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Procurement Lobbying

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Procurement lobbying*

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Abstract

We study lobbying in the context of (competitive) procurement. We propose a tractable, Bayesian model to analyze lobbying-effort and pricing decisions. Lobbying conveys information, even if biased, to the buyer. This allows the buyer to improve the expected value of the match, but it also increases product differentiation, and thus raises prices. When inexpensive, lobbying always happens in equilibrium, even if it is often not profitable. If he anticipates it, the buyer always benefits from a monopolist's lobbying, but he might be hurt by the lobbying of two competing sellers.

1 Introduction

Political scientists and political economists have extensively studied the influence of special or organized interests on political/regulatory decisions, usually under the heading of lobbying.[1] In most of these studies, lobbying is portrayed as a costly activity that influences either or both the probability of obtaining some "prize" and the negotiations about

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[1]See Mazza, 2001, for an excellent survey of the early literature, and Gregor, 2017, for a more recent one.

how to share that prize with the decision maker. Lobbying, however, is a phenomenon that extends well beyond the sphere of policy/political decisions. In particular, lobbying for contracts in – public or private – procurement is common and often perceived to be critical for success. As Nownes, (2006, page 149) puts it, despite many differences in the processes by which agencies purchase goods and services, "... there is one constant in the procurement process: *lobbying.*" Nonetheless, lobbying in this type of setting, and in particular in conjunction with price competition, has received little attention in the literature.

To address this lacuna, in this paper, we put forward a tractable model to study lobbying in procurement. We construe lobbying as a (costly) activity by a seller that generates useful (albeit biased in her favor) information for the buyer about his valuation of the (experience) good/service on offer. This information goes beyond the measurable aspects that the buyer may require the seller to disclose, and for complex products typically involves aspects the buyer may not even anticipate.[2]

Thus, we assume that both Buyer and the seller(s) are uncertain about the fit of Buyer's needs with each seller's product. Buyer's information (a noisy signal he receives) about this relative fit is influenced by the sellers' lobbying on behalf of their products. Lobbying conveys information. That is, the sellers' lobbying increases the information content of the signal. At the same time it also distorts this signal. With signal in hand, Buyer buys from the seller who offers a better *expected fit-price* combination.

We begin by studying the one seller case, where Buyer must decide between Seller's product and the status quo. This model has two equally relevant interpretations, both of which capture a situation where there is no strategic interaction on the lobbying side. The literal take is that of a particular (private) project – with a single possible supplier – that may or may not be undertaken (and so, funded). Alternatively, we can see it as a model of "competitive" (public) procurement: Think of the *federal* government's decision to include a particular weapons system in the budget, where the alternative is a larger

[2] Seeing procurement and lobbying together may bring the idea of corruption to mind. However, we do not consider corruption as a defining characteristic of lobbying, though in an extension (see Section 5.1) we address what we see as their relationship as well.

residual budget to spend on other goods or services, whether in defense or otherwise.

We take for granted that lobbying is not transparent, and thus Buyer cannot observe Seller's lobbying effort (how much smoke she is blowing or how much information value she is providing), so he needs to form conjectures about it.[3] Nonetheless, we show that lobbying is in Seller's interest whatever these conjectures (as long as the marginal cost of lobbying at zero is not too high). In particular, this is true when the conjectures are correct: lobbying is an equilibrium phenomenon. However, lobbying, or rather the fact that Buyer knows that she can engage in lobbying, may not be in Seller's interest. Indeed, when she is expected to lobby, and so to increase the signal's bias in favor of her product, Buyer may be little impressed by signals favorable to her. As a result, he puts more weight on the price: the elasticity of his demand increases, lowering Seller's optimal price unless the increase in informativeness (what shifts demand upwards for favorable signals) compensates. A sufficient condition for the seller not to lose is that lobbying is informative enough so that the information to signal ratio increases with it. Indeed, if lobbying increases the information content of the signal sufficiently, the optimal price (and not only demand) increases with lobbying.

With respect to Buyer, and as long as his conjectures are correct, lobbying is in fact a blessing as long as it improves the information content of the signal, even when it reduces the information to signal ratio. The improved fit will always compensate for the possible increase in price.

We do not allow Seller to choose separately noise (bias) and information precision. We posit that a one-dimensional – instead of a two-dimensional – choice is a better representation of actual lobbying. In Subsection 5.2 we offer a more "micro-founded" version of our model to justify this point, where effort buys bits of "evidence", each characterized by a proportion of bias and information. Seller chooses which bits to spend her effort on, thereby endogenously determining the relationship between bias and precision (subject

[3] How much effort or goodwill is spent with an official at the Pentagon to convince her of the potential of a new weapons system is not observed by the Appropriations Committee member that will sponsor the inclusion of the program in the Defense Bill. Research efforts towards detecting and convincing experts and officials, or towards determining the ways in which the product fullfils Buyer's (perhaps even unnoticed) "needs" are, likewise, difficult to observe.

to the available choice set). We show that she prioritizes noise over information (so that the information content of lobbying is convex in effort) despite the fact that it is the information content of the signal what works to her advantage. That is, once again, the fact that Buyer cannot observe her choices leads Seller to make the counter-intuitive ones.

The (equilibrium) assumption that Buyer's conjectures are correct is not always the best description of reality. For example, a relatively popular view equates lobbying with capture. While these are two different phenomena, certainly there may be a link between the two. Indeed, even if the "official" (in charge of making the purchasing decision) is captured, Seller may need to give that official arguments to justify the choice. That is, "capture" may be better modeled as the official taking the signal at face value.[4] Taking the signal at face value is equivalent to Buyer conjecturing zero lobbying despite Seller's best response being to lobby.

We analyze this case in Subsection 5.1, generalizing to all incorrect conjectures. For instance, and as could be expected from our discussion above, when Buyer underestimates the amount of lobbying and the information to signal ratio is decreasing, Buyer may well be hurt by Seller's lobbying.

We also study procurement lobbying when sellers interact strategically with other sellers. For instance, suppose that two weapons systems compete to be chosen to satisfy some validated capability. In that case, each seller competes not against "the market" but against another strategic agent who, in particular, is also able to lobby. We still find that lobbying is an equilibrium phenomenon. When the lobbying abilities (costs) are different (but everything else is symmetric), the more intensively lobbying seller sells more often and at a higher price. When sellers are also symmetric in the lobbying ability, a decreasing (in lobbying) information to signal ratio is now sufficient (and not even necessary) for the competing sellers to be worse off due to their known ability to lobby. In that sense, competition only makes things worse for sellers when the information to signal ratio is decreasing. Moreover, and at first glance perhaps even more surprisingly, Buyer may now be made worse off when the information to signal ratio is sufficiently higher with lobbying.

[4] That may also be the case when the official cares about the public's perception with regard to his choice, and the "public" is not sophisticated, even though the official is.

In fact, this is a result of a phenomenon mentioned above: a higher information content reduces the elasticity of demand (at the margin) and so increases the expected price for both sellers. The fact that a higher price by the rival induces an additional reduction in the elasticity of each seller's residual demand only makes this problem more severe under competition. As a result, Buyer's better decision on the product that best fits its necessities may not be sufficient to compensate for the higher price. Thus, competition makes it more likely for Buyer and sellers to get hurt by the possibility of lobbying.

1.1 Related literature

There is a large literature that models lobbying as a rent-seeking contest.[5] In a contest, competitors exert (costly) effort to improve their probability of success in appropriating a prize, which may or may not depend on the competitors' efforts. In that literature, the mapping from effort vectors to the probability of success and the value of the prize is treated as a black-box, and information is usually assumed to be symmetric and complete. The latter is obviously a shortcoming for the study of lobbying as a process of information transmission. Another commonly noted shortcoming of this approach is the lack of micro-foundations for that mapping, even when information is assumed asymmetric. The literature has produced some attempts to provide micro-foundations for (the most commonly used of) these mappings, e.g., most related to our problem, Lagerlöf (2007) and Skaperdas and Vaidya, (2012). Yet the approach is too rigid to constitute a promising avenue. Instead, we begin with information and pricing micro-foundations and let the "success" and "prize" mappings be an endogenous consequence of agents´ decisions.

Another strand of the literature on lobbying, focused on political influence, has studied the interplay between voters, special interest, and political parties around the choice of policies (e.g., Grossman and Helpman, 1996) There are two important differences with our object of study: the decision maker is perfectly informed and there is no price competition mediating the final choice.

Our research question is closely related to the literature on advertising. It is customary

[5] See Corchón and Serena, 2018, for a recent survey of the literature on contests.

to distinguish between persuasive and informative advertising. In the first case, preferences of buyers are affected by the sellers' advertising efforts. In this tradition, Bloch and Manceau (1999) study a model where advertising changes the distribution of buyer's "location" on a Hotelling line. The effects on profitability, prices, etc., depend on the shape of the effect. Instead, we model lobbying effort as a (biased) informational process that does not change Buyer's preferences, but his information, so that questions like the effect of lobbying on the buyer's payoff can be posed. In that sense, our approach is more closely related to the literature on informative advertising. A branch of this literature, in particular, studies advertising as a process of informing consumers about horizontally differentiated products' fit to their preferences. Leading examples are Anderson and Renault (2009) and, even closer to our setting, Lewis and Sappington (1994). Advertisements convey information to consumers, but this information is unbiased. Lobbying, we postulate, shares both persuasive and informative aspects: it conveys information without changing preferences, but it does so in a biased way.[6] In fact, the phenomenon of lobbying may be less related to persuasive advertising or informative advertising than to advertising of experience goods, something the other two branches of the literature on advertising typically ignore. Indeed, we postulate that lobbying may affect the buyer's before-purchase conjecture of (relative) quality. Ever since the seminal work of Nelson (1974), the literature on advertising of experience goods[7] has focussed on advertising as an instrument for signaling quality. That is, an action by an informed player that will convey information to another player who observes that action. We depart on both accounts, since we do not assume any information advantage by the sellers and assume that the buyer does not observe (intensity or even existence of) lobbying. Again, for the phenomenon of lobbying, we claim these to be more accurate assumptions.

