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Bidding for talent in sport*

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Abstract

We present a novel micro-structure for the market for athletes. Clubs simultaneously target bids at the players, in (Nash) equilibrium internalizing whether – depending on the other clubs' bids – a player not hired would play for the competition. For low/inelastic talent supply, we support – and generalize to heterogeneous players – the Coasian results of Rottenberg (1956) and Fort and Quirk (1995): talent allocation is efficient and independent of initial "ownership" and revenue sharing arrangements. We also characterize equilibria for high/elastic supply. The analysis uses a non-specific club objective with an endogenously derived trade-off between pecuniary and non-pecuniary benefits.

JEL codes: J4, L1, L2.

Key words: competitive balance, competitive foreclosure, contested workers, labor auction, procurement in sport.

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1 Introduction

Despite the increasing economic significance of the sports industry,[1] the literature on the economics of sport – kicked off by Rottenberg (1956) – continues to be "behind the curve". While there is a generally accepted overall view of the peculiarities of the labor market in this industry – nicely crystallized by Rosen and Sanderson (2001) –, less headway has been made in formal theoretical analysis that not only explains the empirical observations, but can provide insights that are transferable to other industries with oligopolistic competition in both input and output markets.[2] In this paper we take a step in that direction, putting forward a set of original modelling choices, which together form a basic microstructure of the labor market in sport. This framework is amenable to being built upon with the introduction of further institutional details.

We start our journey positing a state-of the-art[3] form for the clubs' objective functions, unifying pecuniary and non-pecuniary motives. The novelty in our formulation is to provide micro-foundations for how the trade-off between utility and money can be endogenously derived, implying the absence of a budget constraint. Our fundamental observation is that both benefactors and supporter-owned clubs have alternative uses for money that also provide them with "utility". A magnate might wish to buy a yacht, a members' club might want to subsidize its other activities/teams. This fact implies that rather than an inflexible (budget) constraint, what the club has to factor into its decision is the shadow utility – or opportunity cost – of the money spent on the team, captured by the slope of the club's indirect utility function. As a result, we can postulate a general objective function, incorpo-

[1]For example, according to Sport England: "*In 2010, sport and sport-related activity generated Gross Value Added (GVA) of £20.3 billion ($30 billion) – 1.9% of the England total. This placed sport within the top 15 industry sectors in England and larger than sale and repair of motor vehicles, insurance, telecoms services, legal services and accounting.*" And this is before taking into account the savings in health care costs, estimated at $16 billion. See also Bryson et al. (2015).

[2]As an example, our model of the player market could be adapted to the market for CEOs, and contribute to an explanation of why they are "overpaid".

[3]For an excellent survey of club objective functions, see Fort (2015). Madden (2015b) (referred to as 'Madden (2012)' by Fort) is the closest to our formulation.

rating both profit and/or non-profit criteria, and still use the first-order approach.

Next, we turn to our main contribution, the modelling of the player market. A crucial question in setting up a game-theoretic model for this is what the clubs' choice variable should be. We claim that it is unhelpful to think of the amount of talent hired as the (strategic) decision of a club.[4] While payoffs are determined by the final distribution of talent, it is clear that neither empirically nor theoretically – just think of the constrained supply case – is it credible that clubs can unilaterally decide the amount of talent they hire.[5] It is their strategic interaction in the player market that leads to the final talent allocation, so we need to model that market with care.

In order to develop a viable microstructure, we note that an essential characteristic of sport is that the clubs are interacting in two different markets. Not only are they competing for the players, but they are also engaged in a tournament – and, in fact, in joint production – on the field/pitch/court. As a result, the willingness to pay of a club for an additional player depends on where this player would go if she were not hired by the club.[6] Moreover, the fundamental issue is that clubs would like to – and, in practice, do – *affect* whether or not they are in direct competition with another club for a player.

We thus posit a market where clubs can decide which players to approach. In actual fact, this may involve complicated forms of multilateral negotiations, but – for simplicity – we model it as the clubs simultaneously making take-it-or-leave-it

[4]This approach leads to difficulties with the interpretation of Nash equilibrium and/or conjectural variations, as the protracted debate (originally kicked off by Szymanski and Késenne, 2004) in the *Journal of Sport Economics* illustrates.

[5]Alternatively, a club *could* unilaterally decide how much to spend on players (see, for example, Madden, 2015a). However, it is unclear how such a game could be implemented in the absence of an "invisible hand", and – as we will see – positing a micro-structure for the market is important.

[6]In a (theoretical) two-team league, one could say that if the supply of talent is low/inelastic then the player will go with the rival, while otherwise they will be unemployed (see Madden, 2011, for a continuous version of this scheme). However, this approach clearly breaks down if there are more than two clubs in the league.

offers to individual players.[7] That is, the clubs' object of choice – their *strategic variable* – is the vector of wage offers (bids) they make. In a Nash equilibrium then each club's vector of bids is a best response to the other clubs' bids (and to the players' acceptance strategies).

The nature of equilibria is determined by the clubs' endogenous demand functions for talent: how much a club values an extra unit of talent depends not just on its own talent level but also on the *distribution* and the *aggregate level* of talent across clubs, at which we wish to evaluate this marginal value. In addition, when considering to hire an additional player – given the other[8] club's (equilibrium) bids – there are two options: the new player could either be attracted away from the rival club – in the case of contested players, who receive an acceptable offer from the other club; or she could be hired from the pool of non-employed talent – in the case of *uncontested* players, for whom the other club does not bid.[9]

Assuming that players are infinitesimal and identical, our first result is to show that in any equilibrium all hired players must be paid the same wage. This wage must equal the marginal willingness to pay for players (of both clubs), otherwise they would have an incentive to hire/poach or let go some players. For uncon-

[7] This can also be construed as a collection of simultaneous sealed-bid auctions, where the clubs bid competitively for each individual player. In Burguet and Sákovics (2017), we use the same bidding model to analyze the competition for inputs among oligopolists. In that paper we concentrate on the input price and quantity enhancing effects of competitive foreclosure and the relevance of (non)anonymity, while there are no issues relating to objectives, competitive balance or revenue sharing. De Fraja and Sákovics (2001) also consider the possibility of competitive bidding in a decentralized market, but they have a random matching environment, rather than targeted offers. Palomino and Sákovics (2004) have both targeted bidding and externalities in a sports context, but they have a single player in each team.

[8] For clarity's sake we set up our framework for a two-team competition. The generalization to more teams is conceptually straightforward. With more teams, however, additional asymmetric equilibria may exist, where clubs compete for players with only a subset of the other clubs.

[9] A similar distinction has often been made in the literature but as exogenous constraints (or conjectural variations) not as the endogenous consequence of the rival's strategy. For example, Dobson and Goddard (2011) consider the cases of closed and open player markets separately. In the former all players are hired and thus contested, by assumption, while in the latter there is an elastic supply of talent and players are uncontested, again by assumption.

tested players this must also coincide with their reservation wage: in the absence of competition, there is no point in paying them more.

Next, we turn to the region of the parameter space (low reservation wage and/or scarcity of talent) where the marginal willingness to pay (of both clubs) for contested players exceeds their reservation wage for all talent distributions such that all talent is hired. We show that an equilibrium with full employment, exclusively comprising contested players, exists and provide a sufficient condition – an upper bound on the reservation wage – for its uniqueness. This equilibrium confirms the Invariance Principle (initial allocation of talent does not affect the final, efficient, allocation). When the equilibrium is not unique, all other equilibria involve involuntary unemployment. When reservation wage is high and/or talent is abundant, we can no longer have full employment. For illustration, we posit a family of revenue functions (micro-founded, based on Falconieri et al., 2004), for which we demonstrate the computation of equilibria.

We also show that – when the Invariance Principle applies – the sharing of (some fraction of) the revenues between the clubs simply dampens the incentives to hire talent, with no effect on its allocation. This does not leave the league without a motive for intervention: when the clubs bid for the same players, they may compete too fiercely, not just bidding up wages, but even hiring too much talent. As revenue sharing softens the competition, it can compensate for this tendency.

To retain ease of comparability with the literature, we carry out the above analysis with homogeneous players. This by no means is a necessary simplification. In Section 6.1 we argue that most of our results directly carry over to a model where there are a continuum of players with different talent levels (and different reservation wages). Practically all that changes is that wages are interpreted "per unit of talent".

Limits on "input usage" (a club can only field the amount of talent that is embodied in the number of its players that the rules allow to play) also make the distribution of talent across individual players an important issue. In Section 6.2 we discuss how our model may accommodate the presence of "stars", players with a positive measure of talent, when these constraints are taken into account.

Finally, we show that if players prefer to play for one club over the other (for the same wage) then these compensating differentials can be straightforwardly incorporated into our analysis, with the Law of One Wage holding for the net payments received.

We end the paper with some concluding remarks, pointing out the numerous issues that we have not addressed in our analysis. Many of these may be avenues for future research based on our model of the market for talent. The proofs of our results are collected in Appendix A, while Appendix B contains the derivation of our illustrative parametric revenue function and the analysis of the corresponding equilibria.

