Lecture 4

Reading: Perloff Chapter 6

August 2015
In this lecture we look at firms and production.

It is the first step in deriving the supply curve we say in the first lecture.
Ownership and Management of Firms - What exactly is a "firm?"

Production - How a firm makes output from their set of inputs.

Short-Run Production - Look at production when the firm has a fixed input.

Long-Run Production - Look at production when there are no fixed inputs.

Returns to Scale - How the size of a firm affects how much it produces.

Productivity and Technical Change - The most output you can get for your inputs varies across firms and across time.
A firm is simply some organization that takes inputs and turns it into outputs.

We can roughly divide these firms into

- private sector
- public
- non-profit firms
Sole proprietorship

- owned by an individual who is responsible for all debts.
- Example: A freelance writer or a bookkeeper.
General partnership

- Jointly owned by multiple people who are together responsible for debts.
- Example: Law office with multiple partners.
Ownership and Management of Firms

- Corporations
  - Owned by shareholders in proportion to the amount of stock they own.
  - limited liability.
  - Example: Microsoft, recently Facebook.
Ownership and Management of Firms

- Private firms have the single goal of maximizing profits.
- **Profit** \((\pi)\) is defined as **total revenue** \((TR)\) minus **total cost** \((TC)\).

\[
\pi = TR - TC
\]

\[
TR = p \times q
\]

- A firm can only maximize profit if it achieves **technical efficiency**.
- Technical efficiency means they get the most output they possibly can from their set of inputs and technology.
A firm takes inputs (factors of production) and turns it into output according to its technology.

We can broadly classify these inputs into capital ($K$), labour ($L$) and materials ($M$) (which we usually ignore for simplicity).
The production function summarizes this process, and tells us exactly how much output the firm can get from their inputs.

For example, suppose our production function is

\[ q = f(L, K) = 2 \times L \times K \]

If the firm employs two units of labour and 4 units of capital it gets 16 units of output (it could produce less, but that would not be efficient).
EXAMPLE

- Suppose a firm produces output using only labour according to the production function $q = L^2$.
- Sketch this production function and identify two production plans, one that is technically efficient and one that is not.
The short run is the period of time that at least one factor of production cannot be changed.

For example, dominoes can decide how many delivery drivers it hires in a month, but can’t decide how many stores to build in this time frame.
The **long run** is a period of time in which all inputs can be varied. The difference between short and long run varies by industry. We call inputs that can’t be changed **fixed inputs**, and ones that can be changed **variable inputs**.
Short-Run Production

- Let's say capital is fixed in the short run, our production function is then

\[ q = f(K, L) \]

- Suppose our production function is \( q = 2KL \), but capital is fixed at \( K = 4 \) in the short-run.

- Our short-run production function becomes \( q = 8L \).

- We can summarize the relationship between output and the amount of labour used by the **total product of labour**, the **average product of labour** and the **marginal product of labour**.
**Total product of labour** is the amount of output that a given amount of labour can produce holding other inputs fixed.

**Marginal product of labour** is the extra output you get from increasing labour by some infinitesimally small amount.

\[ MP_L = \frac{\partial q}{\partial L} \]

**Average product of labour** is the amount of output produced per worker.

\[ AP_L = \frac{q}{L} \]
EXAMPLE

Suppose our production function is

\[ Q = K \times L^2 \]

What is the total product of labour, marginal product of labour and average product of labour if capital is fixed at 50?
If our short run production function is \( q = L + 30L^2 - L^3 \)
Short-Run Production

- We assume the firm can hire fractions of workers (which is why this is smooth)
- The $MP_L$ is the slope of the production function
- The $AP_L$ is the slope of the chord from the origin
The previous production function is typical

\( AP_L \) initially rises because of gains from specialization, but it declines because capital is held constant.
The $MP_L$ always intersects the $AP_L$ at the maximum of the $AP_L$ curve.

- When $MP_L > AP_L$, the average is pulled up.
- When $MP_L < AP_L$, the average is pulled down.
EXAMPLE

Let's prove together that $MP_L = AP_L$ at the maximum of $AP_L$ for the production function $q = L + 30L^2 - L^3$. 
The **law of diminishing marginal returns** is huge in economics.