Finally, Bayesian persuasion has been the workhorse model for truthful biasing of information in many contexts, since the seminal article of Kamenica and Gentzkow (2011). This literature has mostly focused on a single sender, however (though see Gentzkow and Kamenica, 2016). Particularly relevant to our paper are Boleslavsky et al. (2017) and

[6] Some of our results are shared with that literature: more information (increases product differentiation and so) improves the match between consumers and products, but raises prices.

[7] See Renault (2016) fo a recent survey of this and other advertising literatures.

especially Hwang et al. (2019) and Armstrong and Zhou (2021) who analyze a model of competitive persuasion with price competition – though with a continuum of consumers rather than a single buyer. In all this literature the principal assumption is that the senders *commit* to a signal generating process, which has to be a coarsening of the true distribution. In our model, the choice of lobbying effort is not observable and as a result we are able obtain the result where lobbying only exists because Buyer expects it to happen.

In the next section, we present our model of lobbying and procurement competition. In Section 3 we discuss as a benchmark the case with no lobbying. Section 4 contains the main body of the paper, where we analyze the incentives for a firm to lobby and its effects. Section 5 considers several issues and extensions. First, we analyze the case in which the buyer (or perhaps, the agent for the buyer) is naive. As we argue there, this could be a representation of agency capture. Second, we discuss a micro-foundation of our model of lobbying to discuss a seller's incentives to put emphasis on information or on bias. Finally, we introduce competition by assuming that two (strategically related) sellers can lobby and bid for a single project. A final section concludes the paper, and some proofs are relegated to an appendix.

2 Model

We begin by studying the case where there is a unique potential provider (Seller, "she"). The value of Seller's good for Buyer ("he"), $V(\theta) = (2\theta - 1)t$, depends on the realization, θ, of a random variable. Neither party observes θ, whose common prior is uniform in $[0, 1]$. The parameter $t > 0$ measures the – common knowledge – relevance of "fit" for Buyer. The value of not buying (staying with the status quo) is normalized to 0.[8] The game starts with Seller (privately) choosing a lobbying intensity $\alpha \in [0, 0.5]$ – at cost

[8] This model of monopoly, where the alternative to buying is a constant (zero) – as standard – is strategically equivalent to a Hotelling model, where the utility of buying from Seller, located at 1, is $v - (1 - \theta)t$, and that of not buying (choosing the good located at 0) is $v - \theta t$. In Section 5.3 we will use this interpretation to introduce competition.

$c(\alpha)$, $c(0) = 0$, $c' \geq 0$, $c'' \geq 0$ – as well as proposing a price, b. Next, Buyer observes $\widehat{\theta}$, an imperfect signal about θ – whose distribution is affected by α (unobserved by Buyer). Finally, he decides whether to buy from Seller for b or to stay with the status quo.

2.1 Lobbying and information

The signal structure is modelled as follows. Given Seller's lobbying intensity, α, and the realization of the preference parameter, θ, the signal, $\widehat{\theta}$, equals θ with probability $p(\alpha)$ and is uniformly distributed in $[\alpha, 1]$ with probability $1 - p(\alpha)$.[9] That is, with probability $p(\alpha)$ – we call this value precision – the signal is accurate and with the complementary probability it is (biased) noise. We assume $p(.)$ to be differentiable, with $p(0) > 0$, $p' \geq 0$.

Given α, the signal's density function, $\widehat{f}(.;\alpha)$, is easily seen to be

$$\widehat{f}(y;\alpha) = \begin{cases} p(\alpha) & \text{if } y < \alpha \\ \frac{p(\alpha)}{\sigma(\alpha)} & \text{if } \alpha \leq y \leq 1, \end{cases} \tag{1}$$

where

$$\sigma(\alpha) := \frac{p(\alpha)\,(1-\alpha)}{1-\alpha p(\alpha)}$$

is the information-to-signal ratio, conditional on the signal being in $[\alpha, 1]$.[10]

A few words on the particular modelling of lobbying. Intuitively, if lobbying were just smoke (a simple biasing of the signal in Seller's favor), then Buyer would trust the signal more when it pointed against Seller. That is, the information to signal ratio would be higher conditional on unfavorable realizations of the signal. By the same token, Buyer would be more skeptical when the signal were favorable to Seller's product. That is, the information to signal ratio would be lower for such signals. However, lobbying may also improve the information content of the signal (for all signal realizations). That only reinforces the first effect, but may compensate, and even reverse the skepticism of Buyer faced with a signal favorable to Seller. Our model simply captures these ideas, while

[9] Note that this captures the idea that lobbying by Seller concentrates a unifomly distributed noise term towards her "location".

[10] Note that conditional on a signal realization in $[\alpha, 1]$, the probability that the signal is accurate is $\frac{p(\alpha)(1-\alpha)}{1-\alpha p(\alpha)}$.

minimizing the complication that arises from the fact that the information to signal ratio, σ, is a function not only of α but also of $\widehat{\theta}$.[11]

Thus, and as we have put forward in the Introduction, this simple model captures what we see as three defining characteristics of lobbying in procurement – in addition to it being costly for Seller.

First, the intensity of lobbying is not observable and consequently cannot be credibly committed to. While this has no direct effect on the signal received by Buyer, it does affect the method of analysis, as – unlike in the case of information design – Seller's problem cannot be simply stated as that of choosing "the" posterior of Buyer (since the Bayesian update depends on Buyer's – possibly endogenous – beliefs about the lobbying intensity).

Second, despite its intended bias, lobbying can only add information. We capture this by $p' \geq 0$.

Third, lobbying increases the probability that the signal is favorable to Seller. The distribution of θ cannot be affected by lobbying, and so we assume that lobbying affects the distribution of noise: lobbying shifts probability from signals that, at face value, are less favorable to Seller (closer to 0) to signals that are more favorable to her (closer to 1).

The marginal effect of lobbying on the probability density of the signal, for $\widehat{\theta} \in [\alpha, 1]$, is

$$\frac{\partial \widehat{f}\left(\widehat{\theta};\alpha\right)}{\partial \alpha} = \frac{d\frac{p(\alpha)}{\sigma(\alpha)}}{d\alpha} = \left[-\alpha p'(\alpha) + \frac{1-p(\alpha)}{1-\alpha}\right]\frac{1}{1-\alpha}. \tag{2}$$

The second term between the square brackets is positive, but the first term is not: lobbying not only changes the distribution of noise (always in favor of the lobbyist's product) but it also increases the signal's precision. In order to capture the idea that lobbying by a seller

[11] For instance, it may be more elegant to assume that the noise follows a Beta distribution with parameters 1 and $g(\alpha) < 2$, for some increasing function g (with $g(0) = 1$, so that it is uniform when $\alpha = 0$). Recall that this would imply that increasing α moved the density – proportional to $\theta^{g(\alpha)-1}$ – and, thus, the expected value – $\frac{g(\alpha)}{g(\alpha)+1}$ – of the noise towards 1. In addition, the information to signal ratio would indeed be higher for realizations of $\widehat{\theta}$ close to 0 than for realizations closer to 1, and the difference would grow with both α and $\widehat{\theta}$. Our model shares this property with such a Beta model in a computationally very simple way.

ceteris paribus always increases the probability of favorable[12] signals, we bound the slope of the precision function ensuring that the expression in the square brackets is positive:

Assumption 1 $\alpha p'(\alpha) < \frac{1-p(\alpha)}{1-\alpha} = p(\alpha)\frac{1-\sigma(\alpha)}{\sigma(\alpha)}$, for $\alpha \in [0, 0.5]$.

Note that $\frac{p(\alpha)}{\sigma(\alpha)}$ being increasing in α means that even when lobbying increases the posterior probability that the signal is true (the information to signal ratio, σ), it does so by less than it increases the prior probability (p). That is, in the relevant range, lobbying "confounds" the signal with respect to an unbiased signal (with the same, unconditional information content, p).

As already mentioned, $\sigma(\alpha)$ itself may be increasing or decreasing in lobbying intensity. The sign of the derivative depends on the information content of (marginal) lobbying: when $p'(\alpha)$ is large we are in the first case and when it is small in the second (where the threshold is $p(\alpha)\frac{1-p(\alpha)}{1-\alpha} < \frac{1-p(\alpha)}{\alpha(1-\alpha)}$, where the latter value is the upper bound on p' given by Assumption 1).

Of course, in any case, the conditional (on *any* α) expected value of the posterior remains 1/2. Indeed, independently of the precision or the support of the noise – that is, of Seller's lobbying effort – every signal realization either conveys no information, in which case the posterior is the same as the prior, uniform on $[0, 1]$, or it is accurate, in which case it is distributed according to the prior. The fact that lobbying is mean preserving, captures the idea that while lobbyists can blow smoke (noise), they cannot completely misrepresent the truth.

Finally, note that we do not assume any information advantage by Seller regarding the fit of her product with Buyer's preferences.[13] She puts effort into presenting her product in the most favorable way, which allows Buyer to (privately) learn about the product's fit to his needs.

[12] Increasing the density for all values above α, increases the probability of favorable signal: With zero price, the threshold posterior for θ would be 0.5, for any positive price it is clearly above 0.5, so all posteriors favoring Seller are above 0.5. Finally, posteriors are always between the signal and 0.5 (the prior mean).

[13] Unlike in the strand of literature on lobbying that analyzes the – costly – strategic transmission of information that "sellers" (lobbyists) have. See, for instance, Lagerlöf (2007), Skaperdas and Vaidya (2012).

