

Uncertainty

Lecture 9

Reading: Perloff Chapter 17

August 2015

- Everything is uncertain.
- We don't know what *state of nature* we will be in tomorrow.
- We looked at consumer choice over consumption bundles.
- Now we will look at consumer choice over a variety of risky alternatives.

- How much insurance should I buy?
- What is the optimal portfolio given my attitude towards risk?
- Should I cross the street when the light is red?

- **Probability Overview** - How can we measure risk?
- **Decision making under uncertainty** - The decisions people make depend on their attitudes towards risk.
- **Avoiding risk** - People can take steps to avoid risk altogether.
- **Behavioral economics of risk** - Do people behave like we have just described?

Probability Overview

- We don't know what state of nature will be in tomorrow.
- But we might know the relative likelihood of different states of nature.
- Tomorrow is a roll of the dice, it is a random outcome.

Probability Overview

- The gender of the next person you meet, your grade on an exam, the number of times your computer crashes all have an element of randomness.
- An **outcome** is a mutually exclusive result of the random process.
- Your computer might crash once, it might never crash... these are outcomes.

Probability Overview

- The **probability** of an outcome is the proportion of time that outcome occurs in the long run
- The probability of rolling a 4 is $\frac{1}{6}$. If we rolled a dice 1,000,000,000 times, we would get 4 $\frac{1}{6}$ of the time
- We could also express it as the probability of event X is

$$\Pr[X] = \frac{\textit{all events favorable to } X}{\textit{all possible events}}$$

Probability Overview

- A **random variable** is a numerical summary of a random outcome.
- If my computer crashes the variable takes a 1, if not it takes a 0 for example.
- The roll you get from a dice is a random variable.

- A **discrete random variable** takes on only a discrete set of values, like 0, 1, 2, 3.
- A **continuous random variable** takes on a continuum of possible values (any point on some measure of the real line).
- Roll of the dice is a discrete random variable, the amount of rain is a continuous random variable.

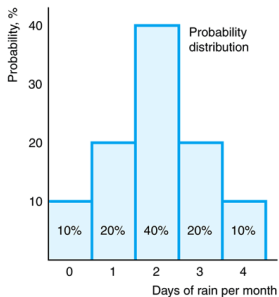
- A **probability distribution** is the list of all possible values of the random variable and the probability they will occur.
- A **cumulative probability distribution** is the probability that the random variable is less than or equal to some value.

EXAMPLE

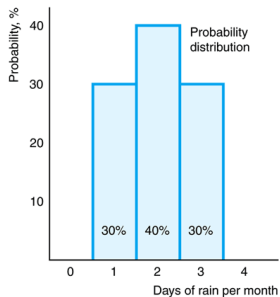
- What is the probability distribution and cumulative probability distribution for a dice role?

Probability Overview

(a) Less Certain



(b) More Certain



Probability Overview

- The **expected value** of a random variable Y is the long run average value the random variable will take over many repeated trials
- It is the weighted average of all possible values (also called expectation or mean)
- If Y is discrete

$$E[Y] = \sum_{i=1}^{\infty} p_i x_i$$

- If Y is continuous

$$E[Y] = \int_{-\infty}^{\infty} x f(x) dx$$

Probability Overview

- Lets say X is the roll of a die, which has 6 possible outcomes all equally likely

$$E[X] = \frac{1}{6}1 + \frac{1}{6}2 + \frac{1}{6}3 + \frac{1}{6}4 + \frac{1}{6}5 + \frac{1}{6}6 = 3.5$$

- If you were to roll a fair die 1,000,000,000,000 times, on average you would get a value of 3.5

EXAMPLE

- I give you £100 if you flip a heads and £0 if you flip a tails. What is your expected payoff from this game?

