A Pasinetti model of savings and growth

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Abstract

This paper develops a two-sector growth model in which institutional investors play a significant role. A necessary and sufficient condition is established under which these investors own the entire capital stock in the long run. The dependence of the long-run growth rate on the behaviour of such investors, and the effects of a productivity increase are analysed.

JEL classification. O41, O43

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1 Introduction

In the Keynes-Kaldor-Pasinetti post-Keynesian growth model [1] two classes of agent, workers and capitalists, save constant proportions of their income. On a balanced growth path the rate of profit is independent of the workers’ savings propensity. Meade (1963)[2] and Samuelson and Modigliani (1966)[3] prove an "anti-Pasinetti" theorem which establishes the existence of an alternative balanced growth path on which pure capitalists cease to exist and all capital is owned by workers. Kaldor (1966)[4] proposed an alternative

In this paper we consider a two-sector model in which institutional investors such as pension funds, unit trusts, insurance companies have an important role. It reflects Pasinetti’s idea that workers must own the capital to which their savings have given rise, but also acknowledges that, in a modern capitalist economy, these savings are typically mediated by institutions such as pension funds. Dinenis and Scott (1993) [21] argue that pension funds are a major vehicle for personal long-run saving in the UK economy. They report that such funds controlled over £250bn of funds in 1989, their total net assets constituting 38% of personal sector net financial wealth. These funds owned 23% of UK equity, 21% of British government securities and 18% of British holdings of foreign equity. Apilado (1972) [20] investigates whether pension savings in the US economy between 1955 and 1970 are a substitute for other forms of saving. He concludes that they were in fact an addition to other forms of saving and that, via an increase in total saving, generated an increase in the growth rate. Pension funds have obligations to pay pensions and in many jurisdictions (e.g. the UK) pensioners are allowed to withdraw a proportion of their pension pot prior to retirement. Workers’ savings/consumption decisions are not explicitly modelled in this paper: rather institutional investors are assumed to invest a proportion $s < 1$ of their income, where $s$ is treated as exogenous.

Van Groezen et. al. (2007) [17] develop a two-sector growth model with a capital intensive commodity sector (with endogenous growth) and a labour intensive services sector. They analyse the effects on economic growth of a switch to a more funded pension scheme. In this model increased savings
resulting from the pension reform generate higher growth in a closed economy provided capital and labour are not strong substitutes. However, the opposite is true for a small open economy. Hachon (2010) [19] analyses the effect of the structure of pension systems on the growth rate. He contrasts "purely Beveridgian" pension systems, where every agent receives the same pension, with "purely Bismarckian" systems, where pensions depend on agents’ wages. Hachon’s focus is on the redistributional effects of pensions, in similar vein to a paper of Docquier and Paddison (2003) [18].

Pasinetti was concerned to provide a normative description of the economic system, focussing on the physical requirements for reproduction. But his insights can be re-interpreted as providing a positive analysis of modern capitalism. In such an economy savings and investment are mediated by institional investors. So two questions arise naturally:

- Will the long-run growth rate in such an economy be determined by the behaviour of institutional investors and, if so, how?
- Will insitutional investors own the entire capital stock in the long run?
- What are the implications for long-run growth and capital ownership, of a one-shot productivity increase?

All three questions are analysed below. A capitalist economy with institutional investors works in a complicated way, but adopting and developing Pasinetti’s insights allows an analysis of these questions which is simple enough to be tractable.

2 Structure of the model

In an economy with institutional investors, investment and hence growth are likely to be influenced by the decisions of such investors. But under modern capitalism there are many high technology firms (e.g. IT, software) which present institutional investors with substantially greater problems of risk and asymmetric information than firms with less dynamic technologies (e.g. consumer durables). It is therefore reasonable to assume a correlation between technological level and the degree to which accumulation is financed from retained profits. We refer to capital accumulated from retained profits as "corporate capital", and that accumulated through institutional investment
as "institutional capital". To capture this distinction in a two-sector model we assume two different production sectors at opposite ends of this "technology spectrum". Sector 1 consists of high technology, capital intensive firms which invest all their profits, and also obtain investment from outside institutional investors. It produces an output $Q_1$ using labour $L_1$ and capital $K_1$. Sector 2 consists of medium technology, less capital intensive firms whose investment expenditure comes exclusively from outside institutional investors. It produces an output $Q_2$ using labour $L_2$ and capital $K_2$. Total output of the economy will be denoted $Q = Q_1 + Q_2$; total labour employed in the economy will be denoted $L = L_1 + L_2$; total capital employed in the economy will be denoted $K = K_1 + K_2$.