As you increase one input, holding all other inputs and technology constant, the marginal returns to that input will decrease eventually.

The second derivative will become negative.

*You can’t grow the world’s food supply in a flower pot.*

This is why Malthus predicted mass starvation (one input -land- is fixed).
Short-Run Production

For the production function

\[ q = L + 30L^2 - L^3 \]

\[ MP_L = \frac{dq}{dL} = 1 + 60L - 3L^2 \]
The marginal product of labour is increasing for low levels of output because of gains from specialization.

Eventually, the marginal product of labour starts to decrease.

\[
\frac{dMP_L}{dL} = 60 - 6L
\]

- \( L < 10 \) \( MP_L \) increases
- \( L > 10 \) \( MP_L \) decreases
EXAMPLE

- Suppose the world’s supply of food is determined by the production function
  \[ F = M^{\frac{1}{2}} L^{\frac{1}{2}} \]
- \( M \) is land and is fixed at 3600.
- \( L \) is labour.
- At what point does this production function exhibit diminishing marginal returns to labour?
In the long run, the firm is free to select as much of any input... nothing is fixed.

Let’s suppose our production function is Cobb-Douglas

\[ q = 3L^{\frac{1}{2}}K^{\frac{1}{2}} \]

Try to graph this.
Long-Run Production

- Now that we have multiple variable inputs, our production function has multiple dimensions.
- Can’t really deal with it graphically.
- We can summarize this production function in 2 dimensions.
An isoquant is a curve that shows the efficient combinations of inputs that produce a single level of output.

This has a very similar interpretation to indifference curves.

Every combination of labour and capital on the same isoquant will produce the same amount of output.
Long-Run Production

The diagram illustrates the relationship between capital (K) and labor (L) in the long run, with different output levels (q) represented by isoclines. The points a, b, c, d, e, and f are marked on the graph, each representing a specific combination of capital and labor that produces a certain output level (q = 6, q = 9, q = 12).
Say our production function is
\[ Q = 10 \times K \times L \]

Draw the isoquant for \( Q = 10 \) and \( Q = 20 \).
Let's now discuss the properties of isoquants.

1. Further from the origin, the greater the output.

2. The more inputs you use, the more output you get if you are producing efficiently.
2. They cannot cross

- Suppose the isoquant where $Q = 20$ and $Q = 15$ cross.
- The firm could produce 15 or 20 units of output for the same input combination.
- The firm would not be efficient if it produced 15 for that input combination.
3. They slope downward

- If they sloped upward, the firm could produce the same level of output with fewer inputs.
4. They are thin

- If they were thick, the firm could decrease its input use and get the same level of output
5. UNLIKE indifference curves, isoquants are a cardinal measure.

- Output is objective, it is not some abstract thing like utility.
Long-Run Production

- The curvature of the isoquant tells us how substitutable/complementary the inputs are in the production process.
- The more "curvy" the isoquant, the greater degree of complementarity there is between inputs.
Long-Run Production

- If they are straight lines, the inputs are perfectly substitutable (apples from Oregon or Washington). This would come from a linear production function

\[ q = x + y \]

- If they are right angles, inputs must be used in fixed proportions (one secretary per phone). This comes from a fixed-proportions production function

\[ q = \min\{x, y\} \]
Long-Run Production
The slope of the isoquant shows the firm’s ability to replace one input with another holding output constant. This is called the **marginal rate of technical substitution** \((MRTS)\).

How much \(K\) can we give up for another unit of \(L\) holding output constant.
Remember that the slope of an indifference curve is the negative ratio of marginal utilities.

The slope of the isoquant \( (MRTS) \) is the ratio of marginal products.

\[
MRTS = \frac{\frac{dq}{dL}}{\frac{dq}{dK}} = -\frac{MP_L}{MP_K}
\]
EXAMPLE

- Let's find the $MRTS$ of

$$q = AL^\alpha K^{1-\alpha}$$

- Is the $MRTS$ constant for this production function?
If we have normal convex isoquants, we have a diminishing marginal rate of substitution.

That is, when a lot of capital and little labour is used, the \textit{MRTS} is really high. When a lot of labour and a little capital is used, it is really low.
Suppose we have a factory with 1,000,000 workers and only one machine.

