1-\gamma^s] \cdot u^s = 0.018$ under segmentation versus $[1-\gamma] \cdot u = 0.016$ under mismatch. So changes in the level of employment (and activity) arise principally through a reduction in the employment of low-skilled workers. Using these insights I consider the impact of the aggregate activity effect on the components of net output. The change in output due to changes in employment of high-skilled workers is $A_{HH} \cdot [[1-\gamma^s] \cdot u^s - [1-\gamma] \cdot u] = 1.564 \cdot 0.002 = 0.003$, while the increase in production costs is $\kappa_H \cdot [[1-\gamma^s] \cdot u^s - [1-\gamma] \cdot u] = 1.008 \cdot 0.002 = 0.002$. Increased aggregate activity on the costs of vacancy posting for high complexity jobs is $c_H \cdot [\theta_H \cdot [1-\gamma] \cdot u - \theta_L^s \cdot [1-\gamma^s] \cdot u^s] = 0.002$. The change in output per capita due to changes in employment of low-skilled workers is $\gamma^s \cdot u^s - \gamma \cdot u = 0.029$, while the change in production costs is $\kappa_L \cdot [\gamma^s \cdot u^s - \gamma \cdot u] = 0.645 \cdot 0.029 = 0.018$. The impact on costs of vacancy posting for low complexity jobs is: $c_L \cdot [\theta_L \cdot \gamma \cdot u - \theta_L^s \cdot \gamma^s \cdot u^s] = 0.645 \cdot [0.796 \cdot 0.692 \cdot 0.052 - 0.226 \cdot 0.785 \cdot 0.082] = 0.009$.

Putting these elements together the net output would be higher at the mismatch levels of activity than at the segmentation levels of activity by $0.003 + 0.029 - [0.002 + 0.002 + 0.018 + 0.009] = 0.001$. This illustrates several key points about the activity effect: (i) it is small relative to the composition effect, (ii) this effect masks stark differences between skills with the size of the effects for low skilled workers being an order of mangnitude greater than for high skilled workers, (iii) for each group of workers the increased production is offset by an associated change in (production and / or posting) costs of vacancies.

So combining the compositional and aggregate activity effects, the former dominate the benefits of mismatch. The compositional effect captures the effect of reallocating labour from segmented to mismatched activities. Although the number of high skilled workers who mismatch to low complexity tasks is twice as large as the number of low-skilled workers who mismatch to high complexity jobs, the increased output of the latter outweigs the decreased output of the former. However, the dominant effect is that the fall in the component of fixed vacancy creation costs that are paid by units currently engaged in production is twice as large for mismatched high skilled owkrers as for mismatched low skilled workers.

While the aggregate activity effect contributes relatively little to the costs of mismatch, it is clear that the underlying changes in activity as a result of eliminating segmentation disproportionately affect low skilled workers.Next I show that this arise through the effect of mismatch on equilibrium vacancy creation.

Tightness in the market for high-complexity positions is lower than in the segmentation equilibrium: $\theta_H^s = 0.347 < 0.503 = \theta_H$. Tightness in the market for low-complexity positions is also lower in the segmentation equilibrium $\theta_L^s = 0.227 < 0.796 = \theta_L$. Mismatch means a substantial number of high-skilled workers join the pool of those seeking low-complexity jobs (which more than offsets the reduced number of low-skilled workers searching for low complexity jobs, since $y_{HL} > y_{LH}$). This increase in the pool of workers would tend to decrease tightness for low-complexity jobs. To explain the observed difference bewteen θ_L and θ_L^s, we need to explore the decision to create new vacancies. Notice that the (3-fold) increase in tightness for low-complexity positions as a result of moving from the segmentation equilibrium to the mismatch equilibrium is greater than the rise in tightness for high-complexity positions. Labour market tightness reflects the impact of profitability of (future) matches on vacancy creation decisions, so changes in the absolute and relative value of labour market tightness offer insight into the behavioural changes that accompany the switch from an equilibrium environment featuring segmentation to one featuring mismatch.

To begin with, consider the impact of segmentation on the decision to create low-complexity vacancies. Given parameters r, K_L, b, P_{LL}, β, δ_L, m_L and α, the free entry condition for low complexity positions determines θ_L^s as the solution to

$$\frac{r + \delta_L + \beta \cdot \theta_L^s \cdot q_L\left(\theta_L^s\right)}{q_L\left(\theta_L^s\right)} = \frac{[1-\beta] \cdot [P_{LL} - r \cdot K_L - b]}{r \cdot K_L}$$