Probability Overview

- The **variance** of some random variable X is a measure of how spread out the distribution is. If the variance is high You are less likely to get a value close to the mean

$$\text{Var}[X] = \sum_{i=1}^n p_i (x_i - E[X])^2$$

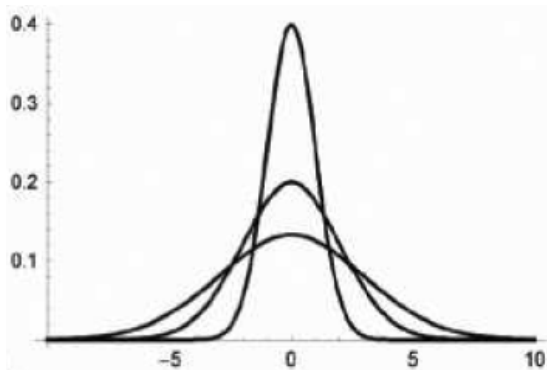
$$\text{Var}[X] = E[X^2] - (E[X])^2$$

- Or if continuous

$$\text{Var}[X] = \int (x - E[X])^2 f(x) dx$$

- The **standard deviation** of X is just the square root of the variance $S.D.[X] = \sigma_x = \sqrt{\sigma_x^2}$

Probability Overview



Decision Making Under Uncertainty

- Remember in chapter 3 we learned about preferences over bundles of goods
- People now have preferences over lotteries
- People will pick the lottery that gives them the highest utility.
- You don't know exactly what utility you will get, however.
- People maximize their **expected utilities**
- The **expected utility** of some action is the probability weighted sum of utilities you get associated with each possible outcome.

Decision Making Under Uncertainty

- Suppose with you the following lottery.
- With probability $\frac{1}{2}$, you win £10 and with probability $\frac{1}{2}$ you win £1
- Your utility function is $U = 10 * w$ where w is wealth.

- Your expected utility is

$$EU = \Pr[\textit{heads}] * U(\textit{heads}) + \Pr[\textit{tails}] * U(\textit{tails})$$

$$EU = \frac{1}{2} * 10(10) + \frac{1}{2} * 10(1) = 55$$

EXAMPLE

- Suppose you have a dice roll and you win the value of the dice roll (if you roll a 6 you get \$6).
- Write down an equation for expected utility of an individual who has a utility function $U = \ln w$ (don't actually calculate the number).

Decision Making Under Uncertainty

- Two people might face the same lottery, but they will get a different expected utility.
- I buy lottery tickets, many people don't.
- The shape of your utility function determines your attitude over risk.

Decision Making Under Uncertainty

- A **fair bet** is a wager with an expected value of 0.
- You pay \$10 to play the lottery and the expected value of the ticket is \$10. This is a fair bet. On expectation you do not lose money.

EXAMPLE

- You win the value of the dice roll (roll a 6 get \$6 for example)
- If you have to pay £4 to play this game, is that a fair bet?

Decision Making Under Uncertainty

- Suppose I offer you a **fair bet**.
- People who are **risk neutral** are indifferent between taking this bet or not
- People who are **risk loving** will always take this bet
- People who are **risk averse** will not.

Decision Making Under Uncertainty

- Let's look at the risk averse people first.
- If somebody is **risk averse**, they have a diminishing marginal utility to wealth.
- Their utility function is concave.
- The extra utility they get from one more dollar is less than they got from the last.

Decision Making Under Uncertainty

- Suppose somebody's utility function is $U = W^{\frac{1}{2}}$.
- This person can play the following game.
 - You win \$2 with probability $\frac{1}{2}$ or you win \$0 with probability $\frac{1}{2}$
 - The expected value of this game is \$1

Decision Making Under Uncertainty

- The person's expected utility from this lottery is

$$EU = .5 * (2)^{\frac{1}{2}} + .5 * (0)^{\frac{1}{2}} = .7$$

- The person's utility of having the \$1 for sure is $(1)^{\frac{1}{2}} = 1$.
- This person gets a higher utility from \$1 for sure than a bet whose expected payoff is \$1.
- This person is risk averse.