To keep output constant, we could trade one machine for a ton of workers (because workers need machines).

As we get more machines, the number of workers we can trade for one more machine will decrease.
For most isoquants, the $MRTS$ is not constant.

As we increase capital and decrease labour, at what rate does the $MRTS$ change?

This is the **elasticity of substitution**... A measure of how "curvy" our isoquants are
Elasticity of substitution \( (\sigma) \) is the percentage change in the capital labour ratio w.r.t a percentage change in the MRTS

\[
\sigma = \frac{d(K/L)}{K/L} = \frac{d(K/L) \ MRTS}{dMRTS \ K/L}
\]

If \( \sigma \) is really high, that means a tiny change in the MRTS results in a big change in \( K/L \), the isoquant is pretty flat.
EXAMPLE

- What is the elasticity of substitution for the following Cobb-Douglas production function?

\[ Q = AL^\alpha K^\beta \]

- What is the elasticity of substitution for a linear production function?
- What does that mean?
Returns to Scale

- If we increase our inputs proportionately, what happens to our output?
- **This is called returns to scale**
- We can have increasing, decreasing or constant returns to scale.
If doubling our inputs leads to exactly double the output, we have **constant returns to scale**

\[ 2f(L, K) = f(2L, 2K) \]
If doubling our inputs leads to more than double the output, we have **increasing returns to scale**

\[ 2f(L, K) < f(2L, 2K) \]

Could be caused by greater specialization
If doubling our inputs leads to less than double the output, we have **decreasing returns to scale**

$$2f(L, K) > f(2L, 2K)$$

- Could be caused by management or organizational problems
What industries do you think have constant, increasing or decreasing returns to scale?

What are some factors that determine returns to scale?
Generally speaking, our production function is homogenous of degree $\gamma$ when

$$f(xL, xK) = x^\gamma f(L, k)$$
EXAMPLE

The following production function is homogenous of degree what?

\[ Q = K^{\frac{1}{2}} L^{\frac{1}{4}} \]
Returns to Scale

- It is possible for production functions to have varying returns to scale.
- Could have increasing returns to scale for low levels of production, and decreasing returns to scale for high levels of production.
- For low levels of production, you have gains to specialization and for large levels of production, you run into management problems.
Returns to Scale

- **Increasing returns to scale**: Path from point $a$ to point $b$.
- **Constant returns to scale**: Path from point $b$ to point $c$.
- **Decreasing returns to scale**: Path from point $c$ to point $d$.

Graph shows the relationship between capital ($K$, units of capital per year) and labor ($L$, work hours per year) with the output ($q$) at different points $(a, b, c, d)$. Dotted lines represent different levels of output ($q = 1, 3, 6, 8$).
Even if two firms are producing efficiency, it is possible that they are not equally as productive.

We can express a firm’s relative productivity by the ratio of the firm’s output $q$ to the amount of output the most productive firm in the industry could have produced from the same inputs $q^*$

$$\rho = \frac{q}{q^*} \times 100$$

If you are the most productive firm in the industry, what is $\rho$?

Estimated that the average productivity of manufacturing firms in the US is 63% to 99%.
It is possible that one firm can produce more today from a given amount of inputs than it could in the past.

An advance in knowledge that allows more output to be produced from the same level of inputs is called technical progress.

Can be neutral or non-neutral.
Neutral technical change means the firm can produce more output using the same ratio of inputs.

\[ q = A(t)f(L, K) \]

For example

\[ q_1 = 10 \times K^{0.5}L^{0.5} \]
\[ q_2 = 100 \times K^{0.5}L^{0.5} \]
Or it can be non-neutral in which innovations alter the proportion of input used.

\[ q_1 = 10 \times K^{0.5} L^{0.5} \]
\[ q_2 = 10 \times K^{0.5} L^{0.8} \]

If a machine is invented that requires only one person to operate it rather than two, this is a non-neutral labour saving technical change.
What is technical efficiency?
What does a production function show you?
What is the difference between the short and long run?
Summary

- What causes diminishing marginal returns?
- What is an isoquant
- What is the marginal rate of substitution and the elasticity of substitution?
- What are returns to scale?
- What is the difference between a neutral and non-neutral technical change?