2.2 Lack of commitment and perfect Bayesian equilibrium

To capture the fundamental characteristic of lobbying that Buyer is uncertain as to how selective the information provided by a lobbyist actually is, we assume that Buyer observes the bid and the signal, but not the lobbying intensity. Thus, the payoff he expects depends on the conjectures that the observables – b and $\widehat{\theta}$ – will lead him to make with respect to the unobservable α. In equilibrium this is straightforward. However, when he is fully rational and realizes that play has veered off the equilibrium path – that is, when the bid surprises him – our concept of equilibrium (perfect Bayesian) imposes no restrictions on his conjectures. In principle, the mapping from out-of-equilibrium bids to these conjectures (that may also depend on $\widehat{\theta}$) could affect the equilibrium. Nonetheless, we do not consider this potential signaling role of pricing important for the problem at hand. Thus, as a first approximation, we restrict attention to equilibria with passive beliefs, so that Buyer will not change his equilibrium conjecture about α no matter what bid he receives (and what signal $\widehat{\theta}$ he observes): he considers unexpected prices to be a mistake.

3 Pricing equilibrium in the absence of lobbying

To establish a benchmark, suppose that – it is common knowledge that – Seller cannot engage in lobbying: $\alpha = 0$.

Buyer's decision depends on the conditional expectation of θ, having observed the signal, $\widehat{\theta}$. Given b, Buyer will be indifferent between buying from Seller or not if and only if

$$b = \left(2E\left[\theta \left| \widehat{\theta} \right.\right] - 1\right) t, \tag{3}$$

where Buyer's posterior expectation is given by[14]

$$E\left[\theta \left| \widehat{\theta} \right.\right] = p(0)\widehat{\theta} + (1 - p(0))\,\frac{1}{2}. \tag{4}$$

Substituting (4) into (3) and solving for $\widehat{\theta}$, we obtain that Buyer buys if and only if the

[14]Note that $\sigma(0) = p(0)$.

signal exceeds

$$\widetilde{\theta}(b) := \frac{1}{2} + \frac{b}{2tp(0)}. \tag{5}$$

Intuitively, the more precise the signal, the more Buyer trusts it, and so the less favorable he needs the signal to be to make him willing to pay b.

The expected payoff for Seller is then[15]

$$\left(1 - \widetilde{\theta}(b)\right) b = \left(\frac{1}{2} - \frac{b}{2tp(0)}\right) b,$$

leading to optimal price, probability of sale and expected profits of

$$b^* = \frac{tp(0)}{2},\ 1 - \widetilde{\theta}(b^*) = 1/4 \text{ and } \pi = \frac{tp(0)}{8},$$

respectively.

Seller's optimal monopoly "output" equates marginal revenue to marginal cost (zero), which as for any linear demand function occurs at the middle point between 0 and the demand at zero price, $\frac{1}{2}$ in this case. The latter is independent of the signal's precision, but Buyer's willingness to pay at "output" $\frac{1}{4}$ is increasing in how much Buyer cares about the fit (t) and the information precision ($p(0)$) the product of which measure the degree of (expected) product differentiation between Seller's product and the status quo.

As in any monopoly problem, the probability of sale (output) is too low from an efficiency viewpoint: market power introduces a price distortion that results in an increase in transportation costs. Indeed, the expected buyer's surplus is

$$\begin{aligned} BS &= \left(1 - \widetilde{\theta}(b^*)\right) \left[\left(2E[\theta \left| \widehat{\theta} > \widetilde{\theta}(b^*)\right] - 1\right) t - b^*\right] \\ &= \frac{1}{4}\left[\left(2\left(p(0)\frac{7}{8} + (1 - p(0))\frac{1}{2}\right) - 1\right) t - \frac{tp(0)}{2}\right] = \frac{tp(0)}{16}, \end{aligned}$$

whereas at the competitive price of 0 it would be $\frac{tp(0)}{4}$. The difference between the two, $\frac{3tp(0)}{16}$, is made up by Seller's expected profits, $\frac{tp(0)}{8}$, and an increase in "transportation costs" of $\frac{tp(0)}{16}$, an efficiency loss.

The inefficiency associated with market power is increasing in $p(0)$, since market power is also increasing in it. Indeed, it is the product differentiation that creates such market

[15] When, due to an excessively high price, $\widetilde{\theta} \geq 1$, the buyer refuses to purchase, so this case will never arise.

power: when $p(0) = 0$, the products are homogeneous even conditional on the signal (which is pure noise), and so price competition results in competitive prices. As the signal becomes more informative, product differentiation increases and so do market power and its consequences.

4 The equilibrium when lobbying is feasible

As Buyer does not observe Seller's lobbying effort, he conjectures an effort $\widehat{\alpha}$, an exogenous parameter that is assumed common knowledge. As discussed in the Introduction, in this way we can capture situations where Buyer is naive, corrupt, or misinformed about the cost of lobbying so that the conjecture $\widehat{\alpha}$ may not be correct. When $\widehat{\alpha}$ coincides with Seller's optimal response to that conjecture, $\alpha(\widehat{\alpha})$, we have a fixed point: an equilibrium.

Just as in the absence of lobbying, Buyer buys when the signal is above (5), except that now he evaluates the (conditional) probability of the signal being correct as $\sigma(\widehat{\alpha})$ rather than $p(0)$.[16] As the resulting cut-off $\widetilde{\theta}(b;\widehat{\alpha}) > 1/2 \geq \alpha$, using (1) we can see that the expected demand at price b is

$$\frac{1}{2}\left(1 - \frac{b}{t\sigma(\widehat{\alpha})}\right)\frac{p(\alpha)}{\sigma(\alpha)}. \tag{6}$$

Again, the demand is linear in b and so the optimal monopoly "output" (probability of sale) is the middle point between 0 and the demand at zero price, $\frac{1}{2}\frac{p(\alpha)}{\sigma(\alpha)}$. That is, the monopoly output (probability of sale) is $\frac{1}{4}\frac{p(\alpha)}{\sigma(\alpha)}$ (i.e., $b(\widehat{\alpha}) = \frac{t\sigma(\widehat{\alpha})}{2}$ and so $\widetilde{\theta}(b;\widehat{\alpha}) \equiv \frac{1}{2} + \frac{b(\widehat{\alpha})}{2t\sigma(\widehat{\alpha})} = \frac{3}{4}$).

That is, given Assumption 1, actual lobbying does not affect the optimal price but shifts the demand (the probability of sale) upwards. Therefore, our first result is that lobbying will always happen, at least as long as its marginal cost at $\alpha = 0$ is not too high.

Proposition 1 *If $c'(0) < \frac{1-p(0)}{8}t\sigma(\widehat{\alpha})$, Seller chooses to lobby ($\alpha > 0$) for any conjecture ($\widehat{\alpha}$) of Buyer.*[17]

[16] Recall that $\sigma(0) = p(0)$.

[17] It is straightforward to see that $\sigma(\alpha) \geq p(0)/2$, so a sufficient condition that is independent of Buyer's conjecture is $c'(0) < \frac{(1-p(0))p(0)}{16}t$.

Proof. Substituting Seller's optimal price into (6), her expected revenues are $t\frac{p(\alpha)}{8\sigma(\alpha)}\sigma(\widehat{\alpha}) > 0$. Therefore, given $c(0) = 0$, a sufficient condition for lobbying to happen is that the costs increase slower than the revenues at $\alpha = 0$: $c'(0) < \frac{t\sigma(\widehat{\alpha})}{8}\left(\frac{p(\alpha)}{\sigma(\alpha)}\right)'|_{\alpha=0} = \frac{t\sigma(\widehat{\alpha})}{8}(1 - p(0))$. ■

In the remainder of this section we focus on equilibrium behavior, where $\widehat{\alpha} = \alpha$. The consequences of possibly incorrect conjectures will be discussed in section 5.1.

Thus, assume that the condition in Proposition 1 is satisfied, so that lobbying indeed happens in equilibrium, and denote the equilibrium lobbying effort by α^*.[18] Our first observation is that despite it being an equilibrium phenomenon, the option to lobby may turn out to be a double-edged sword: Seller might end up worse off than if lobbying was (publicly known to be) not feasible. The reason for this is that the mere fact that Buyer *expects* Seller to lobby may lower Seller's profits across the board (for all α), and thus even if her best response is $\alpha > 0$, she ends up worse off.

Indeed, Seller's expected profits in equilibrium are $\frac{tp(\alpha^*)}{8} - c(\alpha^*)$ with lobbying possible and $\frac{tp(0)}{8} - c(0)$ otherwise. Thus, the option to lobby benefits Seller if and only if the equilibrium gain in precision outstrips the increase in cost by at least a ratio of $t/8$:

$$\frac{p(\alpha^*) - p(0)}{8}t > c(\alpha^*) - c(0). \tag{7}$$

The following corollary of Proposition 1 offers a sufficient condition for this, which applies even when the buyer does not anticipate the correct lobbying effort.

Corollary 1 *When lobbying increases the information-to-signal ratio, $\sigma(\alpha)$, Seller profits from the possibility of lobbying (when costs are affordable) for any $\widehat{\alpha}$ and $\alpha(\widehat{\alpha})$.*

Proof. Seller's profits with $\alpha = \alpha(\widehat{\alpha})$ when $\widehat{\alpha} = 0$ are not lower than without the option to lobby ($\alpha = \widehat{\alpha} = 0$). From the proof of Proposition 1, Seller's revenues are $t\frac{p(\alpha)}{8\sigma(\alpha)}\sigma(\widehat{\alpha})$, strictly increasing in $\sigma(\widehat{\alpha})$. Thus, from (repeated use of) the envelope theorem, Seller's revenues are strictly higher with $\alpha(\widehat{\alpha}) > 0$ than with $\alpha = \widehat{\alpha} = 0$. Thus, unless costs are prohibitive, Seller is better off. ■

[18] When the condition in Proposition 1 is satisfied, $\alpha(0) > 0$ and, since we are assuming $\alpha \leq \frac{1}{2}$, $\alpha(\frac{1}{2}) \leq \frac{1}{2}$. Thus, α^* exists.