Then, for a mismatch equilibrium under the same parameter values, for which $[1-\tau_H] \cdot [P_{HL} - r \cdot K_L] \geq [P_{LL} - r \cdot K_L]$, $\delta_L \geq \delta_H$, $1 > \tau_H \geq 0$

and $1 > \phi_{LL} \geq 0.5$, (as in the calibrated model) it follows that

$$
\begin{aligned}
\frac{r \cdot K_L}{[1-\beta] \cdot q_L(\theta_L)} &= \begin{array}{c} \phi_{LL} \left[\frac{P_{LL} - r \cdot K_L - b}{r + \delta_L + \beta \cdot \phi_{LL} \cdot \theta_L q_L(\theta_L)}\right] + \\ \frac{[1-\phi_{LL}]}{[1-\tau_H]} \left[\frac{[1-\tau_H] \cdot [P_{HL} - r \cdot K_L] - b}{r + \delta_H + \beta \cdot [1-\phi_{LL}] \cdot \theta_L q_L(\theta_L)}\right] \end{array} \\
&\geq \begin{array}{c} \phi_{LL} \left[\frac{P_{LL} - r \cdot K_L - b}{r + \delta_L + \beta \cdot \phi_{LL} \cdot \theta_L q_L(\theta_L)}\right] + \\ \frac{[1-\phi_{LL}]}{[1-\tau_H]} \left[\frac{P_{LL} - r \cdot K_L - b}{r + \delta_H + \beta \cdot [1-\phi_{LL}] \cdot \theta_L q_L(\theta_L)}\right] \end{array} \\
&\geq \begin{array}{c} \phi_{LL} \left[\frac{P_{LL} - r \cdot K_L - b}{r + \delta_L + \beta \cdot \phi_{LL} \cdot \theta_L q_L(\theta_L)}\right] + \\ [1-\phi_{LL}] \left[\frac{P_{LL} - r \cdot K_L - b}{r + \delta_H + \beta \cdot [1-\phi_{LL}] \cdot \theta_L q_L(\theta_L)}\right] \end{array} \\
&\geq \begin{array}{c} \phi_{LL} \left[\frac{P_{LL} - r \cdot K_L - b}{r + \delta_L + \beta \cdot \phi_{LL} \cdot \theta_L q_L(\theta_L)}\right] + \\ [1-\phi_{LL}] \left[\frac{P_{LL} - r \cdot K_L - b}{r + \delta_L + \beta \cdot [1-\phi_{LL}] \cdot \theta_L q_L(\theta_L)}\right] \end{array} \\
&\geq \begin{array}{c} \phi_{LL} \left[\frac{P_{LL} - r \cdot K_L - b}{r + \delta_L + \beta \cdot \phi_{LL} \cdot \theta_L q_L(\theta_L)}\right] + \\ [1-\phi_{LL}] \left[\frac{P_{LL} - r \cdot K_L - b}{r + \delta_L + \beta \cdot \phi_{LL} \cdot \theta_L q_L(\theta_L)}\right] \end{array} \\
&= \frac{P_{LL} - r \cdot K_L - b}{r + \delta_L + \beta \cdot \phi_{LL} \cdot \theta_L q_L(\theta_L)} \\
&> \frac{P_{LL} - r \cdot K_L - b}{r + \delta_L + \beta \cdot \theta_L \cdot q_L(\theta_L)}
\end{aligned}
$$

So the mismatch equilibrium satisfies the following inequality

$$
\frac{r + \delta_L + \beta \cdot \theta_L \cdot q_L(\theta_L)}{q_L(\theta_L)} > \frac{[1-\beta] \cdot [P_{LL} - r \cdot K_L - b]}{r \cdot K_L}
$$

Since $\theta_L \cdot q_L(\theta_L)$ is increasing in θ_L and $q_L(\theta_L)$ is decreasing in θ_L it follows that tightness for low-complexity jobs is higher under mismatch than under segmentation: $\theta_L > \theta_L^s$. Tightness in the market for low-skilled jobs is higher under mismatch than under segmentation because the expected payoff to creating such a vacancy is higher. This occurs for two reasons: (**i**) mismatch means that vacancies cant be filled by more productive high-killled workers ($[1-\tau_H] \cdot [P_{HL} - r \cdot K_L] \geq [P_{LL} - r \cdot K_L]$), which raises the profit rate under segmentation and (**ii**) those profits will be discounted less heavily because matches with high-skilled workers result in less frequent separation $\delta_L > \delta_H$ and because (holding tightness constant) mismatch reduces the effective exit rate for workers in the pool searching for low-complexity jobs from $\theta_L \cdot q_L(\theta_L)$ to $\phi_{LL} \cdot \theta_L \cdot q_L(\theta_L)$. Higher expected profits from vacancy creation means that mismatch leads to more vacancies for low-complexity jobs being posted and hence to lower unemployment rates amongst the low skilled.[20]

6 Conclusion

The results of this paper demonstrate the empirical relevance of mismatch, propose a straightforward approach to embed mismatch in a canonical model and provide a cost-benefit analysis of mismatch.

I have developed a heteroegeneous agent Diamond-Mortensen -Pissarides model of equilibrium unemployment in which vertical skill mismatch can emerge endogenously as one of a number of decentralised equilibria. I calibrate the mismatch equilibrium to US data using a strategy that ensures consistency of skill-specific objects with observable features of the aggregate economy. By evaluating the outcomes of other feasible equilibria at the parameter values of the calibrated mismatch equilibrium I establish that the unemployed workers prefer the equilibrium with both over skilled and under skilled mismatch.