Decision Making Under Uncertainty

- The **risk premium** is the amount that a risk averse person will pay to avoid taking a risk.
- In the previous example, we know the lottery gives us an expected utility of .7.
- To find the risk premium, we need to find the amount of money we would be willing to give up to eliminate risk altogether.
- The lottery in this case has an expected value of \$1, how much of that \$1 would I give up?

Decision Making Under Uncertainty

$$\begin{aligned}(X)^{\frac{1}{2}} &= .7 \\ X &= .49\end{aligned}$$

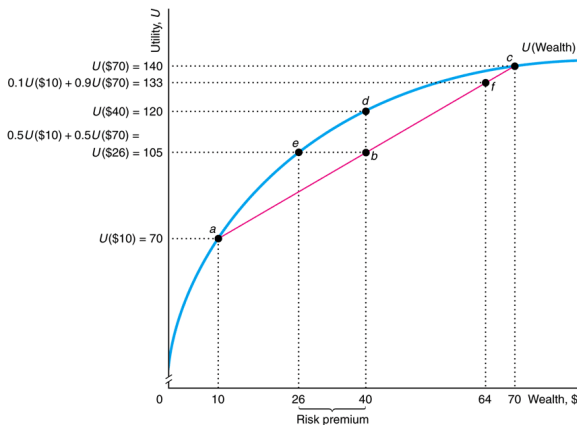
- I am indifferent between playing the lottery with an expected value of \$1 and having \$0.49 cents for sure.
- The risk premium is \$0.51. I would give up \$0.51 to avoid risk altogether.

EXAMPLE

- With probability $\frac{1}{4}$ the car turns out to be bad and is worth \$400.
- With probability $\frac{3}{4}$ the car turns out to be good and is worth \$1600.
- What is the expected **payoff** from buying this car?
- If this person has a utility function $U = W^{\frac{1}{2}}$, what is the expected utility of buying this car?
- What is the risk premium? What is the most this person would be willing to pay for this car?

Decision Making Under Uncertainty

- This is what a risk-averse person's utility function looks like.



- This person would give up 14 dollars to have no risk at all.

Decision Making Under Uncertainty

- Now let's look at a risk neutral person.
- A risk-neutral person has a constant marginal utility over wealth, her utility function is a straight line.
- One extra dollar is worth just as much to her no matter how much money she has.

Decision Making Under Uncertainty

- The utility she gets from the expected value of the bet is exactly equal to the expected utility.
- A risk neutral person will take the bet with the highest expected value.

Decision Making Under Uncertainty

- Suppose somebody has a utility function $U = W$.
- This person can play the following game.
 - You win \$2 with probability $\frac{1}{2}$ or you win \$0 with probability $\frac{1}{2}$
 - The expected value of this game is \$1

Decision Making Under Uncertainty

- Will this person take the bet?
- The expected utility of the bet is

$$\frac{1}{2}2 + \frac{1}{2}0 = 1$$

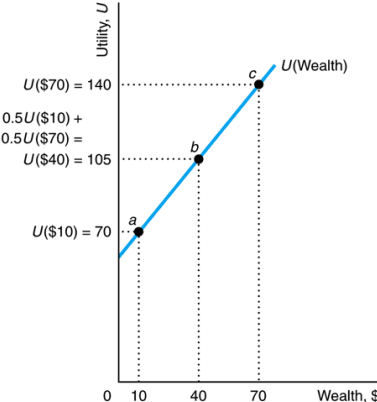
- The utility of having \$1 for sure is exactly the same.
- The presence of risk does nothing.

Decision Making Under Uncertainty

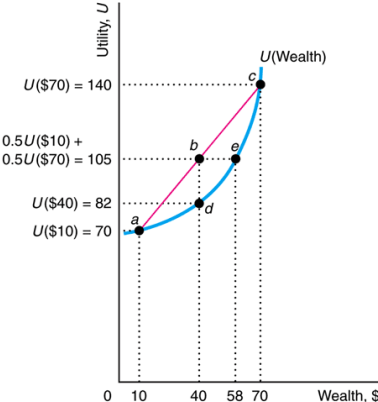
- Now let's look at a risk loving person.
- A risk loving person has an increasing marginal utility of wealth, they will always take a fair bet.
- Their utility function is convex.
- This person has a negative risk premium.
- The expected utility of some bet is higher than the utility she gets from the utility of the expected value.