The proof of the corollary also explains the possibility that Seller might be hurt by her (public) ability to lobby. Indeed, the costs may be low enough for some Buyer's conjecture $\widehat{\alpha} > 0$ (and so for any conjecture lower than $\widehat{\alpha}$, when σ is decreasing), so that Seller is better off lobbying *when he is expected to do so.* However, her revenues would be higher if she were expected to lobby less, as Buyer's demand would then shift upwards. Thus (invoking the envelope theorem), Seller would benefit from lobbying (marginally) less as long as Buyer's conjecture were (marginally) lower.[19] When this effect is sufficiently strong, Seller will prefer not to have the (public) possibility of lobbying. The following example illustrates.

Example 1 *Suppose $p(\alpha) = p$ for all $\alpha \leq 0.5$, so that σ is indeed decreasing in α, and also suppose that $c'(0) < \frac{1-p}{8} t\sigma(0) = \frac{(1-p)p}{8} t$, so that in equilibrium $\alpha^* > 0$. As shown in the proof of Proposition 1, Seller's profits would then be $\frac{tp}{8} - c(\alpha^*)$. If lobbying was not possible, then the profits would be $\frac{tp}{8}$, and so the known option to lobby would hurt Seller.*

From the point of view of Buyer, lobbying has three consequences. First, the precision of the signal is higher, which allows for a higher probability that his choice is informed. Second, the price is higher. Finally, when the signal is noise, Seller is going to be chosen more often than in the absence of lobbying. This is suboptimal, as in expected terms both choices are equally fitted to Buyer's needs, but Seller's comes with a higher price tag. However, the first effect dominates.

Proposition 2 *In equilibrium (and in expectation) Buyer is never worse off and if lobbying is informative, $p(\alpha^*) > p(0)$, he is strictly better off than without the possibility of lobbying.*

Proof. As we have seen, the probability of sale is $\frac{p(\alpha^*)}{4\sigma(\alpha^*)}$, the conditional probability of a precise signal given that the signal realization is above $3/4$ is $\sigma(\alpha^*)$, and the price is

[19] In fact, the argument could be extended to argue that Seller's profits would be higher (than in equilibrium) with conjectures $\widehat{\alpha} = 0$ and actual lobbying $\alpha = \alpha(0)$, but of course this is not sufficient to argue that Seller's profits would be higher with $\widehat{\alpha} = \alpha = 0$, as $\alpha(0) > 0$ in this case.

$\frac{t\sigma(\alpha^*)}{2}$. Also, conditional on not buying, Buyer's payoff is 0. Thus Buyer's expected payoff is given by

$$\begin{aligned} BS &= \left(1 - F(\widetilde{\theta})\right)\left[\left(2E[\theta \,\middle|\, \widehat{\theta} > \widetilde{\theta}] - 1\right)t - b^*\right] \\ &= \frac{p(\alpha^*)}{4\sigma(\alpha^*)}\left[\left(2\left(\sigma(\alpha^*)\frac{7}{8} + (1 - \sigma(\alpha^*))\frac{1}{2}\right) - 1\right)t - \frac{t\sigma(\alpha^*)}{2}\right] \\ &= \frac{tp(\alpha^*)}{16}. \end{aligned}$$

Since her expected payoff is $\frac{tp(0)}{16}$ without lobbying, the result follows. ■

Once more, we may understand this result appealing to basic results in the classic monopoly model with linear demand. From (6), Buyer's demand is

$$D(b) = \frac{1}{2}\left(1 - \frac{b}{t\sigma(\alpha)}\right)\frac{p(\alpha)}{\sigma(\alpha)},$$

where α is (anticipated) lobbying. Or, inverting,

$$b(q) = t\sigma(\alpha)(A - Bq),$$

where q represents the output (probability of sale), $A = 1$ and $B = 2\sigma(\alpha)/p(\alpha)$. (Anticipated) Lobbying shifts (by a factor $\sigma(\alpha)$) and rotates (by a factor $\sigma(\alpha)/p(\alpha)$) the inverse demand. The (parallel) shift by a factor $\sigma(\alpha)$ (equivalent to a change in numeraire) simply multiplies the consumer surplus (and the price) by the same factor $\sigma(\alpha)$. The reduction in (absolute value of) the slope by $\sigma(\alpha)/p(\alpha)$ does not change the vertical intercept of the demand nor the monopoly price, and increases the quantity (and so the consumer surplus) by a factor $p(\alpha)/\sigma(\alpha)$. Thus, the effect of lobbying is to multiply consumer surplus by a factor of $\sigma(\alpha) \times p(\alpha)/\sigma(\alpha) = p(\alpha)$. Consequently, anticipated lobbying indeed increases Buyer's payoff.

Needless to say, the linearity of the demand function is a consequence of assuming uniform distributions. Things may become messier in more general settings, but the basic idea (shifts and rotations of inverse demands) would still determine the effect of lobbying on Buyer's payoffs.

Proposition 2 and the corollary to Proposition 1 immediately imply that lobbying induces an efficiency gain when σ, the information-to-signal ratio, is increasing in lobbying.

Indeed, when this is the case, that is, when lobbying makes the signal more informative at the margin, both Seller and Buyer profit from lobbying, and so total surplus is higher. By the preceding results, both Seller's and Buyer's expected payoffs are higher with lobbying, so total surplus is higher as well. Moreover, as Seller does not incorporate the externality on Buyer when choosing α – and since the expected Buyer surplus is increasing in α for any $\widehat{\alpha}$ – the equilibrium level of lobbying is suboptimal. When σ is decreasing in α no such general conclusions can be drawn.

5 Extensions

We now discuss some ways in which the basic model may be modified to shed light on issues related to lobbying.

5.1 When Buyer is boundedly rational

Let us now abandon the equilibrium constraints on Buyer's conjectures. As it turns out, the interaction between Buyer's conjecture and the actual amount of lobbying depends on how informative lobbying is, that is, on the sign of the slope of σ.[20]

Corollary 2 *If $\sigma(\alpha)$ is increasing (decreasing), then Seller lobbies the more (less) intensely the higher Buyer's expectation of lobbying is.*

Proof. By the envelope theorem, the derivative of Seller's (expected) revenues with respect to $\widehat{\alpha}$ is $t\frac{p(\alpha)}{8\sigma(\alpha)}\sigma'(\widehat{\alpha})$, and so, by Assumption 1 the cross derivative of the revenues with respect to α and $\widehat{\alpha}$ has the same sign as $\sigma'(\widehat{\alpha})$. Since the costs are continuously increasing and independent of $\widehat{\alpha}$, in $\alpha(\widehat{\alpha})$ is increasing (decreasing) in $\widehat{\alpha}$ if $\sigma'(\widehat{\alpha}) > 0$ ($\sigma'(\widehat{\alpha}) < 0$). ■

[20] Note that $\sigma(\alpha)$ is increasing if and only if $p'(\alpha) > p(\alpha)\frac{1-p(\alpha)}{1-\alpha}$.

From the point of view of Buyer, his actual[21] payoff is

$$
\begin{aligned}
BS(\widehat{\alpha}) &= \frac{p(\alpha)}{4\sigma(\alpha)}\left[\left(2\left(\sigma(\alpha)\frac{7}{8}+(1-\sigma(\alpha))\frac{1}{2}\right)-1\right)t-\frac{t\sigma(\widehat{\alpha})}{2}\right] \\
&= \frac{tp(\alpha)}{16}\left(3-\frac{2\sigma(\widehat{\alpha})}{\sigma(\alpha)}\right),
\end{aligned}
$$

where $\alpha = \alpha(\widehat{\alpha})$.

Whether Buyer benefits from the existence of lobbying depends, among other things, on the information content of lobbying and the direction of its bias. Indeed, from the last equation, Buyer benefits from lobbying if, and only if,

$$p(0) < p(\alpha)\left(3-2\frac{\sigma(\widehat{\alpha})}{\sigma(\alpha)}\right).$$

This inequality may be written as

$$p(\alpha)-p(0) > 2\frac{p(\alpha)}{\sigma(\alpha)}\left(\sigma(\widehat{\alpha})-\sigma(\alpha)\right). \tag{8}$$

Thus,

Corollary 3 *When Buyer is* naive, *$\widehat{\alpha} < \alpha$, he is strictly better off with lobbying if $\sigma(\alpha)$ is increasing, but may be worse off otherwise, especially when $p(0)$ is large. When Buyer is* apprehensive, *$\widehat{\alpha} > \alpha$, he is strictly better off with lobbying if $\sigma(\alpha)$ is decreasing, but may be worse off otherwise, especially when $p(0)$ is large.*

Let us now take stock of the information contained in these two corollaries by looking at a case that may resonate with many views on lobbying. Consider the case that lobbying is more smoke than information, so that σ is decreasing. Also, suppose that Buyer, perhaps due to the play of special interest, is very naive, so that $\widehat{\alpha} = 0$. In this case, the incentives for Seller to engage in lobbying are strong, but the consequences for Buyer are quite negative, specially if absent lobbying Buyer would have a pretty good idea of the fit of Seller's product ($p(0)$ is high). Thus, there is a "rent" to be gained by securing a naive Buyer, and so an incentive to find (and fund) the way to do so.