2 Club objectives

We posit a formulation where the clubs' preferences are defined over the final distribution of talent and in which both pecuniary and non-pecuniary benefits are incorporated. We go beyond the state of the art in the (sub)literature on unifying the club objectives – see Fort (2015) for a detailed retrospective – by providing two new ingredients: i) the opportunity cost of spending money on talent is modelled endogenously via an indirect utility function; ii) the latter innovation allows for an endogenous method for deriving the MRS between utility and money as the slope of the indirect utility function, what in turn leads to the observation that budget constraints may be ignored in the analysis.[10]

Since the early days, two competing views have dominated the sports economics arena. On the one hand, more popular among American scholars, it was postulated that clubs maximize profits, just as most firms do. On the other hand, starting with Neale (1964), it has been argued that clubs owned by a "benefactor" – or by

[10]This latter feature is already present in the formulation by Madden (2015b) where he also does away with the budget constraint and uses an exogenous "generosity parameter" to trade off utility against money. Importantly, he obtains an interior solution as well.

a large number of "members" who do not receive dividends, as is often the case with European clubs – do not maximize profits. The usual way of modelling these "utility maximizing" clubs (c.f. Sloane, 1971) is that they hire all the talent they can afford. As the optimal behavior according to the first approach leads to a first-order condition, while the second one is determined by a budget constraint, they often lead to drastically different conclusions, even in terms of comparative statics.

Let us flesh out our argument for a unified club objective by formalizing the two traditional views on it, together with the one we wish to put forward.[11] The ingredients are: the amount of talent hired by the club, t; the distribution of talent in the league, $\boldsymbol{t} = (t_1, t_2)$; an exogenous budget (which may include future income), B; a revenue function, $R(\boldsymbol{t})$; a cost function, $C(t)$; a utility function measuring the non-pecuniary benefits derived from the hired talent, $U(\boldsymbol{t})$; and a (strictly increasing) indirect utility function measuring the utility derived from the next-best use of money, $V(\$)$. We assume additive separability of the two utility functions, and normalize $R(0,.) = C(0) = U(0,.) = 0$.

The traditional formulations are straightforward:

Profit maximization: $\max_t [R(\boldsymbol{t}) - C(t)]$ – equivalently, $\max_t [B + R(\boldsymbol{t}) - C(t)]$; F.O.C. : $\frac{dR(\boldsymbol{t})}{dt} = C'(t)$.

Utility maximization: $\max_t U(\boldsymbol{t})$ s.t. $B + R(\boldsymbol{t}) - C(t) \geq 0$. For any increasing $U(.)$, the solution requires a binding budget constraint, $B + R(\boldsymbol{t}) - C(t) = 0$.

We propose a unified formulation, where a club's objective is the sum of its non-pecuniary benefit from hiring talent, $U(\boldsymbol{t})$, and of the additional benefit that it achieves by spending its net money holding, $B + R(\boldsymbol{t}) - C(t)$, elsewhere. That is,

Our unified approach: $\max_t [U(\boldsymbol{t}) + V(B + R(\boldsymbol{t}) - C(t))]$; F.O.C. :

$$\frac{dU(\boldsymbol{t})}{dt} = V'(B + R(\boldsymbol{t}) - C(t))(C'(t) - \frac{dR(\boldsymbol{t})}{dt}). \quad (1)$$

Looking at (1), note that V' measures the marginal utility of an extra unit of

[11] For simplicity, we assume that all the expected revenue can be invested in talent: there are no credit constraints. This assumption might be relevant for our negative result on the usefulness of revenue sharing to increase competitive balance.

money, and $C' - \frac{dR(\boldsymbol{t})}{dt}$ is the money an extra unit of talent costs the club. At the optimal choice, the product of these two values must, therefore, equal the marginal utility from an additional unit of talent.

It is immediate that, since $V(.)$ is increasing, when non-pecuniary effects are not present – $U(.) \equiv 0$ – the new formulation leads to the same solution as profit maximization. To recover "utility maximization", we would need to make tortuous assumptions on V to recreate the notion of a binding (in both directions) budget constraint (e.g. that V is zero for non-negative values but it is minus infinity for negative ones).

Condition (1) can be rewritten as

$$\frac{dR(\boldsymbol{t})}{dt} + \frac{\frac{dU(\boldsymbol{t})}{dt}}{V'(B + R(\boldsymbol{t}) - C(t))} = C'(t). \tag{2}$$

When the club maximizes profits, the optimal (interior) solution equates the marginal cost of one more unit of talent to the marginal revenue it brings to the club. In general, in our unified approach, the optimal solution equates the marginal cost of one more unit of talent to the marginal increase in the clubs' objectives. The left-hand side of (2) can be viewed as a modified marginal "revenue" function, one that includes not only the direct revenue effect of one more unit of talent, but also the (non-pecuniary) effect on the utility of members/owners, measured in money terms, where the exchange rate between money and utility is given by $V'(B + R(\boldsymbol{t}) - C(t))$.

If neither risk aversion nor wealth effects are significant, the slope of the indirect utility function, $V'(.)$, may be approximated with a constant in the relevant range.[12] We will maintain this assumption throughout the rest of the paper. It will allow us to work with a crisp model of wage determination and talent allocation that focuses on the interaction between clubs without having to disentangle these interactions from less informative income effects.

Despite allowing for the objectives of the club to include both utility and profit considerations, we will continue to refer to the primitive – with respect to its own

[12] See Friedman and Sákovics (2015) for a detailed motivation and analysis of a similar model in a consumer choice context.

talent – of the left hand side of (2), $Z(\boldsymbol{t}) := R(\boldsymbol{t}) + U(\boldsymbol{t})/V'$, as the "revenues".[13] Note that, given our formulation of $Z(\boldsymbol{t})$, not only is there no budget constraint (V' is capturing borrowing costs instead) but there is also no individual rationality constraint to worry about: $Z(\boldsymbol{t}) - C(t) = 0$ has no special economic meaning. The outside opportunities are embodied in the indirect utility function.

Remark 1 *There are a couple of observations worth making. First, note that – unlike $U(\boldsymbol{t}) + V(B + R(\boldsymbol{t}) - C(t))$ – our objective function, $Z(\boldsymbol{t})$, is not a vNM utility function and neither is the convex combination of revenues and utility derived from the hired talent. Instead, it is a monetized value, which is not scalable – and that is why it can be compared across clubs. Second, note that the functions $R(\boldsymbol{t})$ and $U(\mathbf{t})/V'$ are not too different, after all the clubs' revenues come from the payments of fans (for tickets, TV channels etc.) whose preferences are much like the owners'/members'. So both functions depend similarly on total talent level, probability of winning and competitive balance, with only the weights on these factors perhaps being different.*

While the club objectives are defined over the distribution of talent, clubs cannot independently choose their talent level.[14] The final talent allocation is the outcome of the clubs' interaction in the player market. We turn to that topic next.

3 A simple model of the player market

On the supply side, we assume that there is a continuum of talent of measure T, available for hire at (or above) a reservation wage of r. In order to avoid technical difficulties arising from indivisibilities, we treat each infinitesimal unit of talent as a separate entity – that is, a "player" in our non-cooperative game.[15]

[13] Fort (2015) and some of the references therein, also propose a similar functional form, but without the micro-foundations we provide.

[14] Mathematically speaking, the clubs cannot independently evaluate $\frac{dR(\boldsymbol{t})}{dt}$.

[15] In Section 6.1 we discuss the case where players are heterogeneous, be it in the amount of talent they possess or in their reservation wage. In Section 6.2 we investigate the consequences of

The demand comes from two competing clubs, whose gross payoffs are determined by the final allocation of acquired talent. In particular, if t_i units of talent are hired by Club i, it earns "revenue" $Z^i(t_i, t_{3-i})$, for $i = 1, 2$.[16] Then, the payoff functions are $Z^i(t_i, t_j) - C^i$, where C^i is Club i's wage bill (for simplicity, its only cost). We do not write $C^i(t_i)$ – not even $C^i(t_i, t_j)$ – for the wage bill, to emphasize that the cost of hiring t_i units of talent is endogenous, even conditional on (t_i, t_j), as it depends on the clubs' bidding behavior in the player market. To retain the generality of the analysis and also to keep the focus on the market for talent, we treat the relationship between talent distribution and revenues as a black box. This general set-up allows for our results not to be restricted to a specific assumption (like a contest success function, see Szymanski, 2003) about how the talent distribution leads to the (composite) revenues. Instead, we can make do with some generic regularity conditions on the revenue functions Z^i.

Next, we describe how the market for talent operates. Each Club i, simultaneously, sets a – deterministic and Lebesgue measurable – wage schedule, $W_i(\tau)$, $\tau \in [0, T]$, specifying an individual wage offer to each[17] player. Players then accept the highest bid above their reservation wage that they have received – if any. As we are conducting a full game-theoretic analysis, players on the pitch are also players in our game and as such they have strategies that also form part of the Nash equilibria derived. Most notably, their (possibly mixed) strategy upon receiving two identical acceptable offers serves as an endogenous rationing device. Importantly, the clubs are committed to honor all the offers they have made (if accepted).

Take a hypothetical equilibrium, where given the clubs' wage schedules the resulting distribution of talent is (t_i, t_j). To confirm the equilibrium, we need to evaluate whether, holding Club j's strategy – which is its wage schedule, not the amount of talent it hires – fixed, it is in the interest of Club i to change its of-

a player having significantly more talent than the rest.

[16] For ease of exposition, we assume that the revenue functions are twice differentiable in both arguments.

[17] We require that each player receive an offer for mathematical simplicity. If a club wishes not to make an offer to some players, we model it as it offering them a wage below r.

fers/bids. Considering marginal deviations, we observe that Club i's willingness to pay for a marginal unit of talent is equal to its marginal revenue (c.f. (2)), given by $\frac{dZ^i(t_i,t_j)}{dt_i} = \frac{\partial Z^i(t_i,t_j)}{\partial t_i} + \frac{dt_j}{dt_i} \cdot \frac{\partial Z^i(t_i,t_j)}{\partial t_j}$. There are two ways Club i can increase its talent level. If it decides to outbid Club j for a (contested) player then it decreases the rival's talent level by the same amount it increases its own: $\frac{dt_j}{dt_i} = -1$, resulting in $\frac{dZ^i(t_i,t_j)}{dt_i} = Z_1^i(t_i, t_j) - Z_2^i(t_i, t_j)$. If instead it hires an additional uncontested player – that is, one who does not receive an offer from Club j, whose wage schedule we are holding constant –, it does not affect the talent level of Club j: $\frac{dt_j}{dt_i} = 0$. Thus marginal revenue (willingness to pay) is simply $\frac{dZ^i(t_i,t_j)}{dt_i} = Z_1^i(t_i, t_j)$.