Both factors are assumed perfectly mobile, equalising wage and profit rates between the two sectors. Capital is assumed fully employed, but there may be unemployed labour in the economy. Outside institutional investors receive income based on wages (e.g. pension contributions) and from profits earned on their portion of the capital stock. They invest a proportion, $s$, of their income, of which a share, $1 - \theta$, goes to sector 1 and $\theta$, goes to sector 2. We establish conditions under which the growth rate of the economy is independent of the institutional investors’ behaviour: in this case the share of the capital stock funded from retained profits remains strictly positive. There is also a balanced growth path along which the growth rate depends on the behaviour of institutional investors; in this case the share of the capital stock funded from retained profits disappears in the long run and the entire capital stock is owned by institutions.

### 3 Wage and profit rates

Sector 1 consists of high technology firms with capital-output ratio $k_1 = \frac{K_1}{Q_1}$ and output-labour ratio $q_1 = \frac{Q_1}{L_1}$. Sector 2 consists of medium-tech firms with capital-output ratio $k_2 = \frac{K_2}{Q_2}$ and output-labour ratio $q_2 = \frac{Q_2}{L_2}$. It will be assumed that:

$$q_1 > q_2 \text{ and } k_1 > k_2$$  \hspace{1cm} (1)
Together these inequalities imply that:

\[
\frac{K_1}{L_1} > \frac{K_2}{L_2} \tag{2}
\]

Wage profit-frontiers can readily be derived for the two sectors. Let \( w \) denote the wage rate and \( r \) the profit rate. Then:

\[
Q_1 = wL_1 + rK_1 \Rightarrow 1 = \frac{w}{q_1} + rk_1 \tag{3}
\]

And:

\[
Q_2 = wL_2 + rK_2 \Rightarrow 1 = \frac{w}{q_2} + rk_2 \tag{4}
\]

The two wage-profit frontiers are illustrated in figure 1.

![Figure 1. Wage-profit frontiers for sectors 1 and 2.](image)
Inter-sectoral mobility of the two factors ensures that wage and profit rates are determined at the intersection of the two frontiers yielding:

\[ w^* = \frac{k_2 - k_1}{k_2/q_1 - k_1/q_2} \]  \hspace{1cm} (7)

and:

\[ r^* = \frac{q_2 - q_1}{k_2q_2 - k_1q_1} \]  \hspace{1cm} (8)

Note that the wage-profit frontiers do not assume full employment of labour: the availability of labour is never a constraint on growth. Capital is assumed fully employed, and both factors are assumed instantaneously and costlessly mobile between the two sectors.

4 Capital accumulation

Sector 1 (high-technology) firms will be assumed to re-invest all their profits and also to receive a share \( \theta \) of institutional investment. So let \( K_1 = X + Y \) where \( X = " \)corporate capital" (i.e. that portion of sector 1 capital funded from retained profits) and \( Y = " \)institutional capital" (i.e. that portion of sector 1 capital funded by outside institutions). Sector 2 (medium-tech) firms will be assumed to fund their capital accumulation entirely from outside institutional sources. For notational consistency let \( Z = K_2 \). Institutional investors own a portion \( Y + Z \) of the capital stock. Assume all capital depreciates at a rate \( \delta \). It is shown in the Appendix (Proposition 1) that \( X, Y \) and \( Z \) are governed by the linear dynamical system given by equations (9,10,11) below:

\[ \dot{X} = (r^* - \delta)X \]  \hspace{1cm} (9)