[21] Since we are talking about an experience good, what matters for welfare is not the beliefs at the time of purchase but the true value of the product to Buyer (discovered later).

5.2 Micro-foundation of lobbying: the origins of $p(\alpha)$

We are treating a marginal increase of α as an action that has two consequences: increasing the information content of Buyer's posterior, and also increasing the prior probability of bias of the signal. The relation between these two effects is captured by the precision function $p(.)$. We consider $p(.)$as exogenous, rather than assuming that Seller can freely make a two-dimensional choice. We now present a realistic micro-structure for lobbying that justifies this modelling choice.

Suppose that there is a large number of small pieces of "evidence" that Seller could investigate and document (at some cost). Different pieces convey different proportions of information and noise. Or, equivalently, different units of information that could be "embellished" with smoke. Seller decides which pieces of evidence to provide when presenting her product. The mapping of pieces of evidence to p and α are determined by the choices of Seller.

More precisely, let Ω be the set of possible units of evidence and suppose unit $k = 1, 2, ...,$ contains proportion $\mu(k)$ of (accurate) information and $1 - \mu(k)$ noise. As we have been assuming, one unit of "noise" switches the lower bound of the signal's noise content by one unit. That is, if Seller chooses to deliver the set ϖ of evidence, then $\alpha = \lambda \sum_{k \in \varpi}(1 - \mu(k))$, where λ is a (normalizing) "constant" (that depends on ϖ). Similarly, $p = \lambda \sum_{k \in \varpi} \mu(k)$.

Note that, for any price b, Seller's probability of sales (demand) in the relevant range would be

$$\left(\frac{1}{2} - \frac{b}{2t\sigma(\widehat{\alpha})}\right)\frac{1 - \alpha p}{1 - \alpha}.$$

That is, decreasing in p and increasing in α. Therefore, Seller would choose to spend effort on pieces of evidence that are more intensive in smoke before turning to less smoke-intensive ones. Still, once the pieces of evidence are ordered from more smoke to less, larger α means a larger number number of pieces, and thus also a higher precision. Thus, p would be an increasing (and convex) function of α, exclusively determined by the empirical distribution of the available pieces of evidence.

5.3 When there is a direct competitor who also can lobby

We now consider the incentives and effects for lobbying when procurement is competitive by adding a second, potential supplier.[22] The presence of a strategically rival seller introduces competition on two – interactive – levels: in lobbying and in prices.

Thus, we assume that Seller 0 and Seller 1, located at opposite ends of a Hotelling interval $[0,1]$, set bids b_i and lobbying intensities α_i at the start (simultaneously). Buyer buys one, but only one, of the goods offered by these sellers. Sellers' lobbying influences the distribution of noise in the signal by concentrating it – still uniformly – in $[\alpha_1, 1-\alpha_0]$.[23] The precision of the signal is increasing independently in both lobbying intensities. For concreteness, we assume that it is an increasing function of their sum. Buyer's payoff is $v - |\theta - i|\, t - b_i$ when buying from Seller i. Buyer accepts the offer that gives him the higher expected payoff: we assume that v is sufficiently high, so that in equilibrium the status quo is irrelevant. Note that when we fix $\alpha_0 = b_0 = 0$, this model is strategically equivalent to the monopoly analyzed in the previous section.

Let $\boldsymbol{\alpha} := (\alpha_0, \alpha_1)$ and $A := \alpha_0 + \alpha_1$. The signal's density function, $\widehat{f}(.;\boldsymbol{\alpha})$, is now given by[24]

$$\widehat{f}(y;\boldsymbol{\alpha}) = \begin{cases} p(A) & \text{if } y < \alpha_1 \\ \frac{p(A)}{\sigma(A)} & \text{if } \alpha_1 \le y \le 1-\alpha_0 \\ p(A) & \text{if } y > 1-\alpha_0. \end{cases} \tag{9}$$

Similarly as before, we may compute the cut-off $\widetilde{\theta}(\boldsymbol{b};\widehat{\boldsymbol{\alpha}})$ when in $[\alpha_1, 1-\alpha_0]$ as

$$\widetilde{\theta}(\boldsymbol{b};\widehat{\boldsymbol{\alpha}}) = \frac{1}{2} - \frac{b_1 - b_0}{t\sigma(\widehat{A})}, \tag{10}$$

and so, for this case, the probability of sale for Seller 1 as

$$\alpha_0 p(A) + \frac{1}{2}\left(1 - 2\alpha_0 - \frac{b_1 - b_0}{t\sigma(\widehat{A})}\right)\frac{p(A)}{\sigma(A)},$$

which, again, as long as $\widetilde{\theta}(\boldsymbol{b};\widehat{\boldsymbol{\alpha}}) \in [\alpha_1, 1-\alpha_0]$, is increasing in α_1. As the cut-off must always be in $[0,1]$, the only way it is not in $[\alpha_1, 1-\alpha_0]$ is that at least one seller lobbies,

[22] We assume that Buyer cannot multi-source.

[23] Note that a competing lobbyist cannot directly "undo" the lobbying done by its rival.

[24] Recall that $\sigma(A) = \frac{p(A)(1-A)}{1-Ap(A)}$ is the information to signal ratio (in $[\alpha_1, 1-\alpha_0]$).

and in a symmetric equilibrium both do. That is, similarly as in the case of the single seller,

Proposition 3 *If $c_1'(0) = c_2'(0) = 0$, then for any $\widehat{\boldsymbol{\alpha}}$, at least one Seller will lobby in equilibrium.*[25]

Note that if the equilibrium is symmetric then both sellers will lobby. Thus, in (symmetric) equilibrium and for low lobbying costs, competition does not drive out lobbying. Values of the parameters such that in equilibrium $\widetilde{\theta}(\boldsymbol{b}; \widehat{\boldsymbol{\alpha}}) \notin [\alpha_1, 1-\alpha_0]$ (and so, necessarily, one of the sellers does no lobby) are less interesting for our purposes.

In what follows, we restrict attention to equilibrium conjectures, where Buyer conjectures the correct $\boldsymbol{\alpha}$. Also, since lobbying has been established to be a characteristic of equilibrium behavior, we specialize the cost functions to $c_i(\alpha_i) \equiv 0$ for $\alpha_i \in [0, \beta_i]$, and $c_i(\alpha_i) = \infty$ for $\alpha_i > \beta_i$ for some $\beta_i \leq \frac{1}{4}$, $i = 0, 1$. This helps simplify arguments and makes the results more transparent. Given Proposition 3, it is immediate that in equilibrium $\alpha_i = \beta_i$, $i = 0, 1$. Note that we are not imposing that sellers are symmetric, since they may have different cost functions. Nevertheless, we will pay particular attention to the symmetric case. Let $\boldsymbol{\beta} := (\beta_0, \beta_1)$ and $B := \beta_0 + \beta_1$.

5.3.1 The pricing equilibrium

Extending our discussion of monopoly, we have

$$E\left[\theta \left| \widehat{\theta}; \widehat{\theta} \in [\beta_1, 1-\beta_0] \right.\right] = \sigma(B)\widehat{\theta} + (1-\sigma(B))\frac{1}{2}, \tag{11}$$

whereas $E\left[\theta \left| \widehat{\theta}; \widehat{\theta} \notin [\beta_1, 1-\beta_0] \right.\right] = \widehat{\theta}$ leading to $\widetilde{\theta}(\boldsymbol{\beta}) = \frac{1}{2} + \frac{b_1 - b_0}{2t}$ when $\widehat{\theta} \notin [\beta_1, 1-\beta_0]$.[26]

From here, the bidding equilibrium can be characterized (as a function of $\boldsymbol{\beta}$), but the derivation is rather lengthy, so we relegate it to Appendix A. We only state the result – and then we perform some comparative static analysis – here.

[25] Using the results derived later, it can be shown that in equilibrium both sellers lobby (for low enough cost).

[26] Given the discontinuity at the boundaries, for a given b_i, there is an interval of bids $b_{j \neq i}$ for which $\widetilde{\theta} = \beta_1$ (and similarly for $\widetilde{\theta} = 1 - \beta_0$). We discuss this in more detail in the proof of Proposition 4 in the Appendix.

Proposition 4 *For any $\boldsymbol{\beta} \leq (1/4, 1/4)$, there exists a unique bidding equilibrium. In it, the marginal signal is $\widetilde{\theta}^*(\boldsymbol{\beta}) = \frac{1}{2} + \frac{b_1 - b_0}{2t\sigma(B)} \in [\beta_1, 1 - \beta_0]$ and the equilibrium bids for $i = 0, 1$ and $j \neq i$ are*

$$b_i^*(\boldsymbol{\beta}) = t\sigma(B)\left(\frac{\sigma(B)}{p(B)} + \frac{1 - \sigma(B)}{3}(\beta_i - \beta_j)\right). \tag{12}$$

Recall that, under Assumption 1, $\frac{\sigma(B)}{p(B)}$ is decreasing in both components of $\boldsymbol{\beta}$. Thus, when sellers are symmetric, $\beta_i = \beta_j$, and $\sigma(B)$ is decreasing then prices are obviously decreasing in lobbying intensity.

A lower price of course does not imply that unilateral marginal lobbying lowers profits, as it also affects the threshold, to what we turn next.

Corollary 4 *Although the threshold signal is closer to the seller that lobbies more intensively, the (expected) demand for this seller is still higher than her competitor's:*

$$F\left(\widetilde{\theta}^*(\boldsymbol{\beta})\right) = \frac{1}{2} - \frac{1 - \sigma(B)}{6}\frac{p(B)}{\sigma(B)}(\beta_1 - \beta_0).$$

That is, despite the threshold moving towards her – a joint consequence of the known bias and her higher price – the more lobbying seller sells more in equilibrium.