Decision Making Under Uncertainty

(a) Risk-Neutral Individual



(b) Risk-Preferring Individual



Decision Making Under Uncertainty

- To summarize
- Somebody is risk averse if $U(E[X]) > E[U(X)]$
- Somebody is risk averse if $U(E[X]) = E[U(X)]$
- Somebody is risk loving if $U(E[X]) < E[U(X)]$

EXAMPLE

- If you flip a heads I give you £10, if you flip a tails I give you £5.
- What is the most each of the following people would be willing to pay for this game?

① $U = (W)^{\frac{1}{2}}$

② $U = W$

③ $U = (W)^2$

- Draw each person's utility function showing the expected value, expected utility and risk premia.

Decision Making Under Uncertainty

- The curvature of the utility function is what determines somebody's risk aversion
- One commonly used measure of this curvature is the **Arrow-Pratt measure of risk aversion**

$$\rho(W) = -\frac{d^2 U(W) / dW^2}{dU(W) / d(W)}$$

Decision Making Under Uncertainty

- The numerator of this expression is the second derivative.
- If utility is concave, this will be negative
- ρ is positive if somebody is risk averse and it is negative if they are risk loving.

EXAMPLE

- What is the Arrow-Pratt measure of risk aversion for the following utility functions?

$$U = W^{\frac{1}{2}}$$

$$U = 2W$$

$$U = W^2$$

Avoiding Risk

- There are a lot of ways people can avoid or decrease risk
- One such way is through **diversification**.
- Diversification simply means putting your eggs into different baskets.
- The probability that all of the baskets break at the same time is low.

Avoiding Risk

- Another way to deal with risk is through **insurance**.
- Insurance is just contingent consumption.
- You can move some of your payoff from the good state of the world to the bad state of the world.
- Make the bad outcome less bad.

Avoiding Risk

- If your house doesn't burn you keep £1,000,000
- If your house burns down you get \$0
- Insurance would be something like buying a plan that gives you £900,000 if your house doesn't burn down and £100,000 if your house does burn down.

Avoiding Risk

- Suppose that for every \$1 of insurance you buy, you get \$4 if your house burns down.
- The probability your house burning down is .1 and the probability it doesn't is .9.
- By being able to purchase insurance, your expected utility changes from the first expression to the second expression.

$$EU = 0.9U(1,000,000) + 0.1(0)$$

$$EU = 0.9 * U(1,000,000 - x) + 0.1 * U(4x - x)$$

- To find out exactly how much insurance you would buy, you would maximize this with respect to x .

- If our utility function were $U = \ln W$

$$EU = 0.9 * \ln(1,000,000 - x) + 0.1 * \ln(4x - x)$$

- maximise this with respect to x and solve for x .

Avoiding Risk

$$\begin{aligned}\frac{dEU}{dx} &= \frac{0.9}{x - 1,000,000} + \frac{0.1}{x} = 0 \\ \frac{0.1}{x} &= \frac{0.9}{1,000,000 - x} \\ 0.9x &= 100,000 - 0.1x \\ x &= 100,000\end{aligned}$$

EXAMPLE

- You can insure your bike worth \$500 against theft.
- The probability your bike gets stolen is $\frac{1}{2}$.
- For every \$1 of insurance you buy you get \$2 if your bike is stolen.
- If your utility function is $U = W^{1/2}$, how much insurance should you buy?
- What is the intuition behind this result?

- In the previous example, insurance was **actuarially fair**.
- A policy is actuarially fair if the insurance company does not make any profit.
- Expected profit for the insurance company in the previous example for every \$1 of insurance bought is

$$E(\pi) = \frac{1}{2}(\$1) - \frac{1}{2}(\$1 - \$2) = 0$$

EXAMPLE

- Your house burns down with probability 1%.
- For every £1 of insurance you buy the insurance company gives you £50 back.
- Is this actuarially fair?