I have developed estimates of the costs of mismatch based on a comparison of net output in the calibrated mismatch equilibrium versus net output under segmentation. Conventional wisdom views mismatch as a form of misallocation, and hence as costly, however, I have shown that mismatch generates net benefits. In particular, net output is around

[20] Now unlike low-complexity positions, the profit stream from a high-complexity vacancy need not be lower under segmentation: all potential matches are with high-skilled workers and so generate a higher payoff than under segmentation $P_{HH} \geq P_{LH}$, although (holding tightness constant) the impact of on the discount rate of moving to segmentation is ambiguous because while the expected separation rate is lower (as all workers are high-skilled), the exit rate effect again acts to increase the discount rate (holding θ_H constant). Given the limited extent of underskilled mismatch in the calibrated mismatch equilibrium, the impact of segmentation on the expected profitability of future high-complexity matches is likely to be more limited than the impact on low-complexity matches. Hence the overall impact of segmentation on high-complexity tightness is dominated by the increase in the number of workers searching for high-complexity jobs, with a limited effect from the profitability of those jobs, whereas for low-complexity jobs the overall reduction as a result of segmemntation in the number of workers searching for low-complexity jobs is re-inforced by effect of the large(r) decline in the number of low-complexity vacancies caused by the lower profitability of potential matches.

7.5% higher under mismatch than under segmentation. The net benefits of mismatch are consistent with the fact that workers prefer mismatch to segmentation in the decentralied equilibrium.

To shed light on this I presented a decomposition of net output differences between mismatch and segmentation equilibria into compositional effects (holding constant aggregate and skill-specific unemployment) and aggregate activity effects (which take changes in unemployment into account). For the calibrated model, virtually all of the benefits of mismatch arise because of the compositional effect. While there are sizeable differences in the components of net output due to changes in the level of aggregate activity for both low skilled and high skilled workers, these effects cancel out. Although the mismatch equilibrium features (more than) twice as many high-skilled workers mismatched in low complexity jobs as there are low skilled workers mismatched in high complexity jobs, the net benefits of the composition effect arise because the productivity differences between mismatch and segmentation are smaller for high skilled workers and the reduction in vacancy creation costs is larger. In addition the possibility of mismatch appears particularly beneficial for low skilled workers, who exhibit much lower unemployment in a mismatch equilibrium. I have shown that this is because of the effect on the profitability of low complexity jobs and the endogenous response of job creation to mismatch.

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7 Appendix A: Characterising Putative Equilibria

Other feasible equilibria can be constructed as restricted versions of the mismatch equilibrium discussed in Section 2. Rather than clutter the main text with discussion of these alternative equilibria I explore these alternative equilibria below. I describe the conditions that characterised three equilibria: **(i)** the segmentation equilibrium, **(ii)** the equilibrium with over-skilled mismatch (only), **(iii)** the equilibrium with under-skilled mismatch (only). In principle each of these equilibria could be calibrated (afresh) to the data, yielding equilibrium-specific values of the underlying parameters, such as the match-specific productivities, complexity-specific vacancy creation costs and skill-specific separation rates that match values of aggregate unemployment observed in the data. However, in this paper I wish to take the parameter values from the calibration of the mismatch equilibrium in Section 3 as given and instead determine equilibrium objects, such as skill-specific and aggregate unemployment rates, and complexity-specific labour market tightness for each equilibrium. To emphasise this I adopt the following notation when describing each equilibrium: a super-scripted object, x^a represents the value of x in equilibrium $a \in \{s, o, u\}$, whereas an un-super-scripted object, z, represents the value of (parameter) z obtained by calibrating the mismatch equilibrium. This allows a ceteris paribus comparison of mismatch versus segmentation without altering other features of the economy.[21] I also focus on the minimum number of conditions required to characterise eqach equilibrium. To illustrate this, and because I use it as the reference point in evaluating the costs of mismatch, I provide a more detailed discussion in the context of the segmentation equilibrium

[21] This could not be achieved by comparing the calibrated outcome of the mismatch equilibrium with the outcome of the equilibrium with, say, the segmentation calibrated directly to match US data (while imposing $\lambda_{LL} = \lambda_{HH} = \phi_{LL} = \phi_{HH} = s_{LL} = s_{HH} = 1$) because such an approach would lead to different segmentation-equilibrium-calibration-specific values of deep parameters, such as match-specific productivities, $\hat{P}_{jj}$, the value of leisure, $\hat{b}$, and so on. If this approach were used then differences would reflect changes in deep parameters as a result of re-calibration rather than differences in key variables, such as output, unemployment, and skill-requirement-specific labour market tightness.

7.1 Characterising the Segmentation Equilibrium

The segmentation equilibrium arises when high-skilled workers only search for high-complexity jobs and low-skilled workers search only for low-complexity jobs. It requires $\lambda_{LL} = \lambda_{HH} = \phi_{LL} = \phi_{HH} = s_{LL} = s_{HH} = 1$ in the model outlined in Section (2). It can be fully characterised by 7 equations. These include four equations summarising aspects of labour market flows governing: (**i**) the relationship, (36), between aggregate tightness under segmentation and complexity-specific labour market tightness, (**ii**) the relationship, (37), between the exit rate from aggregate unemployment and the exit rates from skill-specific unemployment; (**iii**) the mass balance equation, (38), for low-skilled worker flows and (**iv**) the mass balance equation, (39), for high skilled worker flows.