But what really matter to the sellers are profits.[27]

Corollary 5 *The expected profits for Seller $i = 0, 1$ in an equilibrium with lobbying intensities $\boldsymbol{\beta}$ are*

$$\pi_i(\boldsymbol{\beta}) = \frac{tp(B)}{2}\left[\frac{\sigma(B)}{p(B)} - \frac{(\beta_j - \beta_i)(1 - \sigma(B))}{3}\right]^2 \tag{13}$$

for $j \neq i$. Consequently, the more intensively lobbying seller earns higher profits.

Let us turn to the symmetric case, where $\beta_1 = \beta_2$. In this case, each seller's profits are $\frac{t\sigma(B)}{2}\frac{\sigma(B)}{p(B)}$. The second term is decreasing in B, from Assumption 1. Thus,

Corollary 6 *When $\beta_1 = \beta_2$ and $\sigma(B)$ is decreasing, sellers are made worse off by the possibility of lobbying.*

[27] Without our special cost function we could only talk about revenues here.

Thus, sellers are in a Prisoner's Dilemma situation. In other words, the possibility of lobbying is not to the sellers advantage unless lobbying conveys sufficient information. In equilibrium, what makes lobbying attractive is not biasing the signal, but increasing the expected product differentiation. This phenomenon was present even in the monopoly case. Competition only reinforces it. Indeed, in the monopoly case, an increasing σ was a sufficient (but not necessary) condition for Seller to benefit from the possibility of lobbying. With competition, it is a necessary but not a sufficient, condition. (More on this in Subsection 5.3.3 below.)

The gross buyer surplus in the symmetric case ($\beta_0 = \beta_1$) is

$$v - t\left(\frac{1-p(B)}{2} + \frac{p(B)}{4}\right) = v - t\left(\frac{1}{2} - \frac{p(B)}{4}\right),$$

as Buyer will buy from the seller closer to the signal, which is either noise or the true θ. Thus, the gross surplus is increasing with the signal precision. Since p is non-decreasing in lobbying intensity, the direct effect of lobbying on Buyer is non-negative. However, lobbying also affects the equilibrium prices. From Proposition 4, in a symmetric equilibrium, Buyer's (net) payoff is

$$\begin{aligned} & v - t\left(\frac{1}{2} - \frac{p(B)}{4}\right) - t\frac{\sigma(B)^2}{p(B)} \\ = \; & v - \frac{t}{2} - tp(B)\left[\left(\frac{\sigma(B)}{p(B)}\right)^2 - \frac{1}{4}\right]. \end{aligned} \tag{14}$$

Using (14) and (12) it is straightforward to see that we have three regimes, depending on the shape of the precision function. Note that $\sigma(0) = p(0)$. Thus,

Corollary 7 **i)** *If and only if*

$$\frac{\sigma(0)p(B)}{\sigma^2(B)} < \frac{4 - \left(\frac{p(B)}{\sigma(B)}\right)^2}{3}, \tag{15}$$

then Buyer is hurt by (symmetric) lobbying: the increase in price outweighs the benefit of better information.

ii) *If and only if*

$$\frac{\sigma(0)p(B)}{\sigma^2(B)} \in \left[\frac{4-\left(\frac{p(B)}{\sigma(B)}\right)^2}{3}, 1\right],$$

then there is a price increase, but it is more than compensated for by the improved information; while

iii) *if*

$$\frac{\sigma(0)p(B)}{\sigma^2(B)} > 1,$$

then the price actually decreases, so Buyer benefits from both effects caused by (symmetric) lobbying. This is the case, in particular, if $\sigma(B)$ is decreasing.

As $\frac{p(B)}{\sigma(B)}$ tends to one as $p(B)$ tends to one, for any initial precision strictly less than one there exists a final precision high enough so that Buyer is worse off. At the same time, since $\frac{p(B)}{\sigma(B)} > 1$, for any final precision there exists a high enough initial precision $\sigma(0)$ (still below it) so that Buyer is better off in the lobbying equilibrium. Note that in the monopoly case Buyer always benefited from lobbying (as long as precision was improved).

5.3.2 Asymmetric lobbying

It is worthwhile to analyze the case when lobbying is asymmetric, as it introduces an additional effect. We restrict attention to the behavior of the expected payment as the direct effect on the precision is still positive.

The expected payment is

$$F(\widetilde{\theta})b_0 + (1 - F(\widetilde{\theta}))b_1. \tag{16}$$

Substituting in the equilibrium bids and rearranging, we obtain

$$tp(B)\left(\frac{\sigma(\beta)}{p(B)}\right)^2 \left[1 - (\beta_0 - \beta_1)\frac{1-\sigma(\beta)}{3}\left(1 - 2F(\widetilde{\theta})\right)\right].$$

By Corollary 4, $F(\widetilde{\theta}) > .5$ if and only if $\beta_0 > \beta_1$. Thus, the expected bid payment is higher than in the absence of asymmetry. Indeed, by (12) the difference in lobbying efforts increases the price of Seller 0 by exactly as much as it reduces the price of Seller 1. However, as the demand of the seller exerting more lobbying effort exceeds 50%, Buyer pays the higher price more often.

With respect to transportation cost, the bias resulting from the asymmetry actually *reduces* them: Buyer takes a better decision using a more selective information. Formally, given B and prices, Buyer will take the best decision if all the lobbying is done by the same seller. This, seemingly counterintuitive, finding is easy to understand. As we have pointed out, when the signal happens to be noise, the expected transportation cost is $t/2$ whatever the choice. However, lobbying has two positive effects. The first, is to "clean" the signal from noise for some domain. That reduces transportation cost, whether lobbying is symmetric or not. The second effect is only present when lobbying is asymmetric: conditional on the signal falling in the middle interval, the expected transportation cost is lower. Indeed, in that case the expected transportation cost is lower when Buyer buys from the more active seller: asymmetry moves the middle interval towards her. As with prices, the effect is exactly the opposite for the other seller. But Buyer purchases from the more active seller more often, so takes the better decision more often than absent asymmetry (in which case both options imply the same expected transportation costs).

5.3.3 The effects of competition

It is interesting to compare the results on prices and profits that we obtained with only one seller and with two of them. Comparing their levels is not very enlightening: Buyer's alternative to buy from Seller 1 in one case and the other has a very different value.[28] What is more interesting is to observe the difference in how lobbying affects these levels in one case and the other.

Note that the price may be written as $tp(\alpha)/2$ times $\sigma(\alpha)/p(\alpha)$ in monopoly and as $tp(B)$ times $(\sigma(B)/p(B))^2$ in symmetric duopoly. The multiplying terms, which in the

[28] A way to reconcile the levels is to assign different values to t and the length of the interval (and so the "demand") in one case and the other so that the equilibrium demand is the same number in both cases. Thus, suppose that t is twice as large in the monopoly than in the duopoly, and demand is also double. In that case, prices and profits would be the same in the two cases when firms cannot lobby.

absence of lobbying are just 1, measure the effect of lobbying on prices in each case. Thus, in the direct competition case this effect of lobbying (in reducing prices) is stronger than in the monopoly case: σ/p, which is less than one, is squared. Indeed, by increasing the elasticity of sellers' residual demand (probability of the signal at the margin) lobbying induces a more aggressive bidding behavior by the rival which is matched by a then more aggressive bidding behavior by each firm.

Of course, if competition increases the aggregate lobbying intensity, $\alpha < B$, this effect of lobbying may be more than compensated by an increase in the degree of product differentiation. Finally, the same effect of lobbying combined with competition may be observed with respect to revenues, which can be written as $tp(\alpha)/8$ in the monopoly case and as $tp(B)/2$ times $\sigma(B)/p(B)$ in the competition case.

In turn, this differential effect of lobbying on pricing explains what we have obtained regarding profitability. While an increasing information to signal ratio is a sufficient condition for lobbying to be profitable for the seller in the case of monopoly, it is only a necessary one in case of competition. Likewise, while an improved information content is all that is needed for Buyer to benefit from lobbying in the monopoly case, with competition this is far from sufficient.

6 Concluding remarks

We have presented a tractable model of procurement lobbying, where sellers improve Buyer's information about the fit of their product in a biased – and unobservable – way. Our results deepen our understanding of this phenomenon. When there is a single potential lobbyist, lobbying always[29] happens independently of Buyer's expectations. However, but only if the increase in the precision of the signal as a result of lobbying is sufficiently low Seller may and up worse off than in a situation where influencing Buyer's signal is known to be impossible. At the same time, Buyer is always better off if he anticipates Seller's lobbying effort correctly, but may lose out if his prior information is relatively good and either he underestimates lobbying that is relatively uninformative (overrreacts

[29] Unless it is too expensive.

to the signal) or he overestimates it and lobbying is very informative (underreacts to the signal).

When there is a second lobbyist, these engage in two-dimensional competition. We still have lobbying in equilibrium, but now it is more likely that the sellers are worse off as a result. As their biasing of the information is in opposite directions this is not too surprising. However, now Buyer can be worse off even if he anticipates the lobbying efforts correctly, as the prices might rise to compensate for the more informed choice when the information content in lobbying increases product differentiation sufficiently. Indeed, lobbying reduces the elasticity of demand, just as when there is only one seller, but since prices are strategic complements, the corresponding increase in the rival's price will be an added incentive for each seller to raise her own price.

We have obtained that an important determinant of the effects of lobbying is the information-to-signal ratio, what we call $\sigma(\alpha)$. When lobbying decreases the value of this ratio (at the marginal signal), even though sellers have incentive to lobby, if they could they may prefer to commit not to do so: Buyer's increased reliance on the signal (and so the reduced elasticity of his demand) would be worth it. On the contrary, when lobbying increases the information-to-signal ratio, Buyer may lose as the result of a too informative signal: the better fit of needs to products may not compensate for the increased price tag of this better fit.