\[ \dot{Y} = [(1-\theta)s \left( \frac{tw^*}{k_1q_1} + r^* \right) - \delta]Y + (1-\theta)s \left( \frac{tw^*}{k_2q_2} + r^* \right) Z + \frac{(1-\theta)stw^*}{k_1q_1} X \]  \hspace{1cm} (10)

\[ \dot{Z} = \theta s \left( \frac{tw^*}{k_1q_1} + r^* \right) Y + [\theta s \left( \frac{tw^*}{k_2q_2} + r^* \right) - \delta]Z + \frac{\theta stw^*}{k_1q_1} X \]  \hspace{1cm} (11)
It is further established in the Appendix (Propositions 2 and 3) that this dynamical system has two different types of steady state, depending on whether or not the condition:

$$\frac{sw^*}{(1 - s)} \left[ \frac{1 - \theta}{k_1q_1} + \frac{\theta}{k_2q_2} \right] \leq r^*$$

(12)

is satisfied. Now define shares in the total capital stock:

$$x = \frac{X}{K}; y = \frac{Y}{K}; z = \frac{Z}{K}$$

(13)

We focus on a condition necessary and sufficient for the institutional investors to own the whole economy in the steady state (that is $x = 0$ or $\frac{Y + Z}{K} = 1$ in the steady state).

### 4.1 Steady state 1

It is shown in the Appendix (Proposition 2) that, the dynamical system consisting of equations (9,10 and 11) converges to a steady state in which $x > 0$ and the growth rate is given by $g = r^* - \delta$ if and only if Condition (12) is satisfied. In this steady state, the long-run growth rate does not depend on the savings behaviour of institutional investors, and they do not own the whole economy in the long-run.

### 4.2 Steady state 2

It is shown in the Appendix (Proposition 3) that, the dynamical system consisting of equations (9,10 and 11) converges to a steady state in which $x = 0$ and the growth rate is given by $\lambda_2 = sw^* \left( \frac{1 - \theta}{k_1q_1} + \frac{\theta}{k_2q_2} \right) + sr^* - \delta$ if and only if Condition (12) is violated. In this steady state, the long-run growth rate does depend on the savings behaviour of institutional investors. In particular it increases with $t$ (proportion of the wage bill received by institutional investors), $s$ (the invested proportion of institutional investors’ income) and $\theta$ (the proportion of that investment that goes to sector 1 (high tech) firms). Moreover, in this steady state, institutional investors own the whole economy in the long-run.
5 Numerical simulations

A simple Matlab program was written to simulate the dynamical system of section 4 above. Parameters and initial conditions were set as in Table 1 below:

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>depreciation rate</th>
<th>0.02</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_1 )</td>
<td>capital/output ratio in sector 1</td>
<td>3.5</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>capital/output ratio in sector 2</td>
<td>2.5</td>
</tr>
<tr>
<td>( q_1 )</td>
<td>output/labour ratio in sector 1</td>
<td>10.5</td>
</tr>
<tr>
<td>( q_2 )</td>
<td>output/labour ratio in sector 2</td>
<td>10.0</td>
</tr>
<tr>
<td>( X(1) )</td>
<td>corporate capital (initial value)</td>
<td>45.0</td>
</tr>
<tr>
<td>( Y(1) )</td>
<td>institutional capital in sector 1 (initial value)</td>
<td>25.0</td>
</tr>
<tr>
<td>( Z(1) )</td>
<td>institutional capital in sector 2 (initial value)</td>
<td>30.0</td>
</tr>
</tbody>
</table>