$$\theta^s = \gamma^s \cdot \theta^s_L + [1-\gamma^s] \cdot \theta^s_H \tag{36}$$

$$\theta^s \cdot q^s(\theta^s) = \gamma^s \cdot \theta^s_L \cdot q_L(\theta^s_L) + [1-\gamma^s] \cdot \theta^s_H \cdot q_H(\theta^s_H) \tag{37}$$

$$\theta^s_L \cdot q_L(\theta^s_L) \cdot \gamma^s \cdot u^s = \delta_L \cdot [\mu - \gamma^s \cdot u^s] \tag{38}$$

$$\theta^s_H \cdot q_H(\theta^s_H) \cdot [1-\gamma^s] \cdot u^s = \delta_H \cdot [1 - \mu - [1-\gamma^s] \cdot u^s] \tag{39}$$

Under segmentation, the value of the low-complexity and high-complexity vacancies together with the free entry conditions $\mathcal{V}_j = K_j,\ j \in \{L, H\}$ satisfy

$$r \cdot K_L = [1-\beta] \cdot q_L(\theta^s_L) \left[\frac{P_{LL} - r \cdot K_L - b}{r + \delta_L + \beta \cdot \theta^s_L \cdot q_L(\theta^s_L)} \right] \tag{40}$$

$$r \cdot K_H = [1-\beta] \cdot \frac{q_H(\theta^s_H)}{[1-\tau^s_H]} \cdot \left[\frac{[1-\tau^s_H] \cdot [P_{HH} - rK_H] - b}{r + \delta_H + \beta \cdot \theta^s_H \cdot q_H(\theta^s_H)} \right] \tag{41}$$

The final condition is the government budget constraint which is written as

$$b \cdot u^s = \tau^s_H \cdot w^s_{HH}. \tag{42}$$

where $w^s_{HH} = \beta \cdot [P_{HH} - r \cdot K_H] + \frac{[1-\beta] \cdot r \cdot \mathcal{U}^s_{HH}}{[1-\tau^s_H]}$.

In determining the segmentation equilibrium, I take as given both the equilibrium-invariant general parameters, $\{r, \alpha, \delta, \mu, m_0, \beta\}$, and the parameters identified under the mismatch equilibrium $\{\delta_L, \delta_H, m_L, m_H, A_{LL}, A_{HH}, b, K_L, K_H\}$ covering skill-specific separation rates, δ_j, matching efficiencies, m_j, match-specific productivities, $A_{jj} = P_{jj}$, $j \in \{L, H\}$, unemployment benefits, b, and, K_j , $j \in \{L, H\}$ the cost of creating a vacancy for a j-complexity position. The aim of this approach is to generate values for the following endogenous variables in the segmentation equilibrium: u^s, unemployment, γ^s, the fraction of the unemployed who are low-skilled, θ_L^s, tightness for low-complexity jobs and θ_H^s, tightness for high-complexity jobs and τ_H^s the tax rate. Where the notation X^s is used to refer to an endogenous object in the segmentation equilibrium under the parameters of the mismatch calibration.

Then (40) combined with the functional form assumption $\theta_L^s \cdot q_L(\theta_L^s) = m_L \cdot [\theta_L^s]^{1-\alpha}$ and $q_L(\theta_L) = m_L \cdot [\theta_L^s]^{-\alpha}$, determine θ_L^s, the tightness of the market for low-complexity jobs as the solution to:

$$r \cdot K_L = [1-\beta] \cdot m_L \cdot [\theta_L^s]^{-\alpha} \cdot \left[\frac{P_{LL} - r \cdot K_L - b}{r + \delta_L + \beta \cdot m_L \cdot [\theta_L^s]^{1-\alpha}}\right].$$

Similarly θ_H^s, the tightness of the market for high-complexity jobs is obtained as the solution to

$$rK_H = [1-\beta] \cdot m_H \cdot [\theta_H^s]^{-\alpha} \cdot \left[\frac{P_{HH} - r \cdot K_H - b}{r + \delta_H + \beta \cdot m_H \cdot \theta_H^{1-\alpha}}\right]$$

The flows equations for low-skilled workers under segmentation and for high-skilled workers under segmentation are then a system of 2 equations in γ^s and u^s. These equations can be combined to eliminate γ^s and give the following expression for unemployment under segmentation:

$$u^s = \frac{\delta_L \cdot \mu}{\delta_L + \theta_L^s \cdot q_L(\theta_L^s)} + \frac{\delta_H \cdot [1-\mu]}{\delta_H + \theta_H^s \cdot q_H(\theta_H^s)}.$$

Then the fraction of unemployed workers who are low-skilled in the segmentation equilibrium is given as

$$\gamma^s = \frac{\delta_L \cdot \mu \cdot [\delta_H + \theta_H^s \cdot q_H(\theta_H^s)]}{\delta_L \cdot \mu \cdot [\delta_H + \theta_H^s \cdot q_H(\theta_H^s)] + \delta_H \cdot [1-\mu] \cdot [\delta_L + \theta_L^s \cdot q_L(\theta_L^s)]}$$

While the segmentation equilibrium is characterised by 6 equations, there are only 5 endogenous variables $\{u^s, \gamma^s, \theta_L^s, \theta_H^s, \tau_H^s\}$. To make use of the final equilibrium condition, I allow the cost of high-complexity vacancies to be determined, K_H. It turns out that this makes little difference to the flow equivalent cost of vacancy creation used in the net output calculations.

