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It's Good to be Bad: A Model of Low Quality Dominance in a Full Information Consumer Search Market

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Abstract

This paper examines a consumer search market exhibiting vertically differentiated firms, heterogeneous consumers and endogenous consumer market entry. In an asymmetric information setting high and low quality firms make equal sales and profit in this market. Conversely when there is full information, search frictions induce an unravelling mechanism that leads to a unique refined equilibrium where all consumers approach low quality firms and high quality firms make no sales or profit. This presents a rationale for why low quality firms may disclose their quality and high quality firms may not even when disclosure is costless.

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1 Introduction

The literature on asymmetric information in markets is largely focused on cases where higher quality firms may seek to differentiate themselves from lower quality firms. The

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role of low quality firms has received less attention in this context. Looking at some marketing materials reveals however that some firms go to lengths to communicate that they are low quality to potential customers. A prominent example is Europe's budget airline Ryanair which has been proactive in building a low quality reputation as part of their "no frills" strategy. Ryanair CEO Michael O'Leary has stated to the media that "*Anyone who thinks Ryanair flights are some sort of bastion of sanctity where you can contemplate your navel is wrong. We already bombard you with as many in-flight announcements and trolleys as we can. Anyone who looks like sleeping, we wake them up to sell them things* (The Telegraph 2016)".[1] Other examples are discount stores that specialise in selling "factory seconds" such as Australia's "Reject Shop". These stores can often be seen airing advertisements with phrases that are vague but suggestive of low quality such as "like new", "discontinued", "second-hand" or "refurbished". These advertisements are perplexing because even though some laws may require disclosure of quality information before purchase, no law requires firms to air advertisements disclosing this information to potential customers before they even approach the firm.

Looking at the advertising decisions of lawyers in the United States shows a similar story with many lawyers advertising in undignified ways that are likely to undermine confidence in their ability. The United States has at least three attorneys that call themselves "the hammer" in their advertisements (AboveTheLaw.com 2012). In Texas a criminal defence attorney has been advertising by calling himself the "Texas Law Hawk" and performing stunts likes doing wheelies on a dirt-bike or throwing sticks of dynamite (Wilson 2016). This kind of advertising is strongly discouraged by the American Bar Association who warn that *"lawyers should consider that the use of inappropriately dramatic music, unseemly slogans, hawkish spokespersons, premium offers, slapstick routines or outlandish settings in advertising does not instil confidence in the lawyer or the legal profession (American Bar Association 2016)"*, a view that finds support in the marketing literature (Trebbi et al. 1999).[2] This again presents the question of why lawyers would

[1]This has gone so far as the CEO of Ryanair publicly announcing that extra charges for toilet use were under consideration from the airline (The Economist 2013). These charges were never actually implemented.

[2]In addition the American Bar Association had previously banned all lawyer advertising in 1907

choose forms of advertising which are likely to undermine consumer confidence in their ability.

This paper seeks to explain why firms may disclose that they are of low quality. A model is presented with a high quality firm and a low quality firm. Consumers have heterogeneous marginal utility from quality ("taste") and can choose what firm to go to upon entering the market. The key mechanism presented is one of unravelling. Considering a putative equilibrium where high taste consumers go to the high quality firm and low taste consumers go to the low quality firm, the high quality firm will price to make one of the consumers approaching it indifferent to buying and searching at the low quality firm. The diamond paradox applies to this marginal consumer and they make no surplus. When a consumer can anticipate however that when they arrive at a high quality firm they will be this marginal consumer then that consumer would be better off (by the extent of the search cost) going to the low quality firm ex ante where there will be some lower taste consumer who will be made indifferent to buying and taking their outside option. This will result in fewer and higher taste consumers visiting the high quality firm which will result in this firm raising its prices to make a higher taste consumer indifferent to buying and searching at the low quality firm. This new marginal consumer will again be better off going to the low quality firm ex ante. Through this mechanism we get a full information unravelling result where the sole refined equilibrium is one where all consumers go to a low quality firm and no consumers will visit a high quality firm. Clearly search frictions are key to this result, however the only crucial assumptions over that of a standard consumer search model are that consumers can choose what firm to go to upon entering the market and that consumers have heterogeneous marginal utility from quality.

Aside from presenting a rationale for low quality firms to disclose their quality, a second contribution is to the growing literature considering a firm's choice between signalling and disclosing quality. Our model shows a force which acts to deter high quality firms from disclosing their quality as a known high quality firm is visited by no consumers in equilibrium. Signalling which occurs after a consumer visits a high quality firm remains a

however this ban was overturned by the supreme court in 1977 on freedom of speech grounds (US Supreme Court 1977).

feasible strategy however. Low quality firms on the other hand will choose to disclose their low quality before consumers approach firms but will never want to disclose their quality after consumers have approached firms. This conflict between disclosure and signalling is different from the previous literature that emphasised strategic competitive pricing considerations (Jansson & Roy 2015) or disclosure costs (Daughety & Reinganum 2008). A third contribution is that the model presented has an interesting contrast with Akerlof (1970). Whilst in Akerlof asymmetric information causes markets to unravel with only low quality goods remaining, in this paper the opposite result is shown with unravelling only occurring in the full information setting.