Table 1

This implies equilibrium wage and profit rates of \( w^* = 8.936 \) and \( r^* = 0.043 \) respectively. The economy simulated is thus one which, initially contains a significant high tech sector, with accumulation financed predominantly from retained profits. Initially, it also includes a medium tech sector, financed by institutional investors. Table 2 shows values of \( x \) (share of corporate capital in total capital stock) at \( time = 20 \) and the growth rate at \( time = 20 \) for various values of the parameters. The first four lines of the table relate to parameter combinations which satisfy condition (12), so that the long-run share of corporate capital in the total capital stock \( (x) \) is non-zero, and the long run growth is independent of the investors’ parameters (it is equal to \( r^* - \delta = 0.023 \)). The next two lines of the table relate to parameter combinations which violate condition (12), so that the long-run share of corporate capital in the total capital stock \( (x) \) is zero and the long run growth rate does depend on investors’ parameters, according to the equation of section 4.2 above.
Figures 2 and 3 show output for the simulations described in the first and fourth lines of table 1, corresponding to a low institutional savings ratio (0.3). In the first case the share of corporate capital in sector 1 trends upwards towards an upper limit of 0.87. In the second case the share of corporate capital in sector 1 trends downwards towards a limiting value of 0.40. In both cases the long-run growth rate is independent of institutional investors’ parameters, being equal to $r^* - \delta = 0.023$. Figure 4 shows output for the simulation described in the last line of table 1, corresponding to a high institutional savings rate (0.5). In this case the share of corporate capital in sector 1 trends downwards towards zero. The long run growth rate now depends on institutional investors’ parameters. At 0.036 it is significantly higher than the long-run growth rate of figures 2 and 3.

<table>
<thead>
<tr>
<th>s</th>
<th>t</th>
<th>x(20)</th>
<th>growth rate at time = 20</th>
<th>LR value of x</th>
<th>LR growth rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.05</td>
<td>0.55</td>
<td>0.0121</td>
<td>0.87</td>
<td>$r^* - \delta = 0.023$</td>
</tr>
<tr>
<td>0.3</td>
<td>0.05</td>
<td>0.55</td>
<td>0.0132</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>0.20</td>
<td>0.45</td>
<td>0.022</td>
<td>0.48</td>
<td>0.023</td>
</tr>
<tr>
<td>0.3</td>
<td>0.20</td>
<td>0.45</td>
<td>0.025</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.20</td>
<td>0.35</td>
<td>0.036</td>
<td>0.0</td>
<td>0.029</td>
</tr>
<tr>
<td>0.5</td>
<td>0.20</td>
<td>0.33</td>
<td>0.042</td>
<td>0.0</td>
<td>0.036</td>
</tr>
</tbody>
</table>

Table 2
Fig 2. Output of simulation for $s = 0.3, t = 0.05, \theta = 0.3$
Fig 3. Output of simulation for $s = 0.3, t = 0.2, \theta = 0.9$
6  Increase in productivity

We now consider the effect of an increase in labour productivity in sector 1 (high-tech) with all else (including labour productivity in sector 2) held constant. Figure 5 depicts the wage-profit frontiers for the two sectors. An increase in labour productivity in sector 1 from $q_{11}$ to $q_{12}$ increases the profit rate from $r^*$ to $r^{**}$ and decreases the wage rate from $w^*$ to $w^{**}$. It also affects the income of institutional investors and the proportions of that income arising from the two sectors of the economy. The combined effect on corporate capital over time is shown in figures 6 and 7. For the high pro-
ductivity simulations, $q_1$ is set at 12.5; for the low productivity simulations $q_1$ is set at 10.5. In the former case the wage rate is 6.67 and the profit rate is 0.133. In the latter case the wage rate and profit rate are as in section 6 above, 8.936 and 0.043 respectively. Figure 6 depicts the simulation output for $s = 0.3$, $t = 0.05$, $\theta = 0.3$, corresponding to line 1 of table 1, in which the share of corporate capital increases in the low productivity case and decreases in the high productivity case. Figure 7 depicts the simulation output for $s = 0.5$, $t = 0.2$, $\theta = 0.9$, corresponding to line 6 of table 1, in which the share of corporate capital declines much more rapidly in the high productivity case.