The clubs' choice between hiring contested or uncontested players is at the heart of our analysis. By the above discussion, the value of a contested player is Z_2^i lower than that of an uncontested one. Thus, the determinant of a club's preference is the sign of Z_2^i. If the adversary's hiring of an additional unit of talent hurts Club i ($Z_2^i < 0$) then it will prefer to poach a contested player. On the other hand, if its revenue increases in the other club's talent level ($Z_2^i > 0$) then Club i will prefer to go after an uncontested player (assuming $Z_1^i(t_i, t_j) \geq r$, of course). The sign of Z_2^i is determined by the relative strength of the three factors that affect revenue: total talent level, sporting performance and competitive balance. The first of these pushes Z_2^i up, the second pulls it down, while the direction of the third effect depends on the talent distribution at which we evaluate Z_2^i. At the level of generality of this analysis we cannot assume that any effect dominates, so we will consider all the possibilities.

4 Analysis

4.1 The law of one wage

We now turn to the analysis of the model. Our first result shows that despite the flexibility that our mechanism offers clubs to wage discriminate, in equilibrium not only does each club pay the same wage to all of its players,[18] but the wages paid by the two clubs equalize as well. Note that we need no additional assumptions on the revenue functions for this result to hold.

Lemma 1 *In any Nash equilibrium all hired players are paid the same wage.*

An intuitive way of grasping this remarkable result is to note that each club must be willing to make the same offer to each player it is competitively bidding for, as the maximization problem, given the expected outcome in the competition for the other players, is the same. Also, competition equalizes the offers made to each contested player (otherwise the club offering more would benefit from lowering its wage). Next, note that, in equilibrium, any uncontested player who is hired must be paid r – there is no point in offering a higher wage if there is no competing offer. Finally, observe that if any uncontested player is hired, the wage offered to any (hired) contested players must also be r. Indeed, no club will pay a wage above r to a contested player, at the same time letting another player go uncontested to the competitor for a wage of r. We collect these insights in a more nuanced corollary to Lemma 1.

Corollary 1 *In any Nash equilibrium,*

i. *If any uncontested players are hired, the common wage is r.*

ii. *If only uncontested players are hired but not all players are hired, then the marginal willingness to pay for an uncontested player must equal r for both clubs: $Z_1^1(t_1, t_2) = Z_1^2(t_2, t_1) = r$. (If there is full employment, the marginal willingnesses to pay need not equal either each other or r.)*

iii. *If any contested players are hired, the wage must equal both clubs' marginal willingness to pay for a contested player: $w = Z_1^1(t_1, t_2) - Z_2^1(t_1, t_2) = Z_1^2(t_2, t_1) - Z_2^2(t_2, t_1) \geq r$.*

[18] This is due to all players being identical. In general, the interpretation is that the wage per unit of talent is equalized – at least for contested players (c.f. Section 6.1).

4.2 Equilibria

We split the analysis of the equilibria of our model into two parts. The reason for this is two-fold: first, this way we can directly compare them with existing results; second, the complexity of the analysis is qualitatively different in the two cases.

4.2.1 Preliminaries

By Lemma 1 we can see that despite the complex strategies available to the clubs, the marginal willingness to pay functions described in Section 3 can be usefully thought of as the clubs' (inverse) demands for any constant equilibrium wage. As is standard, we require that this demand function be downward sloping.[19]

Assumption 1 $Z_1^i(t_i, T - t_i) - Z_2^i(t_i, T - t_i)$ *is strictly decreasing in* t_i *for all* t_i, $i = 1, 2$.

In words, when all talent is hired, the incentive to poach a player from the rival is decreasing in the amount of talent the club already has.

Remark 2 *In practice it is likely that for small* t_i *marginal revenue is actually increasing (an issue first pointed out in Madden, 2010): when* t_i *is small (and all talent is employed) competitive balance is low, so the pie to divide is small, so – as the effect of an extra unit of talent on the winning probability is also small – the combined effect is small and therefore smaller than when competitive balance is high (this argument can be formalized using explicit contest success functions). A non-monotonic marginal revenue function leads to a residual demand function with jumps. This is because for any given price the demand is always on the marginal revenue function but when there are multiple talent distributions leading to the same*

[19] Our revenue function is a more complex object than usual, consisting of both a monetary (R) and a non-monetary (U/V') part. For their sum to be concave in t_i, a sufficient condition is that both of these functions are concave in t_i. As the utility function is concave, we are back to requiring concavity of the monetary revenues.

MR, there is one of them selected (the one maximizing $Z^i(t_i, E - t_i) - pt_i$). Thus, in the likely case that the MR curve is single-peaked, we would have a "minimum viable scale" (MVS), given by the talent level where average revenue equals marginal revenue. Note that taking into account fixed costs (that we have assumed away) a MVS would also arise naturally. For simplicity – and following the literature – we do not incorporate these constraints into our model (we assume $MVS = 0$), as it would not affect our main conclusions.

Finally, to facilitate a formal existence proof, we also impose a "common sense" assumption about the range of the demand function:

Assumption 2 [20] *$Z_1^i(0, T) - Z_2^i(0, T) > r \geq Z_1^i(T, 0) - Z_2^i(T, 0)$, $i = 1, 2$;*

Here the first inequality states that, at the maximum of the demand function, willingness to pay exceeds the players' reservation wage, a condition that guarantees that the market may operate – that is, there are gains from trade. The second inequality is the flip side: one team cannot earn benefits from hiring an additional player when the other barely participates.

Next, we calculate the hypothetical "market clearing" wage – and the corresponding talent distribution – when all players are contested.[21] Thus, we let t^* be the solution to

$$Z_1^1(t, T - t) - Z_2^1(t, T - t) = Z_1^2(T - t, t) - Z_2^2(T - t, t), \tag{3}$$

and let

$$w^* = Z_1^1(t^*, T - t^*) - Z_2^1(t^*, T - t^*). \tag{4}$$

By Assumptions 1 and 2, these values are uniquely defined. Note that $(t^*, T - t^*)$ is also the allocation that would result if the league – maximizing the sum of the

[20] In line with Remark 2, we could replace the zeros with the minimum viable scale (and the Ts with $T - MVS$).

[21] There is no claim of equilibrium behavior here. The parameters we calculate should be thought of as primitives of the model.

clubs' revenues – would choose how to allocate talent across the teams.[22] As it turns out whether w^* exceeds r or not,[23] makes an enormous difference not only to the equilibrium set but even to the techniques necessary to characterize it. Let us start with the "easier" case, which is also the one that is more similar to the existing literature.

4.2.2 $w^* \geq r$

This is the situation that can lead to the outcomes that are often put forward in the literature, ever since El-Hodiri and Quirk (1971).

Proposition 1 *When $w^* \geq r$, there exists a unique full-employment equilibrium outcome, with a uniform wage of w^* and the league-optimal allocation of talent: $t_1 = t^*$, $t_2 = T - t^*$.*

While this is a well-known result,[24] we derive it here as a Nash equilibrium of a non-cooperative game, where the clubs can choose which players to bid for. We also explicitly incorporate the effects of hiring contested players into the marginal revenue functions and therefore into willingness to pay. Moreover, we characterize

[22] To see this, note that the league's first-order condition is

$$\begin{aligned}\frac{dZ(t,T-t)}{dt} &= \frac{dZ^1(t,T-t)+Z^2(T-t,t)}{dt} = \\ &= Z_1^1(t,T-t) - Z_2^1(t,T-t) + Z_2^2(T-t,t) - Z_2^1(T-t,t) = 0,\end{aligned}$$

what is equivalent to (3).

[23] For any given revenue functions, this depends on the relative sizes of T and r. Low supply leads to high market-clearing wage.

[24] Whitney (2005) challenges this result on two counts. First, he – correctly – points out that the league-optimal talent allocation does not obtain if clubs hire exclusively uncontested players (in his case by assumption). What we show here is that the clubs will choose not to engage with uncontested players, as long as the aggregate talent available (T) is sufficiently low so that $w^* > r$. Whitney also claims that if the market worked in two stages – first the teams bought (uncontested) players on the open market and then they engaged in bilateral trades – inefficiency would necessarily result. We believe that this is because of his (implicit) assumption that the clubs do not anticipate the second stage when they hire talent in the first.

the rest of the possible equilibria (when they exist) and provide a sufficient condition for this, focal equilibrium to be unique. In Section 6.1 we further argue that the result extends to heterogeneous players.