An important insight of the model is that disclosure can adversely impact market efficiency by resulting in the suboptimal equilibrium where all consumers visit the low quality firm. In addition search frictions can encourage firms to decrease the quality of their good offerings even when it is costless to produce a higher quality good. To avert these outcomes it may thus be desirable to implement minimal quality standards in markets exhibiting search frictions. In addition it may be welfare improving for regulators to prevent low quality firms from voluntarily disclosing their quality. For instance this paper would provide an argument in support of the view that legal bar associations should have the ability to restrict forms of advertising that allow lawyers to disclose their low quality to the market.

2 Background

There is an extensive literature examining the disclosure decisions of vertically differentiated firms where consumers cannot directly observe quality. Many early papers conclude that an unravelling result will prevail in such markets (Grossman 1981, Milgrom 1981). This occurs from the highest quality firm in a putative pooling equilibria wanting to differentiate itself which lowers the expected value of the remaining pool, leading to the next highest quality firm wanting to differentiate itself from the pool and so on. That mandatory disclosure laws are thought necessary in the face of this unravelling result presents somewhat of a puzzle and so later papers look at where this result can fail (Dranove &

Zhe Jin 2010).

Board (2009) presents a duopolistic model where firms are heterogeneous in quality and consumers have heterogeneous marginal utility from quality. He finds cases where disclosure of quality information can result in intensified price competition and lower firm profits. This presents a rationale for mandatory disclosure to increase consumer surplus at the expense of firm profits through better sorting as well as the intensification of price competition among firms. Levin et al. (2009) present a model with two firms offering horizontally and vertically differentiated products. They also find that disclosure can intensify price competition thus deterring firms from disclosing quality information.

There have also been papers that have tried to include signalling and disclosure decisions in a unified framework. Daughety & Reinganum (2008) create a unified model where firms choose between disclosure and signalling. They find that when disclosure is costless all firms will disclose but if disclosure is sufficiently costly firms may signal. Another paper to include signalling and disclosure is that of Caldieraro et al. (2011). Interestingly this paper shows cases where it can be optimal for low quality firms to disclose their quality. Their model includes the possibility of high quality firms signalling their quality by depressing their price which intensifies price competition with the low quality firms. Both high and low quality firms can avert this by disclosure to increase the proportion of consumers that can recognise quality. By informing these consumers the incentives for high quality firms to depress their price for signalling are eroded and both firms are better off while consumer surplus is reduced.

A recent paper to look at the choice between disclosure and signalling in a duopolistic market is that of Jansson & Roy (2015). They examine firms that can be high or low quality (defined by an exogenous probability) where firms interact in a two stage game. In the first stage each firm can credibly disclose their quality to the market whilst in the second stage each firm offers a consumer a price. The consumer then buys from the firm that offers the higher utility after taking into account perceived quality and price. They find equilibria where firms with high quality goods decide to signal rather than disclose due to the strategic effects of disclosure on the other firm's price. When firms price without knowing the quality of their competitor, price competition is less intense. Thus the effects of price competition deter disclosure.

This paper seeks to build upon the literature in a few respects. Whilst Jansson & Roy (2015) offer a mechanism where high quality firms are deterred from disclosure even when it is costless, their mechanism relies on market power and hence would not generalise to markets where there are many firms. On the other hand the mechanism presented in this paper relies on search frictions and hence may be more generalisable to such markets.[3] A second contribution is that in our model, low quality firms proactively disclose that their goods are of low quality. Whilst this possibility has been examined by Caldieraro et al. (2011) two points of difference should be drawn. Caldieraro et al. (2011) present a model where it is beneficial for *all* firms to disclose quality while this paper presents a case where only low quality firms would like to disclose and this disclosure harms high quality firms. Finally the mechanism for low quality firms to disclose in Caldieraro et al. (2011) requires firms to signal high quality by *reducing* their price. While this is supported in their model and may occur in certain markets, it is more intuitive to believe that in general consumers perceive high price to signal high quality. This paper on the other hand presents a rationale for low quality firm disclosure that stems from search fractions and thus may better describe some markets.