- Is this how people actually behave towards risk though?
- Many people make decisions that are inconsistent with expected utility theory.

- One common fallacy is the **gambler's fallacy**
- People believe that past events affect current, independent outcomes.
- If you flip a fair coin 100 times and they all come up heads, the probability the next coin is heads is still .5.
- But many people believe that tails is more likely because it is "due."

- People often overestimate their probability of winning.
- In one survey, gamblers estimated their chance of winning a bet is 45% when the objective probability was 20%.

- Many people put excessive weight on outcomes they consider to be certain.
- A group of subjects were asked to choose between two options in an experiment.

Behavioral Economics of Risk

Option A: You receive \$4,000 with probability .8 and \$0 with probability .2

Option B: You receive \$3,000 with certainty

- 80% of people choose option B

Behavioral Economics of Risk

- They were then given a separate set of options

Option C: You receive \$4,000 with probability .2 and \$0 with probability .8

Option D: You receive \$3,000 with probability .25 and \$0 with probability .75

- 65% of people choose C

Behavioral Economics of Risk

- These choices are inconsistent
- Choosing B over A implies the expected utility of B is greater than A
- $U(3,000) > .08U(4000)$
- Or $\frac{U(3,000)}{U(4000)} > .08$

Behavioral Economics of Risk

- Choosing C over D implies that $.2U(4000) > .25U(3,000)$
- This implies $\frac{U(3,000)}{U(4000)} < .08$
- This is inconsistent with expected utility theory

Behavioral Economics of Risk

- The way you present the same lottery can change people's preferences.
- The Avian Flu will kill 600 people, the government is proposing two programs to combat it

Program A: 200 people will be saved

Program B: There is a $1/3$ probability that 600 people will be saved and a $2/3$ probability nobody will be saved.

- 72% opted for program A

Behavioral Economics of Risk

- The Avian Flu will kill 600 people, the government is proposing two programs to combat it

Program C: 400 people will die

Program D: There is a $1/3$ probability that nobody dies and a $2/3$ probability that 600 people will die.

- 78% opted for program D

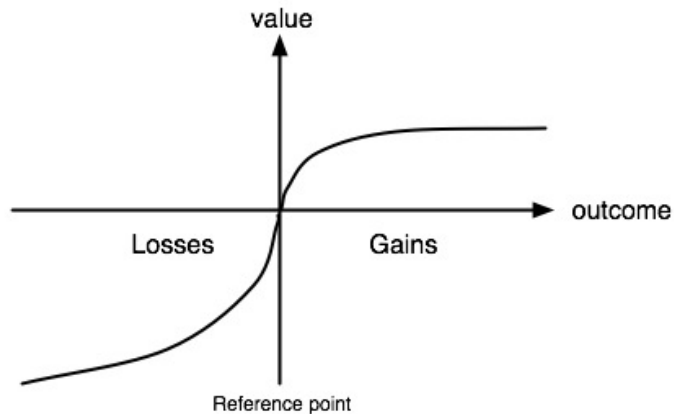
- A and C are the same and B and D are the same!
- Framing is a huge issue in economics.

- **Prospect theory** is an alternative approach to decision making under uncertainty.
- Empirically, it is a better match of people's behaviors.
- People treat gains and losses differently in prospect theory.
- Small changes are evaluated differently than big changes.

Behavioral Economics of Risk

- The basic tenants of prospect theory are
 - ① Everything is evaluated according to some reference point.
 - ② People are more sensitive to small gains than to large gains. For example the difference between \$1 and \$2 is larger than \$1001 and \$1002.
 - ③ People hate taking losses more than they like taking gains.

Behavioral Economics of Risk



Summary

- What is the expected value of a random variable?
- What is expected utility?
- What are the different attitudes towards risk and who will take a fair bet?
- What is the risk premium?

Summary

- What is fair insurance?
- What are inconsistencies with expected utility theory in the real world?
- What is prospect theory?