In this equilibrium both clubs offer the equilibrium wage – equalling their marginal revenue – to all the players and the players accept each club's offer with the probability corresponding to the equilibrium proportion of talent hired by that club, t_i/T.[25] As all the players that are hired are contested, each club knows that if it lets a player go, this player will end up playing for the other team. It is *as if* there was a technological constraint requiring that trades can only happen between clubs. Thus, we have verified the insight of Rottenberg/Coase:

Corollary 2 *When $w^* \geq r$,[26] the final allocation of talent (in the full-employment equilibrium) will be the same as if clubs started with an arbitrary initial allocation of players, but they were allowed to frictionlessly trade among themselves.*

Uniqueness of the equilibrium outcome cannot be guaranteed without further assumptions on the revenue functions. $w^* \geq r$ only guarantees (existence and) uniqueness among full-employment equilibria. While it is tempting to think that the condition does imply at least that, "since" the aggregate demand for contested players (weakly) exceeds supply, there is no opportunity to hire any uncontested player (Club i cannot prevent Club j from bidding for a player Club i would like to have an exclusive deal with), this argument only holds when the clubs make a serious offer to every player – and thus full employment is guaranteed, see below. In general, we cannot draw conclusions about the behavior of the demand functions when there is no full employment from their behavior under full employment (e.g.

[25] Alternatively, different players could accept offers from different clubs, in a way that the aggregate measures of acceptance are t_i/T.

[26] Note that $w^* \geq r$ is only a sufficient condition. The only scenario where the equivalence breaks down is where there are uncontested players hired in equilibrium. For that to happen, we must have that the marginal benefit of hiring an unemployed player is higher than attracting one away from the rival. That is, the concerns about aggregate revenue must outweigh the concerns about performance on the pitch. For example, when the revenues are shared in a non-performance-related manner. See Section 5.

$Z_1(., t_2)$ can be very different from $Z_1(., t_2/2)$). Indeed, even with $w^* \geq r$, equilibria can exist where some uncontested players are hired (and, therefore, by Corollary 1, the common wage is $r = Z_1^1(t_1, t_2) = Z_1^2(t_2, t_1)$).

Let us now consider the possibility that some players are (involuntarily) unemployed.[27] These players would be happy to accept the market wage, but they are not approached by either club. This can happen in our model as, due to the personalized nature of offers, the clubs can choose to approach the same subset of players (in equilibrium). Indeed, equilibria can exist where the two clubs hire $t_1 + t_2 < T$ (contested) players and the common wage paid to the hired (contested) players is given by

$$w(t_1, t_2) = Z_1^1(t_1, t_2) - Z_2^1(t_1, t_2) = Z_1^2(t_2, t_1) - Z_2^2(t_2, t_1). \tag{5}$$

Moreover, in addition to not all players being employed, some equilibria may exhibit both contested and uncontested hires. Of course, unemployment can only be part of equilibrium if no one is interested in hiring some of the uncontested players:

4.2.3 Full employment

In order to present a sufficient condition that ensures that there exists no unemployment in equilibrium, let us define the highest wage at which there is always (that is, at any distribution of talent) demand for an uncontested player by at least one club.

Definition 1 *Let* $\widehat{w} = \sup[w \in \mathbb{R} \,|\max\{Z_1^1(t_1, t_2), Z_1^2(t_2, t_1)\} \geq w \;\forall\, (t_1, t_2) : t_1 + t_2 \leq T]$.

With this definition in hand, the following lemma is immediate, as when $\widehat{w} > r$ at least one club would strictly want to hire an unemployed player (at wage r) at any interior – that is, $t_1 + t_2 < T$ – talent distribution.[28]

[27] Not getting hired by either club need not mean that the player is literally unemployed. For example, a basketball player not hired in the NBA might play in Europe (or a lower league, say, ABA 2000). Nonetheless, for ease of exposition we will label them as unemployed.

[28] When $t_1 + t_2 = T$ there are no unemployed players left to hire.

Lemma 2 *When $\widehat{w} > r$, there can be no unemployment in equilibrium.*

When $w^* \geq r$, we have already argued that with full employment no uncontested players will be hired. This leads to our uniqueness result:

Corollary 3 *When $\widehat{w} > r$, the equilibrium outcome of Proposition 1 is unique.*

4.2.4 $w^* < r$

When the reservation wage exceeds w^*, an equilibrium where all the players are contested (and thus hired) is no longer possible, since by Assumption 1 at such r at least one club would prefer not to contest some players. This observation shows that – similarly to the case where $\widehat{w} < r \leq w^*$ and both contested and uncontested players are hired in equilibrium – the analysis becomes more complex as the interplay between contested and uncontested offers make the problem multi-dimensional. The situation is simplest when $\widehat{w} > w^*$. Then, by Lemma 2, in the region $r \in (w^*, \widehat{w})$ any equilibria must be of the form of splitting the player market between the clubs. Given full employment and not all players being contested, by Corollary 1, any equilibrium will exhibit a wage of r. It is the particular form of the Z^i, what determines the mix of contested and uncontested players hired.

When $r > \max\{w^*, \widehat{w}\}$, only under somewhat unrealistic revenue functions can we retain full employment (with uncontested players) in equilibrium.[29] The rest of possible equilibria are similar to what we have seen above, but always with unemployment. We can have only contested players being hired for a wage satisfying (5). We can have only uncontested players hired for wage r. Finally, it may be also that some hired talent is contested and some is not. This requires that all players are hired at the wage r, from Corollary 1. There are two possible types of these equilibria. First, it may be that a club hires only contested players and the other only uncontested players. This would correspond to a situation where one club benefits

[29] Basically, we would need that the talent distributions for which $\max\{Z_1^1(t_1, t_2), Z_1^2(t_2, t_1)\} < r$ are not the ones near full employment, implying that the willingness to pay is sometimes increasing in the talent already hired.

from an larger talent pool of the competitor – e.g., due to a large competitive imbalance –, whereas the other club suffers a negative externality from a higher pool of the rival. That is, Z_2^i is positive for a club and negative for the other. Second, it may be that both clubs hired both contested and uncontested players. This corresponds to a rare situation where there is a talent distribution such that both the pros and cons of poaching as opposed to hiring uncontested workers exactly compensate for both clubs: Z_2^i is zero for both clubs in equilibrium. Moreover, in this case an equivalent (i.e., with the same talent allocation and payoffs) equilibrium always exists where both clubs hire only contested players: both clubs send offers to all hired players and these accept according to the distribution of talent in the original equlibrium.

4.2.5 Example

We now present an exemplary revenue function to illustrate our results. This (parametric) example incorporates all three revenue generating motives mentioned before. Thus, it may also serve as a workhorse model to discuss issues regarding the relative strength of these motives. In Appendix B we provide some justification for the choice of the functional form and parameters and show that this revenue function satisfies our assumptions. We also display the arguments sustaining our claims about the equilibrium set. We let

$$Z^i(t_i, t_j) = m_i \frac{t_i t_j^\alpha}{t_i + t_j}, \ i = 1, 2 \text{ with } \alpha \in (0, .5).$$

Here, m_i is a parameter capturing the possible asymmetries between clubs, like "fan drawing powers". We analize the simplest case here. Thus, we normalize at $T = 1$, $m_1 = m_2 = 1$.

In Appendix B we argue that all equilibria must be symmetric. Thus, to calculate w^* we substitute $t_1 = .5$ into $Z_1^1(t_1, 1-t_1) - Z_2^1(t_1, 1-t_1) = (1-t_1)^{\alpha-1}(1-(1+\alpha)t_1)$ to obtain $w^* = .5^\alpha(1-\alpha)$. We also show that $\widehat{w} = Z_1^1(.5, .5) = .5^{\alpha+1} < .5^\alpha(1-\alpha) = w^*$. We also note that $Z_2(t, t) < 0$ for all t, and so we do not have equilibria with uncontested players being hired.

We may compute now the symmetric equilibrium with lowest wage (and thus maximum equilibrium talent display) for different parameters.

i. If and only if $r \leq w^* = .5^\alpha(1-\alpha)$ ($\simeq .4547$, when $\alpha = .4$), there exists a symmetric equilibrium with wage w^*, full employment and all players contested. Any other equilibrium has a higher wage (and perhaps lower aggregate hired talent). If $r \leq .5^{\alpha+1}$, ($\simeq .3789$, when $\alpha = .4$) this equilibrium outcome is unique.

ii If $r > w^*$, then we cannot have an equilibrium with full employment. The lowest wage that is consistent with equilibrium (and with the highest level of talent employed) is r. In fact, there exists an equilibrium where both clubs send offers r to the same talent $t = \left(\frac{w^*}{r}\right)^{\frac{1}{1-\alpha}}$, and half of these players accept each of the clubs' offers.

5 Revenue sharing

There are a host of interventions a league can employ in order to increase its fan appeal (or simply aggregate profits). Our model of the player market can be used to evaluate the consequences of these. Due to space constraints, we content ourselves with showcasing only one of these, but perhaps the most controversial one.

One of the most debated questions with regard to the player market (c.f. Fort and Quirk, 1995) is whether teams with high revenues should be forced to share them with poorer teams – presumably – in order to increase the overall quality of the league (due to competitive balance considerations). The resolution of the problem of optimal revenue sharing could also help in determining whether imposing the collective sale of TV rights – a procedure which makes redistribution much more practical – is a good idea.[30] We cannot provide a full answer in this paper, but we wish to highlight a few implications of our approach.