Using this information I can determine **(i)** net output under mismacth $\omega^s = y^s - c_H \cdot \frac{V_H^s}{N} - \kappa_H \cdot y_{HH}^s - c_L \cdot \frac{V_L^s}{N} - \kappa_L \cdot y_{LL}^s$, used in Section 5 to compute the costs of mismatch and **(ii)** the value of unemployment for high-skilled and low-skilled workers used in Section 4 to assess the mismatch equilibrium:

$$r\mathcal{U}_{HH} = \frac{[r+\delta_H]\,b + [1-\tau_H^s] \cdot \beta \cdot \theta_H^s \cdot q_H(\theta_H^s) \cdot [P_{HH} - r \cdot K_H]}{r + \delta_H + \beta \cdot \theta_H^s \cdot q_H(\theta_H^s)},$$

$$r\mathcal{U}_{LL}^s = \frac{[r+\delta_L] \cdot b + \beta \cdot \theta_L^s \cdot q_L(\theta_L^s) \cdot [P_{LL} - r \cdot K_L]}{r + \delta_L + \beta \cdot \theta_L^s \cdot q_L(\theta_L^s)}.$$

7.2 Characterising the Equilibrium with Over-skilled Mismatch (only)

The equilibrium featuring over-skilled mismatch (only) arises when high-skilled workers search for both high-complexity jobs and low-complexity jobs, while low-skilled workers search only for low-complexity jobs. It requires $\lambda_{LL} = \phi_{HH} = s_{LL} = 1$ in the model outlined in Section (2). It can be fully characterised by 10 equations. These include six equations summarising aspects of labour market flows governing: **(i)** the relationship, (43), between aggregate tightness under segmentation and complexity-specific labour market tightness, **(ii)** the relationship, (44), between the exit rate from aggregate unemployment and the exit rates from skill-specific unemployment; **(iii)** the mass balance equation, (45), for low-skilled worker flows and **(iv)** the mass balance equation, (46), for high skilled worker flows, **(v)** the fraction of all those searching for low complexity jobs who are low-skilled, (48) **(vi)** the fraction of high-skilled workers who are not mismatched, (48).

$$\theta^o = \theta_L^o \cdot [\gamma^o + [1-\lambda_{HH}^o] \cdot [1-\gamma^o]] + \theta_H^o \cdot \lambda_{HH}^o \cdot [1-\gamma^o] \tag{43}$$

$$\theta^o \cdot q^o(\theta^o) = \begin{array}{c} \theta_L^o \cdot q_L(\theta_L^o) \cdot [\gamma^o + [1-\lambda_{HH}^o] \cdot [1-\gamma^o]] \\ +\theta_H^o \cdot q_H(\theta_H^o) \cdot \lambda_{HH}^o \cdot [1-\gamma^o] \end{array} \tag{44}$$

$$\delta_L \cdot [\mu - \gamma^o \cdot u^o] = \theta_L^o \cdot q_L(\theta_L^o) \cdot \gamma^o \cdot u^o \tag{45}$$

$$\delta_H \cdot [1-\mu-[1-\gamma^o] \cdot u^o] = [[1-\lambda_{HH}^o] \cdot \theta_L^o \cdot q_L(\theta_L^o) + \lambda_{HH}^o \cdot \theta_H^o \cdot q_H(\theta_H^o)] \cdot [1-\gamma^o] u^o \tag{46}$$

$$\phi_{LL}^o = \frac{\gamma^o}{\gamma^o + [1-\gamma^o] \cdot [1-\lambda_{HH}^o]} \tag{47}$$

$$s_{HH}^o = \frac{\lambda_{HH}^o \cdot \theta_H^o \cdot q_H(\theta_H^o)}{[1-\lambda_{HH}] \cdot \theta_L^o \cdot q_L(\theta_L^o) + \lambda_{HH}^o \cdot \theta_H^o \cdot q_H(\theta_H^o)} \tag{48}$$

One arbitage condition for unemployed high-skilled workers, which ensures that they mismatch rather undertake segmented search, (49):

$$\mathcal{U}_{HL}^o = \mathcal{U}_{HH}^o \tag{49}$$

Two free entry conditions for low-complexityvacancies, (50), and high-complexity vacancies, (51) (together with the free entry conditions $\mathcal{V}_j = K_j$, $j \in \{L, H\}$):

$$r \cdot K_L = [1-\beta] \cdot q_L(\theta_L^o) \cdot \begin{bmatrix} \phi_{LL}^o \cdot \frac{[P_{LL} - r \cdot K_L] - b}{r + \delta_L + \beta \cdot \phi_{LL}^o \cdot \theta_L^o \cdot q_L(\theta_L^o)} + \\ [1-\phi_{LL}^o] \cdot \frac{[1-\tau_H^o] \cdot [P_{HL} - r \cdot K_L] - b}{[1-\tau_H^o] \cdot [r + \delta_H + \beta \cdot [1-\phi_{LL}^o] \cdot \theta_L^o \cdot q_L(\theta_L^o)]} \end{bmatrix} \tag{50}$$

$$r \cdot K_H = [1-\beta] \cdot q_H(\theta_H^o) \cdot \frac{[1-\tau_H^o] \cdot [P_{HH} - r \cdot K_H] - b}{[1-\tau_H^o] \cdot [r + \delta_H + \beta \cdot \theta_H^o \cdot q_H(\theta_H^o)]}. \tag{51}$$

and the government budget constraint, (52):