Fig. 5. Wage-profit frontiers for low and high productivity in sector 1.
Figure 6 Share of corporate capital for high and low productivity in sector 1
Figure 7 Share of corporate capital for high and low productivity in sector 1

7 Conclusions

We analyse an economy in which corporate saving is undertaken by high-tech firms with high capital/labour ratios and correspondingly high labour productivity. Institutional saving is undertaken by pension funds, insurance companies and unit trusts which invest in high and medium tech firms (the latter having lower capital/labour ratios and correspondingly lower labour productivity). In high profit economies, the steady state growth rate is independent of the behaviour of institutional investors and the steady state share of corporate capital is non-zero (i.e. the institutions do not own the whole economy in the long run). In low profit economies the steady state growth rate does depend on the behaviour of institutional investors. In particular
it increases with $t$ (proportion of the wage bill received by institutional investors), $s$ (the invested proportion of institutional investors’ income) and $\theta$ (the proportion of that investment that goes to sector 1 (high tech) firms). Moreover, the steady state share of corporate capital is zero (i.e. the institutions do own the entire economy in the long run). There is a parallel here with the "anti-Pasinetti" theorem of Meade (1963)[2] and Samuelson and Modigliani (1966)[3] which establishes the existence of a balanced growth path on which pure capitalists eventually cease to exist, and all capital is owned by workers. In all cases the share of corporate capital declines more rapidly or rises more slowly if labour productivity rises in sector 1.

8 Appendix

In this appendix we establish the central propositions referred to in the main text.

8.1 Capital accumulation

**Proposition 1** The variables $X$, $Y$ and $Z$ are governed by the linear dynamical system given by:

$$
\dot{X} = (r^* - \delta)X 
$$

$$
\dot{Y} = [(1-\theta)s \left(\frac{tw^*}{k_1q_1} + r^*\right) - \delta]Y + (1-\theta)s \left(\frac{tw^*}{k_2q_2} + r^*\right) Z + \frac{(1-\theta)stw^*}{k_1q_1}X 
$$

$$
\dot{Z} = \theta s \left(\frac{tw^*}{k_1q_1} + r^*\right) Y + \left[\theta s \left(\frac{tw^*}{k_2q_2} + r^*\right) - \delta\right] Z + \frac{\theta stw^*}{k_1q_1}X
$$

**Proof.** Equation (14) follows immediately from the definitions. Institutional investors will be assumed to have an income $V$, consisting of a proportion $t$ of the wage bill (e.g. pension contributions) and the profits they earn on their portion of the capital stock. We therefore have:

$$
V = tw^* [L_1 + L_2] + r^* [Y + Z] 
$$

Using the definitions of section 3, this yields:

$$
V = t \left[\frac{w^* K_1}{k_1q_1} + \frac{w^* K_2}{k_2q_2}\right] + r^* [Y + Z]
$$
Using the definitions of section 4, this yields:

\[
V = t \left[ \frac{w^*(X + Y)}{k_1q_1} + \frac{w^*Z}{k_2q_2} \right] + r^* [Y + Z] \tag{19}
\]

It is now simple to derive accumulation equations for \(X, Y\) and \(Z\):

\[
\dot{X} = (r^* - \delta)X \tag{20}
\]

\[
\dot{Y} = (1 - \theta)sV - \delta Y \tag{21}
\]

\[
\dot{Z} = \theta sV - \delta Z \tag{22}
\]

Substituting (19) into (21) and (22) and rearranging gives yields equations (15) and (16)

Equations (14), (15) and (16) constitute a linear dynamical system in \(X, Y\) and \(Z\). Equation (14) can solved independently to give:

\[
X(t) = X(0)e^{(r^* - \delta)t} \tag{23}
\]

Now define the following shares in the total capital stock:

\[
x = \frac{X}{K}; \quad y = \frac{Y}{K}; \quad z = \frac{Z}{K} \tag{24}
\]

We focus on conditions necessary and sufficient for the institutional investors to own the whole economy in the steady state (that is \(x = 0\), or \(Y + Z = 1\), in the steady state).