Let us denote the net revenues accrued to Club i after revenue sharing by S^i, and consider a simple revenue sharing scheme, where a proportion $1 - \beta$ of each

[30] See Falconieri et al. (2004).

club's (monetary) revenues is transferred to the rival.[31] That is, taking into account non-pecuniary – and therefore non-transferable – benefits:

$$
\begin{aligned}
S^1(t_1, t_2; \beta) &= \frac{U^1(t_1, t_2)}{V_1'} + \beta R^1(t_1, t_2) + (1-\beta) R^2(t_2, t_1), \\
S^2(t_2, t_1; \beta) &= \frac{U^2(t_2, t_1)}{V_2'} + \beta R^2(t_2, t_1) + (1-\beta) R^1(t_1, t_2).
\end{aligned}
\tag{6}
$$

Note that $\beta = 1$ corresponds to no revenue sharing, while at the other extreme, $\beta = 1/2$ captures full sharing of the (expropriable) revenues. We may define the analogue of w^* when these are the new "revenue" functions, as

$$
\begin{aligned}
w^*(\beta) &= S_1^1(t_1^*, T - t_1^*; \beta) - S_2^1(t_1^*, T - t_1^*; \beta) = \\
&\quad S_1^2(T - t_1^*, t_1^*; \beta) - S_2^2(T - t_1^*, t_1^*; \beta).
\end{aligned}
\tag{7}
$$

We can now apply Proposition 1 to generalize the irrelevance result of Fort and Quirk (1995):

Proposition 2 *As long as $w^*(\beta) \geq r$ and everyone is hired, revenue sharing has no effect on the talent distribution, while it decreases the market wage: $w^*(\beta) = w^*(1) - (1-\beta)\left(\frac{dR^1(t_1^*(\beta), T - t_1^*(\beta))}{dt_1} - \frac{dR^2(T - t_1^*(\beta), t_1^*(\beta))}{dt_1}\right)$.*

When all talent is hired, the only effect of revenue sharing is to redistribute revenue from players to clubs. That revenue sharing does not affect the allocation of talent follows from the combined effect of two facts. One is that it is the marginal revenues being equal that defines equilibrium.[32] The other is that in equilibrium all players are contested, implying that the marginal increase of one club's talent level leads to the same marginal decrease of that of the other club ($\frac{\partial t_i}{\partial t_j} = -1$). Together,

[31] Note that setting $\beta = 1 - \frac{\alpha}{2}$ this is equivalent to the clubs sharing a fraction α of the total revenue equally.

[32] The reason why it has been claimed that the irrelevance result does not hold with "utility maximizing" clubs is that in those models demand is not determined by marginal revenue, but average revenue. With average revenues the effects of a transfer would not be equal on both teams' demand functions as $\frac{R^i}{t_i} \neq -\frac{R^i}{t_j}$. See below.

these imply that

$$\begin{aligned}
\frac{d^2S^1(t_1,T-t_1)}{d\beta dt_1} &= \frac{dR^1(t_1,T-t_1)}{dt_1} - \frac{dR^2(t_2,T-t_2)}{dt_1} \qquad (8)\\
&= R_1^1(t_1,T-t_1) - R_2^1(t_1,T-t_1) - R_2^2(t_2,T-t_2) + R_1^2(t_2,T-t_2)\\
&= -\frac{dR^1(t_1,T-t_1)}{dt_2} + \frac{dR^2(t_2,T-t_2)}{dt_2} = \frac{d^2S^2(t_2,T-t_2)}{d\beta dt_2}.
\end{aligned}$$

Thus, the transferred revenue has exactly the same (negative) effect on the marginal revenues of both the giving and the receiving team. Therefore, if the marginal revenues were equal to start with for a given talent distribution, they will continue to be so following redistribution and thus the same talent distribution still arises in equilibrium after the transfer.

As the clubs' incentives to win – and thus their willingness to pay for talent – are unambiguously reduced by revenue-sharing, wages are the lower the fuller the revenue sharing arrangement is.

When not all talent is hired in equilibrium, revenue sharing may affect competitive balance, in conjunction with a – normally negative[33] – change in the aggregate amount of talent hired by the league. When supply is elastic, the same forces that drive willingness to pay down with fixed supply are still operational and, as a result, the overall amount of talent hired in equilibrium normally decreases with revenue sharing. However, competitive balance might increase or decrease, depending on the specifications of the revenue functions. Therefore, while it often does, revenue sharing does not necessarily reduce the "quality" of the league, as the effect of revenue sharing on competitive balance may compensate for the lower aggregate talent level.

6 Heterogeneity

Up to now, we have assumed that talent is the unit of measure, that all talent has the same reservation wage, and that all that matters for clubs' revenues is how many

[33] When the amount of talent that one club hires changes, this also changes the other club's willingness to pay for talent. Therefore, the talent-reducing effect may not dominate for all revenue functions.

units of that measure they hire. There are two ways in which player heterogeneity can affect the market outcome. The first has to do with the way the market operates: players, and not units of talent, are the recipients of offers. Consequently, with heterogeneous players the targeting of offers becomes even more relevant, while the calculation of marginal revenue is also altered. The second channel affects the clubs' valuations, as they may depend not only on the total amount, but also on the composition of the talent hired. We will have a brief look at both of these added complexities in turn. Our third extension relates to the heterogeneity of clubs. While we have allowed for arbitrary asymmetries in their objective functions, we have not considered the possibility that players have preferences over the clubs *ceteris paribus*. When compensating differentials exist, our analysis has to be adjusted. As it turns out, the adjustment is straightforward and does not alter our results.

6.1 Only aggregate talent matters

Let T continue to be the measure of both the players and their aggregate talent, but let $f(x)$, for $x \in [0, T]$, be the measure (density) of the talent Player x has. Thus, $\int_0^T f(x)dx = T$ and our baseline model corresponds to the case when all players have one unit of talent: $f(x) \equiv 1$.[34] Once we consider this heterogeneity, assuming that players are homogeneous in their reservation wage adds no simplification at all, as what will matter is the *per unit of talent* (PUT) reservation wage. So, let $r(x)$ be Player x's reservation wage and assume – without loss of generality – that $r(x)/f(x)$ is weakly increasing in x.[35]

We can now show that our main results carry over to this scenario. Note that now

[34] We thank an anonymous referee for suggesting this extension.

[35] It is likely that this implies that the reservation wage per unit of talent is weakly decreasing in the talent of a player and thus the clubs hire players in decreasing order of talent. It sounds very reasonable: Just think of Lionel Messi (whose base salary is over \$30 million): his reservation wage per unit of talent is practically zero – the minimum wage divided by a very large number. Even if his outside option were to play in another market, we would assume that currently he is efficiently placed so that his salary in the outside option would decrease by more relative to his talent than that of the second best player.

we need to calculate the marginal effect on supply and demand of hiring an additional player (rather than an additional unit of talent). Thus, we do not differentiate the revenue function with respect to talent, t, but with respect to number of players, n. Assuming that the players with the lowest reservation wages are the ones hired by two clubs, and letting $t_1 + t_2 = \int_0^{x'} f(x)dx$, willingness to pay for the marginal uncontested player, x', becomes

$$\frac{dZ^i(t_i, t_j)}{dn_i} = \frac{dt_i}{dn_i} \cdot \frac{dZ^i(t_i, t_j)}{dt_i} = f(x')Z_1^i(t_i, t_j),\ j \neq i = 1, 2,$$

since $\frac{dt_j}{dn_i} = 0$. Similarly, for contested player x''

$$\begin{aligned}\frac{dZ^i(t_i, t_j)}{dn_i} &= \frac{dt_i}{dn_i} \cdot \frac{dZ^i(t_i, t_j)}{dt_i} + \frac{dn_j}{dn_i} \cdot \frac{dt_j}{dn_j} \cdot \frac{dZ^i(t_i, t_j)}{dt_j} \\ &= f(x'')\left(Z_1^i(t_i, t_j) - Z_2^i(t_i, t_j)\right),\ j \neq i = 1, 2,\end{aligned}$$

where we have used the facts that $\frac{dt_i}{dn_i} = \frac{dt_j}{dn_j} = f(x'')$ and that $\frac{dn_j}{dn_i} = -1$. As a club's willingness to pay PUT is always the same for all contested players, the identity of x'' is irrelevant. Finally, note that we can write $r(x)$ as $f(x)\frac{r(x)}{f(x)}$, so that all our equations are as before, with the partial derivatives of the Z^i re-interpreted as willingness to pay PUT. Consequently, our main proposition directly generalizes to this environment.

Proposition 3 *(1') When $w^* \geq \frac{r(T)}{f(T)}$, there exists a unique full-employment equilibrium outcome, with a uniform PUT wage of w^* and the league-optimal allocation of talent: $t_1 = t^*$, $t_2 = T - t^*$. When $\widehat{w} \geq \frac{r(T)}{f(T)}$ this equilibrium outcome is unique.*

Thus, the Law of One Wage extends to this equilibrium, with the PUT interpretation. In general, all contested players will always receive the same PUT wage, equal to the reservation PUT wage of the marginal contested player. However, it is possible that in equilibrium some uncontested players may also be hired. As before, that will require that they are paid their own reservation wages, which now can be different. In order for this to be compatible with equilibrium, it is necessary that the PUT wage of hired uncontested players is no lower than that of the (common) PUT wage of contested players.That is, equilibrium behavior also requires that firms compete for the players who would accept the lowest offers. In addition such mixed hiring will also require that a club hiring uncontested players has no incentive to poach the rival (instead). That is, that Z_2^i is positive for that club at the equilibrium talent distribution.

6.2 Team composition matters

In order to simplify the analysis, we have assumed that the revenues of each club were a function (only) of their total talent (and that of their rivals). Our baseline model works well when talent is evenly distributed across (relevant) players. Indeed, in that case, discussing wages in terms of talent instead of in terms of players is simply a shortcut that allows us to use the first-order approach. Only vexing integer-issues are added, if we consider the discrete nature of the "packs of talent".

In team sports, though, it is not only the aggregate talent level of a club that matters for performance, but also its distribution among individual players. Indeed, one of the most salient phenomena that characterize sports is the relevance of big stars for the outcome (not to talk about fan appeal, c.f. Buraimo and Simmons, 2015): Michael Jordans are very exceptional even if we only consider players in top teams in the NBA. This exceptionality is relevant because only five players may be on court at any point during a match: if rival teams could simultaneously field ten players who together incorporated Jordan's talent (and who would cost approximately what Jordan costs), that exceptionality would be of no relevance to our analysis.