$$b \cdot u^o = \tau_H^o \cdot [w_{HH}^o \cdot s_{HH}^o + w_{HL}^o \cdot [1 - s_{HH}^o]]. \tag{52}$$

where $w_{HH}^o = \beta \cdot [P_{HH} - r \cdot K_H] + \frac{[1-\beta] \cdot r \cdot \mathcal{U}_{HH}^o}{[1-\tau_H^o]}$ and $w_{HL}^o = \beta \cdot [P_{HL} - r \cdot K_L] + \frac{[1-\beta] \cdot r \cdot \mathcal{U}_{HL}^o}{[1-\tau_H^o]}$. The values of unemployment for high-skilled and low-skilled workers used in Section 4 to assess the mismatch equilibrium are:

$$r \cdot \mathcal{U}^o_{LL} = \frac{[r + \delta_L] \cdot b + \beta \cdot \phi^o_{LL} \cdot \theta^o_L \cdot q_L(\theta^o_L)[P_{LL} - r \cdot K_L]}{r + \delta_L + \beta \cdot \phi_{LL} \cdot \theta_L q_L(\theta_L)}, \quad (53)$$

$$r \cdot \mathcal{U}^o_{HH} = \frac{[r + \delta_H] \cdot b + [1 - \tau^o_H] \cdot \beta \cdot \theta^o_H \cdot q_H(\theta^o_H) \cdot [P_{HH} - r \cdot K_H]}{r + \delta_H + \beta \cdot \theta^o_H \cdot q_H(\theta^o_H)}, \quad (54)$$

$$r \cdot \mathcal{U}^o_{HL} = \frac{[r + \delta_H] \cdot b + [1 - \tau^o_H] \cdot \beta \cdot [1 - \phi^o_{LL}] \cdot \theta^o_L \cdot q_L(\theta^o_L) \cdot [P_{HL} - r \cdot K_L]}{r + \delta_H + \beta \cdot [1 - \phi^o_{LL}] \cdot \theta^o_L \cdot q_L(\theta^o_L)} \quad (55)$$

7.3 Characterising the Equilibrium with Under-skilled Mismatch (only)

The equilibrium featuring over-skilled mismatch (only) arises when high-skilled workers search for both high-complexity jobs and low-complexity jobs, while low-skilled workers search only for low-complexity jobs. It requires $\lambda_{HH} = \phi_{LL} = s_{HH} = 1$ in the model outlined in Section (2). It can be fully characterised by 10 equations. These include six equations summarising aspects of labour market flows governing: (i) the relationship, (56), between aggregate tightness under segmentation and complexity-specific labour market tightness, (ii) the relationship, (57), between the exit rate from aggregate unemployment and the exit rates from skill-specific unemployment; (iii) the mass balance equation, (58), for low-skilled worker flows and (iv) the mass balance equation, (59), for high skilled worker flows, (v) the fraction of all those searching for low complexity jobs who are low-skilled, (47) (vi) the fraction of high-skilled workers who are not mismatched, (61).

$$\theta^u = \theta_L^u \cdot \lambda_{LL}^u \cdot \gamma^u + \theta_H^u \cdot [[1 - \lambda_{LL}^u] \cdot \gamma^u + [1 - \gamma^u]], \quad (56)$$

$$\theta^u \cdot q^u(\theta^u) = \frac{\theta_L^u \cdot q_L(\theta_L^u) \cdot \lambda_{LL}^u \cdot \gamma^u +}{\theta_H^u \cdot q_H(\theta_H^u) \cdot [[1 - \lambda_{LL}^u] \cdot \gamma^u + [1 - \gamma^u]]}, \quad (57)$$

$$\delta_L \cdot [\mu - \gamma^u \cdot u^u] = [\lambda_{LL}^u \cdot \theta_L^u \cdot q_L(\theta_L^u) + [1 - \lambda_{LL}^u] \cdot \theta_H^u \cdot q_H(\theta_H^u)] \cdot \gamma^u \cdot u^u, \quad (58)$$

$$\delta_H \cdot [1 - \mu - [1 - \gamma^u] \cdot u^u] = \theta_H^u \cdot q_H(\theta_H^u) \cdot [1 - \gamma^u] \cdot u^u, \quad (59)$$

$$\phi_{HH}^u = \frac{[1 - \gamma^u]}{\gamma^u \cdot [1 - \lambda_{LL}^u] + [1 - \gamma^u]}, \quad (60)$$

$$s_{LL}^u = \frac{\lambda_{LL}^u \cdot \theta_L^u \cdot q_L(\theta_L^u)}{\lambda_{LL}^u \cdot \theta_L^u \cdot q_L(\theta_L^u) + [1 - \lambda_{LL}^u] \cdot \theta_H^u \cdot q_H(\theta_H^u)}. \quad (61)$$