Appendix A

The derivation of the bidding equilibrium under competition

Proposition 4 For any $\boldsymbol{\beta} \leq (1/4, 1/4)$, there exists a unique bidding equilibrium. In it, the marginal signal is $\widetilde{\theta}^*(\boldsymbol{\beta}) = \frac{1}{2} + \frac{b_1 - b_0}{2t\sigma(B)} \in [\beta_1, 1-\beta_0]$ and the equilibrium bids for $i = 0, 1$ and $j \neq i$ are

$$b_i^*(\boldsymbol{\beta}) = t\sigma(B)\left(\frac{\sigma(B)}{p(B)} + \frac{1-\sigma(B)}{3}(\beta_i - \beta_j)\right).$$

Proof The expected revenues for Sellers 0 and 1, respectively, can be written a

$$F\left(\widetilde{\theta}(\beta, b)\right) b_0 \quad \text{and} \quad \left(1 - F\left(\widetilde{\theta}(\beta, b)\right)\right) b_1,$$

where, from (10) and taking into account that the equilibrium cut-off signal might not be in $[\beta_1, 1-\beta_0]$, $\widetilde{\theta}(\boldsymbol{\beta}, b)$ is given by[30]

$$\widetilde{\theta}(\boldsymbol{\beta}, b) = \begin{cases} \frac{1}{2} + \frac{b_1-b_0}{2t} & \text{if } \frac{1}{2} + \frac{b_1-b_0}{2t} < \beta_1 \\ \beta_1 & \text{if } \widetilde{\theta}^*(B,b) < \beta_1 \leq \frac{1}{2} + \frac{b_1-b_0}{2t} \\ \widetilde{\theta}^*(B,b) & \text{if } \beta_1 \leq \widetilde{\theta}^*(B,b) \leq 1-\beta_0 \\ 1-\beta_0 & \text{if } \frac{1}{2} + \frac{b_1-b_0}{2t} \leq 1-\beta_0 < \widetilde{\theta}^*(B,b) \\ \frac{1}{2} + \frac{b_1-b_0}{2t} & \text{if } 1-\beta_0 < \frac{1}{2} + \frac{b_1-b_0}{2t}, \end{cases}$$

where $\widetilde{\theta}^*(B,b) = \frac{1}{2} + \frac{b_1-b_0}{2t\sigma(B)}$. Note that there are five possible scenarios: three interior solutions – in $[0,\beta_1)$, $(\beta_1, 1-\beta_0)$ or $(1-\beta_0, 1]$ – and two corner solutions – β_1 or $1-\beta_0$ – for the threshold signal in terms of the bids – dropping the (in equilibrium impossible) extreme cases where one firm is chosen for all signal realizations.

Next, we derive a necessary condition for (pure-strategy) equilibrium, leading to a unique putatively optimal price vector. Then we will show that there is no profitable deviation from that pair of prices, completing the proof.

Let us start with characterizing the best response for Seller 0**.** Seller 0's expected revenues can be written as

$$F\left(\widetilde{\theta}\right) b_0 = b_0 \begin{cases} \left(\frac{1}{2} + \frac{b_1-b_0}{2t}\right) p(B) & \text{if } \frac{1}{2} + \frac{b_1-b_0}{2t} < \beta_1 \\ \beta_1 p(B) & \text{if } \widetilde{\theta}^* < \beta_1 \leq \frac{1}{2} + \frac{b_1-b_0}{2t} \\ \beta_1 p(B) + \left(\widetilde{\theta}^* - \beta_1\right) \frac{p(B)}{\sigma(B)} & \text{if } \beta_1 \leq \widetilde{\theta}^* \leq 1-\beta_0 \\ 1-\beta_0 p(B) & \text{if } \frac{1}{2} + \frac{b_1-b_0}{2t} \leq 1-\beta_0 < \widetilde{\theta}^* \\ 1-\left(\frac{1}{2} - \frac{b_1-b_0}{2t}\right) p(B) & \text{if } 1-\beta_0 < \frac{1}{2} + \frac{b_1-b_0}{2t}. \end{cases}$$

We can rewrite this as a function of $b_1 - b_0 := \Delta$:

$$F\left(\widetilde{\theta}\right) b_0 = b_0 \begin{cases} \left(\frac{1}{2} + \frac{\Delta}{2t}\right) p(B) & \text{if } \Delta \leq -(1-2\beta_1)t \\ \beta_1 p(B) & \text{if } \Delta \in (-(1-2\beta_1)t, -(1-2\beta_1)t\sigma) \\ \beta_1 p(B) + \left(\widetilde{\theta}^* - \beta_1\right) \frac{p(B)}{\sigma(B)} & \text{if } \Delta \in [-(1-2\beta_1)t\sigma, (1-2\beta_0)t\sigma] \\ 1-\beta_0 p(B) & \text{if } \Delta \in ((1-2\beta_0)t\sigma, (1-2\beta_0)t) \\ 1-\left(\frac{1}{2} - \frac{\Delta}{2t}\right) p(B) & \text{if } \Delta \geq (1-2\beta_0)t\ . \end{cases} \tag{17}$$

Here is an illustration, depicted for a fixed value of b_1

[30] Note that, since $\sigma(B) < 1$, if and only if $b_1 < b_0$ then $\widetilde{\theta}^*(\beta, b) < \frac{1}{2} + \frac{b_1-b_0}{2t}$.

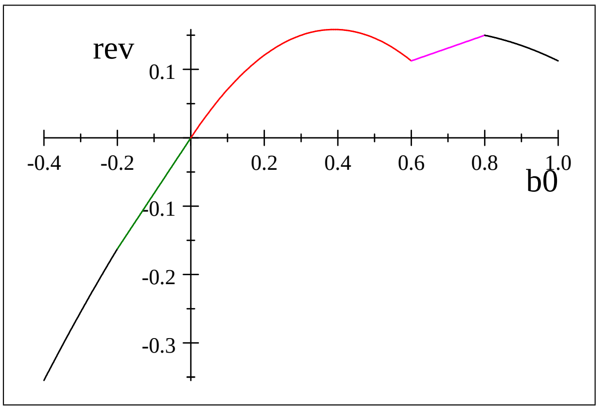

Figure 1 $\beta_i = 1/4$, $p(1/2) = 3/4$, $t = 1$, $b_1 = 0.3$

From (17), the derivative of Seller 0's revenue function with respect to b_0 is

$$\frac{F\left(\widetilde{\theta}\right)b_0}{db_0} = \begin{cases} \left(\frac{1}{2} + \frac{b_1 - 2b_0}{2t}\right)p(B) & \text{if } \Delta \leq -(1-2\beta_1)t & R1 \\ \beta_1 p(B) & \text{if } \Delta \in (-(1-2\beta_1)t, -(1-2\beta_1)t\sigma(B)) & R2 \\ \beta_1 p(B) + \left(\frac{1}{2} + \frac{b_1-2b_0}{2t\sigma} - \beta_1\right)\frac{p(B)}{\sigma(B)} & \text{if } \Delta \in [-(1-2\beta_1)t\sigma(B), (1-2\beta_0)t\sigma(B)] & R3 \\ 1 - \beta_0 p(B) & \text{if } \Delta \in ((1-2\beta_0)t\sigma(B), (1-2\beta_0)t) & R4 \\ 1 - \left(\frac{1}{2} - \frac{b_1-2b_0}{2t}\right)p(B) & \text{if } \Delta \geq (1-2\beta_0)t\ , & R5 \end{cases}$$

where we named the different regions.[31] Therefore, it is straightforward that, given $\boldsymbol{\beta}$ and the bid of the competitor, out of the five regions in (17), both sellers' revenue functions are linear and strictly increasing in $R2$ and $R4$ – and they are (piece-wise) strictly concave in each region of the rest of the domain.

Next we show that Seller 0's revenue function is strictly decreasing in $R1$ – and, therefore, Seller 1's revenue function is strictly decreasing in $R5$. A sufficient condition for the best response never to be in the interior of $R1$ is that the slope of the revenue function at the upper boundary of the region (i.e., at the lowest value of b_0 in the region) is non-positive. That is,

$$\left(\frac{1}{2} + \frac{b_1 - 2\left[b_1 + (1-2\beta_1)\,t\right]}{2t}\right)p(B) \leq 0,$$

or

$$b_1 \geq t\,(4\beta_1 - 1)\,, \tag{18}$$

[31] Note that in Figure 1 the regions show up in reverse order (as they are defined for Δ, not b_0.

what, given $\beta_i \leq 1/4$ is always satisfied.