We may generalize our model to discuss specific issues that the existence of stars raise in sports.[36] One of these issues is whether we should expect a star's wages be simply proportional to the number of efficiency units of talent (s)he possesses.

To take into account, among other nuances, the relevance of stars once not all the talent can be fielded simultaneously, let us generalize our baseline model by supposing that every player, n, is characterized by an amount of talent $f(n)$, and

[36] See Rosen (1981) for the pioneering work on this topic.

the aggregate talent a club fields on the pitch is a function of the vector of talent $\boldsymbol{t}_i = (f(n_1), f(n_2), ..., f(n_{N_i}))$ embodied in the set of players N_i hired by club i: $\mu(\mathbf{t}_i)$. This function takes into account the intensity with which the player may be used in the competition. Stars, with large values of $f(n)$ will play regularly, whereas players with relatively low value of $f(n)$ will be used mainly as substitutes. The revenues of a club then depend on the weighted amounts of talent,

$$Z^i\left(\mu(\mathbf{t}_i), \mu(\mathbf{t}_j)\right).$$

Our baseline model is a special case of this formulation where the talent levels are simply added up: $\mu(\mathbf{t}_i) = \sum_{j \in N_i} f(n_j)$.

Now, suppose that there is only one star, with a discrete amount of talent, τ, whereas there is a continuum of "normal" players with identical and small amount of talent, that we treat as infinitesimal (just as in our baseline model). Thus, let $\mu_\tau(t)$ denote the normal talent that a club with the star and normal talent t can put on the field, and let $\mu_\tau(t) + \tau$ be the total talent that this club puts on the field. Also, let $\mu_0(t)$ denote the talent that the club without the star can field. Naturally, if the teams are assumed *ex ante* similar (same Z), we expect in equilibrium that $\mu_\tau(t) + \tau > \mu_0(t) > \mu_\tau(t)$. In other words, the team with the star is stronger than the team without, but the teammates of the star together put less talent on the pitch than the starless team. Also, both functions should be increasing in t. To simplify the discussion, we model competition for talent sequentially, so that clubs compete simultaneously bidding for the star first, and then, after the star has signed with a club, they compete for normal talent as in our baseline model.[37]

The amounts of normal talent hired by the clubs in the second stage are given by (first-order) conditions similar to the ones we have obtained before. For example, suppose that there is a total of T normal talent. Also, define t^* for this case as the solution in t to

$$\begin{aligned} & Z_1\left(\mu_\tau(t) + \tau, \mu_0(T-t)\right)\mu_\tau'(t) - Z_2\left(\mu_\tau(t) + \tau, \mu_0(T-t)\right)\mu_0'(T-t) \qquad (9) \\ = \;& Z_1\left(\mu_0(T-t), \mu_\tau(t) + \tau\right)\mu_0'(T-t) - Z_2\left(\mu_0(T-t), \mu_\tau(t) + \tau\right)\mu_\tau'(t), \end{aligned}$$

[37] See Palomino and Sákovics (2004) for a similar sequential auction.

where $\mu_i'(.)$ represents the derivative of μ_i. Finally, denote the corresponding value of either side of the above equation by w^*. The following lemma is then a straightforward corollary of Proposition 1 and the fact that in the auction for the star the winning club will have to pay the difference between the continuation payoff with or without the star.

Lemma 3 *If the clubs are ex ante identical, both teams bid the same for the star player and she plays for either team with equal probability. If, moreover, $w^* > r$, then, (assuming, without loss of generality, that Club 1 signs the star) the distribution of normal talent is $(t^*, T - t^*)$ and the wage is w^*. The equilibrium condition determining the star's wage is*

$$w^s = Z\left(\mu_\tau(t^*) + \tau, \mu_0(T - t^*)\right) - Z\left(\mu_0(T - t^*), \mu_\tau(t^*) + \tau\right) - w^*(2t^* - T). \quad (10)$$

Having characterized the equilibrium, we may ask questions as, for instance, whether there is a "superstar" bias in the wage of these sport stars. The answer may depend on the functional forms of $Z(.)$ and $\mu(.)$, but we should expect that at the solution to (9), $2t^* - T < 0$. In words, mantaining that both clubs are otherwise symmetric – i.e., have the same revenue function –, the team without the star hires more normal talent in equilibrium.[38] Then, according to (10), the star is paid above her marginal revenue product: w^s is above the difference in revenues that having the star brings to the club (as it also brings on a reduction in the wage bill for the normal talent).[39]

The analysis of this sort of biases or the efficiency of talent distribution for particular forms of $Z(.)$ and $\mu(.)$ may then be accommodated in our general framework. Although that analysis is beyond the scope of the present paper, we believe that the modeling tools that we have developed here offer an appropriate workhorse model for this task.

[38] This would clearly happen when the $\mu(.)$ are linear, as in that case the final talent allocations would be equal between the two clubs.

[39] This could have an exacerbating effect on the negative externalities that some teams obsessively seeking a league championship can have on the rest (c.f. Whitney, 1993).

6.3 Compensating differentials

In the above analysis, we have taken for granted that contested players accept the highest wage offer. In practice, the situation is more complicated. As different teams tend not to be in the same location, playing for one club may lead to different amenity values and/or local price/tax levels and/or chances of sporting success than playing for the other.[40] As a result, players may accept the lower of two competing wages if the compensating differential outstrips the wage difference. We can incorporate this to our model by assuming that Club 1 has an advantage of y over Club 2 and that the reservation wage r is defined for Club 1 (and thus the reservation wage at Club 2 is $r + y$).[41] This difference is also present for contested players: wage competition will lead the clubs to make offers that lead players to be indifferent between them. This also has an effect on the equilibrium talent distribution as now marginal willingnesses to pay do not equalize, Club 2 has to pay more to make it competitive, so it has to have a higher equilibrium marginal willingness to pay. To illustrate: equations (3) and (4) become

$$Z_1^1(t, T-t) - Z_2^1(t, T-t) + y = Z_1^2(T-t, t) - Z_2^2(T-t, t),$$

and

$$\begin{aligned} w_1^* &= Z_1^1(t^*, T-t^*) - Z_2^1(t^*, T-t^*), \\ w_2^* &= Z_1^2(T-t^*, t^*) - Z_2^2(T-t^*, t^*) = w_1^* + y. \end{aligned}$$

The way to reconcile this with the Law of One Wage is that the net wage is common – even if the gross wages are different.

7 Conclusion

We have proposed a new modelling framework for the analysis of labor markets in professional sports. A major advantage of our set-up is that it allows for a proper

[40] We thank an anonymous referee for suggesting this extension.

[41] A multiplicative advantage can be accommodated in the analysis in a similar way.

game-theoretic analysis. A number of our findings have concurred with previously existing results, now basing them on a realistic micro-structure and Nash behavior.

There are several considerations that we have not addressed, despite their importance. They should provide topics for extending our work. Let us enumerate the most important avenues here:

1. The result that the initial allocation of talent does not matter (Corollary 2) crucially depends on the Coasian nature of bargaining. If there are frictions, like switching costs or asymmetric information, then they introduce a wedge which needs careful analysis (c.f. Burguet et al., 2002).

2. We have also assumed that the players are free agents. If they were not, the club holding their contract could extract surplus from the poaching club by charging a transfer fee. This situation is qualitatively similar to the one where players have a preference over which club to play for as the poaching club needs to pay extra.

3. Our model and analysis – as the vast majority of the literature – is static (except for the star player model). Dynamic approaches, taking into account the evolution of talent distribution, like Grossman et al. (2010), can further add to our understanding. Similarly, the length of contract also may be a factor (c.f. Buraimo et al., 2015).

4. By assuming that the clubs could use their future revenues for hiring today, we have also abstracted away from credit market imperfections and asymmetries. To the extent that revenue sharing improves competitive balance in practice, it is likely to operate through the alleviation of those.

5. While our set-up does extend to a league of many teams in a straightforward manner, not all our results do. With more teams, they may in principle compete for players in endogenously partitioned submarkets, what may result in more than one wage paid to contested players.

6. While we have analyzed the consequences of revenue sharing, we have not identified the optimal rule that a league would impose, neither have we investigated the effects of other interventions.

7. We have not conducted a welfare analysis either. Even if the league has significant autonomy, it is of interest to see how the market equilibrium (possibly modified by league imposed restrictions) performs relative to a social optimum. The key ingredient here would be the incorporation of consumer surplus into the welfare function (see, for example, Dietl et al., 2009).

Finally, we have not made an attempt to capture the myriad institutional details that vary across different sports and leagues. In fact, we have made an effort to minimize the specificity of our model. The goal of this paper is to propose a general approach to modelling the player market that, for transparency, we have presented in the simplest possible scenario. Our observations are posited based on sufficient conditions, which continue to apply if additional restrictions are imposed.

Appendix A

Proof of Lemma 1 and Corollary 1:

Proof. First, observe that any uncontested player hired must be paid r in any equilibrium: in the absence of competition, from any higher offer, it would be profitable to deviate and make a lower offer that still exceeds r (and thus will be accepted). If there are also contested players hired in equilibrium these cannot be paid a wage above r: the rival of a club hiring uncontested players would profit from deviating and outbidding its rival (by bidding $r+\varepsilon$) for some of their originally uncontested players, while letting the same amount of contested players go to the rival. The resulting talent distribution would be the same, but the wage bill would be lower. This proves **i.**.