An arbitage condition for unemployed low-skilled workers, which ensures that they mismatch rather undertake segmented search, (62),

$$\mathcal{U}_{LL}^u = \mathcal{U}_{LH}^u \quad (62)$$

Two free entry conditions for low-complexity vacancies, (63), and high-complexity vacancies, (64) (together with the free entry conditions $\mathcal{V}_j = K_j$, $j \in \{L, H\}$):

$$r \cdot K_L = [1 - \beta] \cdot q_L(\theta_L^u) \cdot \frac{P_{LL} - r \cdot K_L - b}{r + \delta_L + \beta \theta_L^u q_L(\theta_L^u)}, \quad (63)$$

$$r \cdot K_H = [1 - \beta] \cdot q_H(\theta_H^u) \begin{bmatrix} \phi_{HH}^u \cdot \frac{[1-\tau_H^u] \cdot [P_{HH} - r \cdot K_H] - b}{[1-\tau_H^u] \cdot [r + \delta_H + \beta \cdot \theta_H^u \cdot q_H(\theta_H^u)]} + \\ [1 - \phi_{HH}^u] \cdot \frac{[P_{LH} - r \cdot K_L] - b}{r + \delta_L + \beta \cdot [1 - \phi_{HH}^u] \cdot \theta_L^u \cdot q_L(\theta_L^u)} \end{bmatrix} . \quad (64)$$

and the government budget constraint, (65):

$$b \cdot u^u = \tau_H^u \cdot w_{HH}^u \quad (65)$$

where $w_{HH}^u = \beta \cdot [P_{HH} - r \cdot K_H] + \frac{[1-\beta] \cdot r \cdot \mathcal{U}_{HH}^u}{[1-\tau_H^u]}$. The values of unemployment for high-skilled and low-skilled workers used in Section 4 to assess

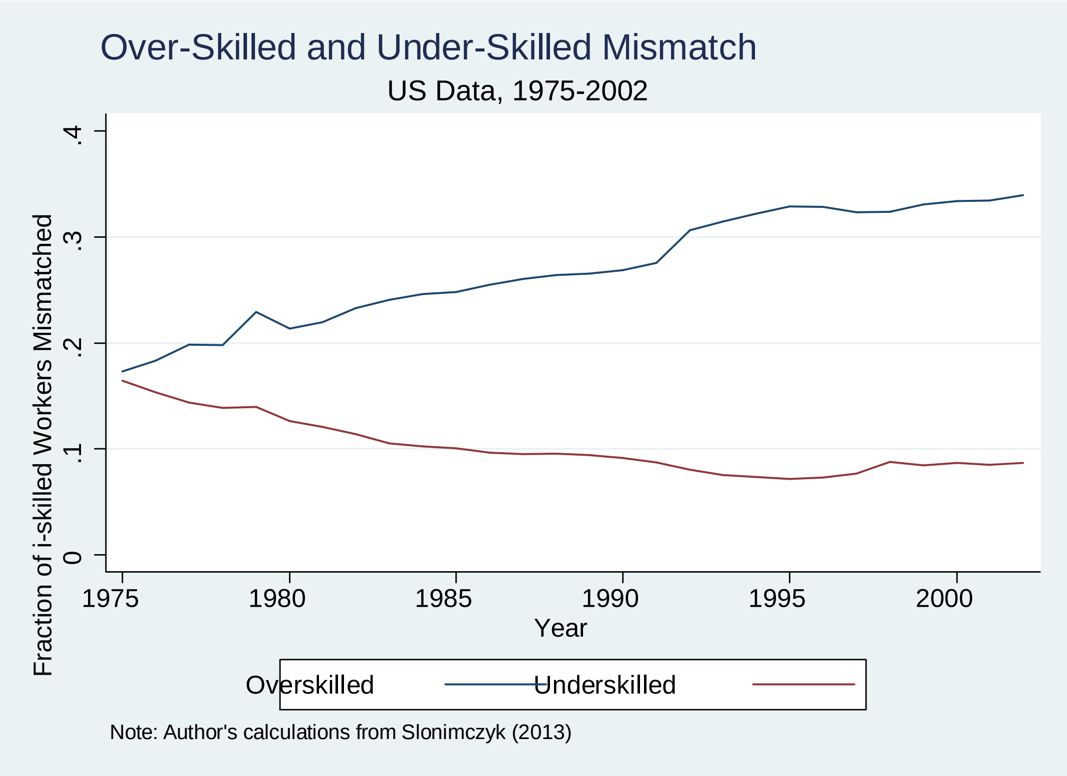

Figure 1:

the mismatch equilibrium are:

$$r \cdot \mathcal{U}_{HH}^{u} = \frac{[r+\delta_H] \cdot b + [1-\tau_H^u] \cdot \beta \cdot \phi_{HH}^u \cdot \theta_H^u \cdot q_H(\theta_H^u) \cdot [P_{HH} - r \cdot K_H]}{r + \delta_H + \beta \cdot \phi_{HH}^u \cdot \theta_H^u \cdot q_H(\theta_H^u)},$$

$$r \cdot \mathcal{U}_{LL}^{u} = \frac{[r+\delta_L] \cdot b + \beta \cdot \theta_L^u \cdot q_L(\theta_L^u) \cdot [P_{LL} - r \cdot K_L]}{r + \delta_L + \beta \cdot \theta_L^u \cdot q_L(\theta_L^u)},$$

$$r \cdot \mathcal{U}_{LH}^{u} = \frac{[r+\delta_L] \cdot b + \beta \cdot [1-\phi_{HH}^u] \cdot \theta_H^u \cdot q_H(\theta_H^u) \cdot [P_{LH} - r \cdot K_H]}{r + \delta_L + \beta \cdot [1-\phi_{HH}^u] \cdot \theta_H^u \cdot q_H(\theta_H^u)}.$$