In sum, there could be no equilibrium in the interior or regions $R1$, $R2$, $R4$, or $R5$. Moreover,

Lemma 1 *For any $\boldsymbol{\beta}$, no equilibrium exists at any boundary point of regions $R2$ or $R4$*

Proof. Suppose prices are such that $1-\beta_0 = \widetilde{\theta}^*$. That is, given the price b_1, Seller 0 sets a price at the lower bound of region $R4$. At that point, the right derivative of Seller 0´s profits with respect to b_0 is positive, and so Seller 0 is not best responding. Consider now the upper bound of $R4$ and note that this would coincide with the lower bound of the $R2$ if we had defined the regions in terms of Seller 1's profits, instead of Seller 0's. Thus, at that point, the right derivative of Seller 1's profits with respect to b_1 is positive, and so Seller 1's is not best responding. The exercise to exclude the corners of $R2$ follow a symmetric reasoning, inverting the roles of Seller 0 and 1. ■

As no equilibrium could exist with zero sales by one seller, only an interior equilibrium in region $R3$ is possible. The first-order conditions are

$$\begin{aligned} F\left(\widetilde{\theta}^*\right) + f\left(\widetilde{\theta}^*\right)\frac{\partial\widetilde{\theta}^*}{\partial b_0}b_0 &= 0, \\ 1 - F\left(\widetilde{\theta}^*\right) - f\left(\widetilde{\theta}^*\right)\frac{\partial\widetilde{\theta}^*}{\partial b_1}b_1 &= 1 - F\left(\widetilde{\theta}^*\right) + f\left(\widetilde{\theta}^*\right)\frac{\partial\widetilde{\theta}^*}{\partial b_0}b_1 = 0. \end{aligned}$$

Adding the two equations, we can write this system as

$$\begin{aligned} F\left(\widetilde{\theta}^*\right) + f\left(\widetilde{\theta}^*\right)\frac{\partial\widetilde{\theta}^*}{\partial b_0}b_0 &= 0, \\ 1 + f\left(\widetilde{\theta}^*\right)\frac{\partial\widetilde{\theta}^*}{\partial b_0}(b_1+b_0) &= 0. \end{aligned} \tag{19}$$

Substituting into (19), we obtain from the first equation

$$\frac{1}{2} + \frac{b_1 - 2b_0}{2t\sigma(B)} = \beta_1(1-\sigma(B))$$

and from second equation

$$b_1 = 2t\frac{\sigma(B)^2}{p(B)} - b_0.$$

Solving, we obtain that

$$\begin{aligned}
b_0^* &= \frac{2t\sigma(B)}{3}\left(\frac{\sigma(B)}{p(B)} - \beta_1(1-\sigma(B)) + 1/2\right) \\
&= t\frac{\sigma(B)^2}{p(B)} - \frac{2t\sigma(B)}{3}\left(\frac{\sigma(B)}{2p(B)} + \beta_1(1-\sigma(B)) - 1/2\right) \\
&= t\frac{\sigma(B)^2}{p(B)} + \frac{t\sigma(B)}{3}(1-\sigma(B))(\beta_0 - \beta_1)
\end{aligned}$$

and

$$b_1^* = t\frac{\sigma(B)^2}{p(B)} + \frac{t\sigma(B)}{3}(1-\sigma(B))(\beta_1 - \beta_0).$$

For these values to be in the interior of the middle region, we need to check that

$$b_1^* - b_0^* = \frac{2t\sigma(B)}{3}(1-\sigma(B))(\beta_1 - \beta_0) \in [-(1-2\beta_1)t\sigma(B), (1-2\beta_0)t\sigma(B)] \tag{20}$$

or

$$2\beta_1 - 1 < \frac{2}{3}(1-\sigma(B))(\beta_1 - \beta_0) < 1 - 2\beta_0$$

When $\frac{1}{4} \geq \beta_1 \geq \beta_0$ the first inequality is trivially satisfied, and the second is also satisfied, since the right-hand side is at least $\frac{1}{2}$ and the left hand side is decreasing in $\sigma(B)$ and so it is always lower than $\frac{2}{3} \cdot \frac{1}{4} = \frac{1}{6}$. The exercise is symmetric when $\frac{1}{4} \geq \beta_0 > \beta_1$.

To prove that (b_1^*, b_0^*), indeed constitute an equilibrium, all we have left to prove is that – given that the competitor plays b_j^* – there does not exist a profitable deviation for Seller i to outwith $R3$. Again we take Seller 0's problem, Seller 1's is symmetric.

We already know the best response cannot be in the interior of $R1$, since the objective function is strictly decreasing. A sufficient condition for Seller 0's best response never to be in $R5$ – is that the slope of the revenue function at the upper boundary (in b_0) of that region is positive. That is,

$$1 - \left(\frac{1}{2} - \frac{b_1^* - 2\left[b_1^* - (1-2\beta_0)\,t\right]}{2t}\right)p(B) > 0,$$

or

$$b_1^* < t\left(\frac{2}{p(B)} + 1 - 4\beta_0\right). \tag{21}$$

Substituting in (and dividing by t), we need

$$\frac{\sigma(B)^2}{p}+\frac{\sigma(B)}{3}(1-\sigma(B))(\beta_1-\beta_0)<\frac{2}{p(B)}+1-4\beta_0.$$

Since $\beta_0 \leq 1/4$, the right-hand side exceeds 2. As $\sigma(B) \leq p(B)$, the first term on the left-hand side is less than 1, while the second is less than 1/12. Thus the inequality is satisfied.

Given that the profit function is strictly concave in $R5$ and its slope is positive at the highest value of b_0 in the region, we can guarantee that the slope is also positive in all of $R5$. Neither can the best response be in $R4$, where the profit is increasing in b_0. Thus the only remaining possibility is at the highest value of b_0 in $R2$. That is, a price of $b_0 = b_1^* + (1-2\beta_1)t$. Therefore, we need to show that $\pi' = \beta_1 \left(b_1^* + (1-2\beta_1)t\right) p(B)$ is less than the hypothetical equilibrium profit, $\pi^* = F\left(\widetilde{\theta}^*(B, b^*)\right) b_0^*$. As

$$\pi^* = \beta_1 p(B) b_0^* + \left(\widetilde{\theta}^* - \beta_1\right) \frac{p(B)}{\sigma(B)} b_0^*,$$

we need

$$\frac{p(B)}{\sigma(B)}\left(\widetilde{\theta}^* - \beta_1\right) b_0^* - \beta_1 \left(b_1^* - b_0^* + (1-2\beta_1)t\right) p(B) \geq 0 \tag{22}$$

or

$$\begin{aligned}&\left(\frac{1}{2}+\frac{(1-\sigma(B))(\beta_1-\beta_0)}{3}-\beta_1\right)\left(\frac{\sigma(B)}{p(B)}+\frac{1}{3}(1-\sigma(B))(\beta_0-\beta_1)\right)\\ \geq\ &\beta_1\left(\frac{2\sigma(B)}{3}(1-\sigma(B))(\beta_1-\beta_0)+1-2\beta_1\right).\end{aligned}$$

Denoting $\frac{(1-\sigma(B))(\beta_1-\beta_0)}{3}$ by W

$$\left(\frac{1}{2}+W-\beta_1\right)\left(\frac{\sigma(B)}{p(B)}-W\right) \geq \beta_1 \left(2\sigma(B)W + 1 - 2\beta_1\right).$$

Since $\sigma(B) < 1$, it is sufficient if,

$$\left(\frac{1}{2}+W-\beta_1\right)\left(\frac{\sigma(B)}{p(B)}-W\right) \geq \beta_1 \left(2W + 1 - 2\beta_1\right),$$

or

$$\left(\frac{1}{2}+W-\beta_1\right)\left(\frac{\sigma(B)}{p(B)}-W-2\beta_1\right)\geq 0.$$

The first term is clearly positive, as $W<1/12$. The second term is positive if

$$\frac{\sigma(B)}{p(B)}-W>2\beta_1.$$

The left-hand side is easily seen to be decreasing in β_0,[32] so it is lowest when $\beta_0=1/4$. In this case W is non-positive, while we have $\frac{\sigma(B)}{p(B)}\geq\sigma(B)>1/2\geq 2\beta_1$.

Q.E.D.

Appendix B

Proof of Corollary 4. Substituting into $\widetilde{\theta}^*(B,b)=\frac{1}{2}+\frac{b_1-b_0}{2t\sigma(B)}$ from (20)

$$\widetilde{\theta}^*(B,b)=\frac{1}{2}+\frac{(1-\sigma(B))(\beta_1-\beta_0)}{3}.$$

Therefore,

$$\begin{aligned}
F\left(\widetilde{\theta}^*(B,b)\right) &= \beta_1 p(B)+\left(\frac{1}{2}+\frac{(1-\sigma(B))(\beta_1-\beta_0)}{3}-\beta_1\right)\frac{p(B)}{\sigma(B)}\\
&= \frac{\left(\frac{1}{2}+\frac{(1-p(B))(\beta_1-\beta_0)}{3(1-Bp(B))}\right)(1-Bp(B))-\beta_1(1-p(B))}{1-B}\\
&= \frac{\frac{1-Bp(B)}{2}-\frac{(1-p(B))(\beta_1+B)}{3}}{1-B}\\
&= \frac{3-2B-Bp(B)-2\left(1-p(B)\right)\beta_1}{6(1-B)}=1/2+\frac{\beta_0-\beta_1+p(B)(2\beta_1-B)}{6(1-B)}\\
&= 1/2+(\beta_0-\beta_1)\frac{1-p(B)}{6(1-B)}.
\end{aligned}$$

■

[32] The derivative is $(1-p(B))\left(1/3-1/(1-Bp(B))^2\right)$.

Proof of Corollary 5. We obtain π_0 by simply substituting in for $F(\widehat{\theta}^*)$ and b_0^*.

$$
\begin{aligned}
& F\left(\widetilde{\theta}^*(B,b)\right) b_0(\beta) \\
= & \left(\frac{1}{2} - \frac{1-\sigma(B)}{6}\frac{p(B)}{\sigma(B)}(\beta_1 - \beta_0)\right)\left(t\frac{\sigma(B)^2}{p} + \frac{t\sigma(B)}{3}(1-\sigma(B))(\beta_0 - \beta_1)\right) \\
= & t\sigma(B)\left[(\beta_1 - \beta_0)^2(1-\sigma(B))^2\frac{p(B)}{18\sigma(B)} - (\beta_1 - \beta_0)\frac{1-\sigma(B)}{3} + \frac{\sigma(B)}{2p(B)}\right] \\
= & \frac{t\sigma(B)}{2}\left[(\beta_1 - \beta_0)(1-\sigma(B))\sqrt{\frac{p(B)}{9\sigma(B)}} - \sqrt{\frac{\sigma(B)}{p(B)}}\right]^2.
\end{aligned}
$$

■

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