Next, suppose that only uncontested players are hired, and some players are not hired (in equilibrium). Assume that for one of the clubs, say 1, their marginal willingness to pay for an uncontested player $Z_1^1(t_1,t_2) \neq r$. If it is higher than r then, by the continuity of $Z_1^1(t_1,t_2)$, they would prefer to hire some of the unemployed players. If it is lower, then by the same argument they would prefer to shed some of their hired players. Note that in case there are no unemployed players even if a club would prefer to hire another uncontested player there would not be any. This proves **ii.**.

Finally, assume that some of the players hired are contested and that for one of the clubs, say 1, at the equilibrium talent distribution for a positive measure of players their wages are not equal to $Z_1^1(t_1,t_2) - Z_2^1(t_1,t_2)$. If the wage is higher, then by the continuity of $Z_1^1(t_1+x,t_2-x) - Z_2^1(t_1+x,t_2-x)$ in x, the club would profit from hiring ε less contested players (and thus "transferring" them to Club 2). If the wage is lower then the club would profit from outbidding its rival for ε more players. Thus, if contested players are hired in equilibrium the wage must be common and equal to the clubs' marginal willingness to pay for contested players. This proves **iii.**. The three cases together prove Lemma **1.** ■

Proof of Proposition 1:

Proof. Suppose $w^* \geq r$, and suppose that Club 2 offers every player a wage w^*. Club 1's best response amounts to choosing how much talent t_1 to hire at wage w^* (or perhaps infinitesimally above w^*) letting the rest of talent $T - t_1$ go to the rival. The optimal choice satisfies the first-order condition $Z_1^1(t_1^*, T - t_1^*) - Z_2^1(t_1^*, T - t_1^*) = w^*$, that we already know that has a (unique) solution. As, by Assumption 1, the left-hand side of this condition is decreasing in t_1, that first order condition is also sufficient, confirming that we indeed have an equilibrium, as long as players accept Club i's offer with probability $\frac{t_i^*}{T}$, what is a best response for them as they are indifferent.

Suppose now that we have another full-employment equilibrium, with $t_1 > t^*$. That implies that not all players are contested, since $Z_1^1(t, T - t) - Z_2^1(t, T - t)$ is decreasing in t. Therefore, the wage must be r. Also, $Z_1^2(T - t_1, t_1) - Z_2^2(T - t_1, t_1) > w^* > r$, since the expression, by Assumption 1, is increasing in t_1, which cannot be equilibrium unless $t_1 = 0$, a contradiction of $t_1 > t^*$. For $t_1 < t^*$ (and so $T - t_1^* > T - t^*$) the proof is similar. ∎

Proof of Proposition 2:

Proof. Substituting the value of S^1 and S^2 into (7), we have

$$\begin{aligned} & \frac{1}{V_1'}\frac{dU^1(t_1, T - t_1)}{dt_1} + \beta\frac{dR^1(t_1, T - t_1)}{dt_1} + (1 - \beta)\frac{dR^2(T - t_1, t_1)}{dt_1} \qquad (11) \\ = \; & -\frac{1}{V_2'}\frac{dU^2(T - t_1, t_1)}{dt_1} - \beta\frac{dR^2(T - t_1, t_1)}{dt_1} - (1 - \beta)\frac{dR^1(t_1, T - t_1)}{dt_1} = w^*(\beta). \end{aligned}$$

Collecting terms,

$$\begin{aligned} & \frac{1}{V_1'}\frac{dU^1(t_1, T - t_1)}{dt_1} + \frac{dR^1(t_1, T - t_1)}{dt_1} = -\frac{1}{V_2'}\frac{dU^2(T - t_1, t_1)}{dt_1} - \frac{dR^2(T - t_1, t_1)}{dt_1} \\ = \; & w^*(\beta) + (1 - \beta)\left(\frac{dR^1(t_1, T - t_1)}{dt_1} - \frac{dR^2(T - t_1, t_1)}{dt_1}\right), \qquad (12) \end{aligned}$$

and recalling that $Z_j^i(t_i, t_j) = \frac{1}{V_i'}\frac{\partial U^i(t_i, t_j)}{\partial t_j} + \frac{\partial R^i(t_i, t_j)}{\partial t_j}$, yields

$$Z_1^1(t_1, T - t_1) - Z_2^1(t_1, T - t_1) = Z_1^2(t_1, T - t_1) - Z_2^2(t_1, T - t_1), \qquad (13)$$

for any β. This equation is (4), so the equilibrium talent distribution, $(t^*, T - t^*)$ is unchanged by revenue sharing. That is, the first line of (12) is independent of β.

The last equality in (12) then characterizes the equilibrium wage, $w^*(\beta)$. Finally note that, when $w^* > r$, $\frac{dR^1(t_1^*, T-t_1^*)}{dt_1} > 0$ and $\frac{dR^2(T-t_1^*, t_1^*)}{dt_1} < 0$, so $w^*(\beta)$ is indeed increasing. ■

Appendix B

Here we argue that the revenue function we use in our example depicts a sensible model of the "downstream market" of sport competition. We do not claim that it is the "best" or the "only" representation, we only suggest it for illustration. It builds on the micro-founded Cobb-Douglas match value function developed in Falconieri et al. (2004) for TV viewers and corrects it for the missing factor of performance related revenue and allows for asymmetry in the size of followers of each club by incorporating a "drawing factor". We thus arrive at $Z^i(t_i, t_j) = m_i \frac{t_i t_j^\alpha}{t_i + t_j}$ with $\alpha \in (0, .5)$.

The restriction on the value of α can be argued straightforwardly. Denote the total talent employed by $E = t_i + t_j$. We can write $Z_1^i(t_i, t_j) = m_i \frac{t_j^{\alpha+1}}{E^2}$ and $Z_2^i(t_i, t_j) = m_i t_i t_j^{\alpha-1} \frac{\alpha t_i - (1-\alpha) t_j}{E^2}$. Now note that the bliss point of this revenue function (in an equilibrium where all players are contested) is at the talent distribution where $Z_1^i - Z_2^i = 0$. Writing γt_i for t_j the equality becomes $\gamma^2 = \alpha - (1-\alpha)\gamma$. Solving for γ, we obtain $\gamma = \alpha$. As empirical evidence for fans gives bliss values of win percent of around two thirds (corresponding to $\gamma = .5$), we would expect an even lower value to account for performance related revenue that the fans do not internalize.

To see that this revenue function satisfies our assumptions, note that:

$$
\begin{aligned}
Z_{11}^i(t_i, t_j) &= -2m_i \frac{t_j^{\alpha+1}}{E^3}, \\
Z_{12}^i(t_i, t_j) &= Z_{21}^i(t_i, t_j) = m_i t_j^\alpha \frac{(\alpha+1)t_i - (1-\alpha)t_j}{E^3} \quad \text{and} \\
Z_{22}^i(t_i, t_j) &= m_i t_i t_j^{\alpha-2} \frac{\alpha(\alpha-1)t_i^2 + \alpha(\alpha^2-\alpha-2)t_i t_j + (\alpha-1)(\alpha^2-\alpha-2)t_j^2}{E^3}.
\end{aligned}
$$

Assumption 1 is satisfied if

$$
\begin{aligned}
& Z^i_{11}(t_i,t_j) - 2Z^i_{12}(t_i,t_j) + Z^i_{22}(t_i,t_j) \\
= \; & \frac{m_i}{E^3}\left(\begin{array}{c} -2t_j^{\alpha+1} - 2t_j^{\alpha}((\alpha+1)t_i - (1-\alpha)t_j) + t_i t_j^{\alpha-2}(\alpha(\alpha-1)t_i^2 + \\ \alpha(\alpha^2-\alpha-2)t_i t_j + (\alpha-1)(\alpha^2-\alpha-2)t_j^2) \end{array}\right) < 0.
\end{aligned}
$$

This is equivalent to

$$
\begin{aligned}
& -2\alpha t_j^{\alpha+1} + \big((\alpha-1)(\alpha^2-\alpha-2) - 2(\alpha+1)\big)\, t_i t_j^{\alpha} + \\
& t_i t_j^{\alpha-2}(\alpha(\alpha-1)t_i^2 + \alpha(\alpha^2-\alpha-2)t_i t_j) \\
= \; & -2\alpha t_j^{\alpha+1} + \left(\alpha^3 - 2\alpha^2 - 3\alpha\right) t_i t_j^{\alpha} + t_i t_j^{\alpha-2}(\alpha(\alpha-1)t_i^2 + \alpha(\alpha^2-\alpha-2)t_i t_j) \\
< \; & 0,
\end{aligned}
$$

which is satisfied for all (t_i, t_j), as each coefficient is negative.

Finally, Assumption 2 is satisfied as $Z^i_1(0,T) - Z^i_2(0,T) = 1$ and $Z^i_1(T,0) - Z^i_2(T,0) = 0$.

Next, we offer the arguments/computations that sustain the results for the equilibria presented in the main text.