Notation	**Meaning**	**Value**	**Source / Rationale**
	Aggregate Parameters		
u	Unemployment rate	0.052	Data
$\theta \cdot q(\theta)$	Exit rate from unemployment	0.037	Shimer flows data 1973-2002
θ	Aggregate labour market tightness	0.720	Pissarides (2009)
α	Elasticity of matching function	0.500	Petrongolo & Pissarides (2001)
β	Worker bargaining power	0.500	Pissarides (2009)
r	Real interest rate	0.004	Data: annual real rate = 4%
	Skill Specific Parameters		
μ	Labour force share of low-skilled	0.528	Data
u_L	Unemployment rate of low-skilled	0.068	Data
s_{LL}	Fraction of low skilled workers in low complexity jobs	0.899	Slominczyk (2013)
s_{HH}	Fraction of high skilled workers in high complexity jobs	0.742	Slominczyk (2013)
δ_{HL}	Ratio of separation rates for high skilled : low skilled	0.574	Fallick and Fleischmann (2001)
χ	Wage premium high skilled: low skilled	1.68	Acemoglu and Autor (2012)
P_{LL}	Low-skilled worker productivity in low complexity job	1	Normalisation
P_{LH}	Low-skilled worker productivity in high complexity job	P_{HL}	Normalisation
K_H	Fixed cost of creating high complexity vacancy	$P_{HH} \cdot K_L$	Normalisation
τ_L	Income tax rate for low skilled workers	0	Normalisation

Table 1: Parameter Values

Name	Meaning	Value	Equations
δ	Aggregate separation rate	0.037	(3)
γ	fraction of unemployed who are low-skilled	0.692	(1)
δ_L	Low-skilled separation rate	0.041	(3) (6), (4) & (5)
δ_H	High-skilled separation rate	0.023	(27)
λ_{LL}	Fraction of L-unemployed searching for low complexity jobs,	0.922	(4), (5), (9) & (10)
λ_{HH}	Fraction of H-unemployed searching for high complexity jobs,	0.684	
$\theta_L q_L(\theta_L)$	Exit rate from unemployment to low complexity jobs,	0.546	
$\theta_H q_H(\theta_H)$	Exit rate from unemployment to high complexity jobs,	0.726	
ϕ_{LL}	Fraction of low-complexity job -searchers who are low-skilled,	0.868	(7)
ϕ_{HH}	Fraction of high-complexity job -searchers who are high-skilled,	0.796	(8)
θ_L	Tightness for low-complexity jobs,	0.796	(2),(11),(18),(19), (20),(21) & (25)
θ_H	Tightness for high-complexity jobs,	0.503	
b	Unemployment benefits,	0.164	
τ_H	Tax rate on high-skilled wages,	0.032	
P_{HL}	Productivity of high-skilled workers in low complexity jobs,	1.430	
P_{HH}	Productivity of high-skilled workers in high complexity jobs,	1.564	
$r \cdot K_L$	Cost of creating a low-complexity job,	0.645	
m_L	Efficiency of matching technology for low complexity jobs,	0.696	(28)
m_H	Efficiency of matching technology for high complexity jobs,	1.164	(29)

Table 2: Calibrated Values and Calibration Strategy

	$\mathcal{U}_{HH} > \mathcal{U}_{HL}$	$\mathcal{U}_{HH} = U_{HL}$
$\mathcal{U}_{LL} > \mathcal{U}_{LH}$	Segmentation	Over-skilled mismatch
$\mathcal{U}_{LL} = \mathcal{U}_{LH}$	Under-skilled mismatch (only)	Mismatch

Table 3: Workers Search Patterns and Feasible Equilibria

Skill	**Mismatch** $\mathcal{U}_{ii}$	**Segmentation** $\mathcal{U}_{ii}^{s}$	**Under-skilled Mismatch,** $\mathcal{U}_{ii}^{u}$	**Over-skilled Mismatch,** $\mathcal{U}_{ii}^{o}$
High	0.5052	0.4948	0.4993	0.5054
Low	0.3248	0.3103	0.3150	0.3248

Table 4: Skill-Specific Capitalised Value of Unemployment

	Mismatch	**Segmentation**
u	0.052	0.083
γ	0.692	0.786
θ_H	0.503	0.347
θ_L	0.796	0.227
τ_H	0.032	0.055
y_{LL}	0.400	0.463
y_{LH}	0.049	-
y_{HL}	0.118	-
y_{HH}	0.338	0.454
y	1.211	1.174
ω	0.432	0.401

Table 5: Outcomes under Mismatch and Segmentation

Component of net output difference	**Composition** $\omega - \hat{\omega}$	**Aggregate Activity** $\hat{\omega} - \omega^s$	**Total** $\omega - \omega^s$
output of high skilled	-0.016	0.003	-0.013
output of low skilled	0.021	0.029	0.050
vacancy costs (*production*) high skilled	-0.043	0.002	-0.041
vacancy costs (*production*) low skilled	0.018	0.018	0.036
vacancy costs (*posting*) high complexity	-0.001	0.002	0.001
vacancy costs (*posting*) low complexity	0.001	0.009	0.010
Total net output	0.030	0.001	0.031

Table 6: Decomposition of Net Output Differences