### 8.2 Dynamics of the model

We first establish:

**Proposition 2** The dynamical system consisting of equations (14), (15) and (16) converges to a steady state in which \(x > 0\) and the rate of growth is given by \(g = r^* - \delta\), if and only if

\[
\frac{stw^*}{1 - s} \left[ \frac{1 - \theta}{k_1q_1} + \frac{\theta}{k_2q_2} \right] \leq r^*.
\]
Proof. The dynamical system clearly has $g$ as one eigenvalue. Let the other two eigenvalues be $\lambda_1$ and $\lambda_2$. The determinant of the dynamical system is the product of the eigenvalues and the trace is their sum. Thus we may write:

\[
\lambda_1 \lambda_2 = \delta(\delta - (1 - \theta)a_2 - \theta a_3) \tag{25}
\]

\[
\lambda_1 + \lambda_2 = (1 - \theta)a_2 + \theta a_3 - 2\delta \tag{26}
\]

where:

\[
a_2 = s \left( \frac{tw^*}{k_1 q_1} + r^* \right) \text{ and } a_3 = s \left( \frac{tw^*}{k_2 q_2} + r^* \right) \tag{27}
\]

The solution of these equations, which are symmetric in $\lambda_1$ and $\lambda_2$, is:

\[
\lambda_1 = -\delta \tag{28}
\]

\[
\lambda_2 = (1 - \theta)s \left( \frac{tw^*}{k_1 q_1} + r^* \right) + \theta s \left( \frac{tw^*}{k_2 q_2} + r^* \right) - \delta = \tag{29}
\]

\[
stw^* \left( \frac{1 - \theta}{k_1 q_1} + \frac{\theta}{k_2 q_2} \right) + sr^* - \delta \tag{30}
\]

Solutions of the dynamical system take the form:

\[
X(t) = X(0)e^{gt} \tag{31}
\]

\[
Y(t) = b_1 e^{\lambda_1 t} + b_2 e^{\lambda_2 t} + b_3 e^{gt} \tag{32}
\]

\[
Z(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + c_3 e^{gt} \tag{33}
\]

It follows that the system will tend to steady state growth at a rate $g$ if and only if $\lambda_2 \leq g$, since $\lambda_1 < 0$. We have $g = r^* - \delta$, so that $\lambda_2 \leq g$ if and only if:

\[
(1 - \theta)s \left[ \frac{tw^*}{k_1 q_1} + r^* \right] + \theta s \left[ \frac{tw^*}{k_2 q_2} + r^* \right] \leq r^* \iff \tag{34}
\]

\[
stw^* \left[ \frac{1 - \theta}{k_1 q_1} + \frac{\theta}{k_2 q_2} \right] \leq (1 - s)r^* \iff \tag{35}
\]

\[
stw^* \left[ \frac{1 - \theta}{k_1 q_1} + \frac{\theta}{k_2 q_2} \right] \leq r^* \tag{36}
\]

which is the required condition. From equations (31), (32) and (33) it is clear that, if $\lambda_2 \leq g$, $x = \frac{X}{K} \rightarrow \frac{X(0)}{b_3 + c_3 + X(0)} > 0$ as $t \rightarrow \infty$. That is $x > 0$ in the steady state, as required. \[\blacksquare\]
So, provided the condition (36) of proposition 1 is satisfied, the steady state growth rate is independent of the behaviour of institutional investors, and the long-run share of corporate capital in the total capital stock is non-zero.

We now characterise the long-run growth rate when condition (36) is violated.

**Proposition 3**  The dynamical system consisting of equations (14), (15) and (16) converges to a steady state in which \( x = 0 \), and the rate of growth is given by

\[
\lambda_2 = stw^* \left( \frac{1 - \theta}{k_1q_1} + \frac{\theta}{k_2q_2} \right) + sr^* - \delta \text{, if and only if condition (36) is violated.}
\]

**Proof.** By the argument of Proposition 1, \( \lambda_2 > g \) if and only if condition (36) is violated. Then, from equations (31), (32) and (33), the steady state growth rate must be equal to \( \lambda_2 \). Moreover, since \( \lambda_2 > g \) and \( \lambda_1 < 0 \), \( x = \frac{X}{K} \to 0 \) as \( t \to \infty \). That is, \( x = 0 \) in the steady state, as required.

So, if the condition (36) of proposition 1 is violated, the steady state growth rate does depend on the behaviour of institutional investors. In particular it increases with \( t \) (proportion of the wage bill received by institutional investors), \( s \) (the invested proportion of institutional investors’ income) and \( \theta \) (the proportion of that investment that goes to sector 1 (high tech) firms). Moreover, in this steady state, institutional investors own the whole economy in the long-run.

**References**