Lemma 4 *In our symmetric example all equilibria are symmetric and involve only contested hires.*

Proof. We first observe that equilibria may involve contested hires only (or be equivalent to that) or one team hiring only contested, and the other only uncontested players. Indeed, as we discussed in the text, only in knife-edge cases (here it would require $\alpha = .5$) may we have symmetric equilibria with both types of hires, and then an equivalent equilibrium exists with the same wage (r) and talent distribution in which all players receive two offers. In an asymmetric equilibrium with both types of hires, we would need that at the equilibrium talent distribution for one team its contested marginal valuation is higher and for the other its uncontested marginal valuation is. This would imply that the first only hires contested players and the second only hires uncontested players. These require that

$$
Z^1_2(t_1,t_2) < 0 \leq Z^2_2(t_2,t_1), \tag{14}
$$

or

$$\frac{t_1}{t_2} > \frac{1-\alpha}{\alpha}. \tag{15}$$

We also need (by Corollary 1) that

$$Z_1^1(t_1,t_2) - Z_2^1(t_1,t_2) = Z_1^2(t_2,t_1).$$

In our case, this means

$$1 - \alpha\frac{t_1^2}{t_2^2} + (1-\alpha)\frac{t_1}{t_2} = \frac{t_1^{\alpha+1}}{t_2^{\alpha+1}}. \tag{16}$$

Substituting $\frac{t_1}{t_2} = \frac{1-\alpha}{\alpha}$, the LHS is 1, while the RHS is more than 1 (since $\alpha < 1$). Moreover, the derivative of the LHS is negative in $\frac{t_1}{t_2}$ for $\frac{t_1}{t_2} > \frac{1-\alpha}{\alpha}$, while the derivative of the RHS is positive. Therefore (15) and (16) are incompatible, proving that the asymmetric equilibrium with both types of hires does not exist in our example.

Next, note that, by Corollary 1, in an equilibrium with only uncontested players we must have $Z_1^1(t_1,t_2) = Z_1^2(t_2,t_1)(= r)$. Substituting in for the revenue function, we have $\frac{t_2^{\alpha+1}}{(t_1+t_2)^2} = \frac{t_1^{\alpha+1}}{(t_1+t_2)^2}$, implying that the equilibrium must be symmetric. Moreover, $Z_2^i(\tau,\tau) = \tau^\alpha \frac{(2\alpha-1)\tau}{4\tau^2} < 0$ for $\alpha < .5$, and so equilibria with only uncontested hires may not occur with these revenue functions (as the value for the marginal contested player is higher than that of the marginal uncontested one at any symmetric talent distribution).

Finally, turning to the equilibria with only contested hires, note that, by Corollary 1, we must have $Z_1^1(t_1,t_2) - Z_2^1(t_1,t_2) = Z_1^2(t_2,t_1) - Z_2^2(t_2,t_1)$. This equation has only symmetric solutions. Indeed, note that for any E and $t_2 = t_1 = E/2$, the equation is trivially satisfied. Now, for any $\delta > 0$, and from Assumption 1, satisfied by our example, the LHS is larger and the RHS smaller at $(E/2-\delta, E/2+\delta)$. Therefore, and since any distribution of talent may be obtained from some E and some δ, the result follows. Thus, we can restrict our attention to equilibria, where both clubs hire the same amount of contested players, by Lemma 1, making the same offer to all of them. ■

Now, in search for equilibrium, and without loss of generality, suppose Club 2 makes t offers with wage $w \geq r$. We compute the best response for Club 1. That is, taking into account that by slightly overbidding the rival Club 1 can choose how many of those t players it hires at wage w, we study the solution to the following parametric optimization problem,

$$\begin{aligned} & \max_{\gamma,\beta} \frac{(\gamma+\beta)(t-\gamma)^{\alpha}}{t+\beta} - w\gamma - r\beta \qquad (17) \\ s.t. \quad 0 & \leq \gamma \leq t \\ 0 & \leq \beta \leq 1-t, \end{aligned}$$

where γ and β are the resulting measure of contested and uncontested players, respectively, hired by Club 1. The derivatives of the objective function with respect to γ and β, respectively, are

$$\frac{(t-\gamma)^{\alpha-1}}{t+\beta}(t-\gamma-\alpha(\gamma+\beta)) - w \quad \text{and} \qquad (18)$$

$$\frac{(t-\gamma)^{\alpha+1}}{(t+\beta)^2} - r. \qquad (19)$$

For an equilibrium with only contested players to exist, we need $\beta^* = 0$. When $t = 1$, this is true since that is the only point in Club 1's choice set of β. When $t < 1$, it is necessary that (19) – which is decreasing in β – is negative at $\beta = 0$. We analyze both cases in turn.

7.0.1 $t = 1$

Consider (18) with $\beta = 0$ and $t = 1$. We can write the first order condition for the optimal γ as

$$(1-\gamma)^{\alpha-1}(1-(1+\alpha)\gamma) - w = 0.$$

As $\alpha < 1$, the LHS is decreasing in γ for $\gamma \in (0, 1)$. Thus, – the sufficient second order conditions are satisfied and – there is a unique optimal value of γ. This solution is $\gamma = .5$, if and only if

$$w = w^* = \frac{1-\alpha}{2^{\alpha}}.$$

When this value is above r, this is an equilibrium.

7.0.2 $t < 1$

In this case the first-order condition (with $\beta = 0$) can be written as

$$(t - \gamma)^\alpha (1 + \alpha) - t\alpha(t - \gamma)^{\alpha - 1} - tw = 0.$$

The LHS is still decreasing in γ, leading to a unique solution. This solution is $\gamma = .5t$, only if

$$w = t^{\alpha-1} 2^{-\alpha}(1 - \alpha). \tag{20}$$

Note that any such equilibrium leads to a wage above w^*. Solving for t, we obtain

$$t = w^{\frac{1}{\alpha-1}} \left(\frac{1}{2}\right)^{\frac{\alpha}{1-\alpha}} (1 - \alpha)^{\frac{1}{1-\alpha}}. \tag{21a}$$

We also need that the optimal $\beta = 0$ given $\gamma = .5t$. That is, from (19)

$$.5^{\alpha+1} t^{\alpha-1} < r. \tag{22}$$

Note that the limit of the (decreasing) LHS as $t \to 1$ is $\widehat{w}$. Indeed, there is no $t < 1$ for which $r \leq \widehat{w}$ is compatible with equilibrium.

We are not done yet. We still need to check for the profitability of a global deviation from $(t/2, t/2)$ and $\beta = 0$. Such a deviation would imply $\beta > 0$ and $\gamma \neq t/2$.

Letting $x = t - \gamma$, we can now write (18) as

$$x^\alpha (1 + \alpha) - (t + \beta)\alpha x^{\alpha - 1} - (t + \beta)\, w,$$

which is still increasing in x (decreasing in γ). Thus, for each β and t, there is a unique (perhaps corner) solution to the first-order condition and thus a unique optimal value of γ.

When t is as given by (21a), (19) is smaller, for any non-negative (β, γ), than

$$t^{\alpha-1} - r = w\frac{2^\alpha}{1 - \alpha} - r.$$

Thus, a sufficient condition for a fully interior best response not to exist is that $w < \frac{1-\alpha}{2^\alpha} r$. As $\frac{1-\alpha}{2^\alpha} > 1$ for $\alpha > 0$, $w = r$ is sufficient.

As we have already considered $\beta = 0$, the response involving $\gamma = 0$ is the only remaining case to check – note that $\gamma = t$ can never be a solution, as then the rival would have no players, leading to zero payoff to both clubs. Then β must either satisfy (19)$= 0$ or be $1 - t$ if (19)> 0 at $\beta = 1 - t$. That means that the optimal β (with $\gamma = 0$) is the non-negative solution to

$$\frac{t^{\alpha+1}}{(t+\beta)^2} - r = 0,$$

when $t^{\alpha+1} < r$ and $1 - t$ otherwise. Club 1 will prefer not to choose this deviation if $Z^1(\beta^*, t) - r\beta^* \leq Z^1(\frac{t}{2}, \frac{t}{2}) - w\frac{t}{2}$, where

$$\beta^* = \max\{0, \min\{1 - t, t^{\frac{\alpha+1}{2}} r^{\frac{-1}{2}} - t\}\}.$$

Let us first look at the case $1 - t \geq t^{\frac{\alpha+1}{2}} r^{\frac{-1}{2}} - t$. That is, $r \geq t^{\alpha+1}$. We need to compare the objective funcion evaluated at $\gamma = 0$ and $\beta = t^{\frac{\alpha+1}{2}} r^{\frac{-1}{2}} - t$ – when this is smaller than $1 - t$ – with the objective function evaluated at $\gamma = t/2$ and $\beta = 0$. That is,

$$Z^1(t^{\frac{\alpha+1}{2}} r^{\frac{-1}{2}} - t, t) - r\left(t^{\frac{\alpha+1}{2}} r^{\frac{-1}{2}} - t\right) = t(t^{\frac{\alpha-1}{2}} - r^{\frac{1}{2}})^2 \tag{23}$$

with

$$Z^1(\frac{t}{2}, \frac{t}{2}) - w\frac{t}{2} = \frac{t}{2}\left(t^{\alpha-1}\left(\frac{1}{2}\right)^\alpha - w\right) = \alpha t^\alpha 2^{-(\alpha+1)}, \tag{24}$$

where we have substituted (20) into the second equality. Thus, for equilibrium we need that

$$\alpha t^{\alpha-1} 2^{-(\alpha+1)} \geq (t^{\frac{\alpha-1}{2}} - r^{\frac{1}{2}})^2. \tag{25}$$

It is straightforward to show that the inequality is always satisfied (for $\alpha \leq .5$) when $w = t^{\alpha-1} 2^{-\alpha}(1 - \alpha) = r$.

When $t^{\alpha+1} > r$ (and so $\beta = 1 - t$), (23) becomes $(1 - t)(t^\alpha - r)$, and (25) turns into

$$\alpha t^\alpha 2^{-(\alpha+1)} \geq (1 - t)(t^\alpha - r). \tag{26}$$

Again, it can be shown that when $w = t^{\alpha-1}2^{-\alpha}(1-\alpha) = r$, the inequality is always satisfied. Thus, when t is given by (21a), $\gamma = t/2$ and $\beta = 0$ is indeed an equilibrium with the lowest wage, $w = r$.

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