3 The Model

There are two firms in a market, one of which sells a product of quality H (the "high firm") and one of which sells a good with a lower quality of L (the "low firm"). Both firms produce their goods costlessly. There is a unit measure of consumers. Consumers have heterogeneous marginal utility from quality described by a "taste" parameter. A consumer with a taste parameter of a_i, with an offer for a good with an expected quality

[3] Whilst for simplicity the model of this paper analyses the case of two firms, in an extension (section 4.4) it is shown that the result where low quality firms benefit from full information extends to the setting of many firms.

of Q and a price of P gets an expected utility from purchase of:

$$a_i Q - P \tag{1}$$

The taste parameter is uniformly distributed on $[0, 1]$ with a cdf given by:[4]

$$\text{Prob}[a_i < x] = x \qquad \text{for} \qquad x \in [0, 1] \tag{2}$$

Either firm can choose to disclose the qualities of both firms to the market in which case both qualities are observable and there is full information.[5] If neither firm chooses to disclose then there is asymmetric information and both goods appear identical to consumers. In the case of full information a fraction $1 - \psi$ of "*directed*" consumers (orthogonal to taste) can choose to approach either the low or high quality firm upon entering the market. The complementary fraction, ψ, of consumers are "*undirected*" and approach either firm with 50% probability. Upon entering the market undirected and directed consumers are identical.

The timing is as follows. Firms choose whether or not to disclose and the information setting is realised. Consumers then proceed to approach one of the two firms. Firms offer a price to all consumers with no price discrimination possible. Consumers can then decide to buy, search at the other firm with a search cost of s or leave the market to get an exogenous outside option of value 0. We assume $L > 0$ and thus a low quality firm recognised as being low quality will still be able to make some sales if visited by consumers with sufficiently high taste. We adopt the indifference rule that where $a_i Q - P_Q < 0$ for both firms for a consumer with taste a_i that consumer will approach the firm with a higher value of $a_i Q - P_Q$.

We assume that all consumers enter the market costlessly.[6] Consumers face a search

[4]The simple form of the utility from consumption $a_i Q - P$ and the uniform distribution of taste parameters are useful for tractability reasons, however the central unravelling mechanism (Proposition 1) extends to other taste distributions (including discrete distributions and distributions exhibiting gaps) and utility function forms, u, exhibiting $\frac{\partial u}{\partial a} > 0$, $\frac{\partial u}{\partial Q} > 0$, $\frac{\partial^2 u}{\partial a \partial Q} > 0$.

[5]Alternatively this can be though of as one firm disclosing the relative qualities of its good in comparison to the other.

[6]One justification is that consumers may benefit from the experience of visiting a single firm but get

cost for visiting the other firm once they are in the market however. We assume that this search cost is strictly nonzero and positive but sufficiently small such that the best alternative to buying for the marginal consumer at the high firm is to buy from the low firm. This assumption depends on equilibrium pricing decisions and thus will be formalised later on.[7]

Note that the mass of undirected consumers ensures that each firm is always approached by a positive mass of consumers. This ensures that in the putative equilibrium where all consumers visit one of the firms the beliefs of the other firm regarding the taste of deviating consumers visiting them out of equilibrium do not need to be established. The methodology of analysing this game will be to first examine the Perfect Bayesian Equilibrium (PBEs) at any given level of ψ. For the equilibrium that we will call refined we will then take the limit of a sequence of PBEs as $\psi \to 0$.[8]

These equilibrium concepts can be formally described as:

Definition 1 (Equilibrium Concepts). *A Perfect Bayesian Equilibrium (PBE) in this game is defined as a pricing and disclosure strategy for firms, a search strategy for consumers and quality beliefs of consumers such that no consumers or firms have a profitable deviation and all beliefs are supported by Bayes rule in equilibrium.*

A Refined Perfect Bayesian Equilibrium (RPBE) is the limit of a sequence of PBEs as $\psi \to 0$.

The possible search paths open to consumers in the full information setting are summarised in figure 1. This paper will first analyse the full asymmetric information subgame in section 3.1 before the full information subgame is examined in section 3.2. The dis-

bored thereafter. For instance the first test-ride of car at a dealership may be enjoyable whilst the process of looking for similar cars at other dealerships may be dull. To take an alternate example in the legal services market, a consumer facing criminal charges may benefit from hearing one lawyer's opinion on their case (while that lawyer provides a quote) but gets no additional information from visiting other lawyers (and getting other quotes). An alternate justification of this assumption may include undirected consumers randomly running into shops as they go about their normal day while directed consumers may be those able to plan their movements to choose what stores they run into. For instance a directed consumer intending to visit a beach may choose to go to one near a low/high quality shopping centre in order to visit a low/high quality firm at the same time. In this setting an undirected consumer is one that lives near only one beach so they cannot choose what firm to visit for free.

[7] This is done on page 12 just before lemma 3.

[8] Thus the refined equilibrium we present can be interpreted as a trembling hand refinement (Selten 1975) of an equilibrium without undirected consumers, with the start of search strategy being trembled.

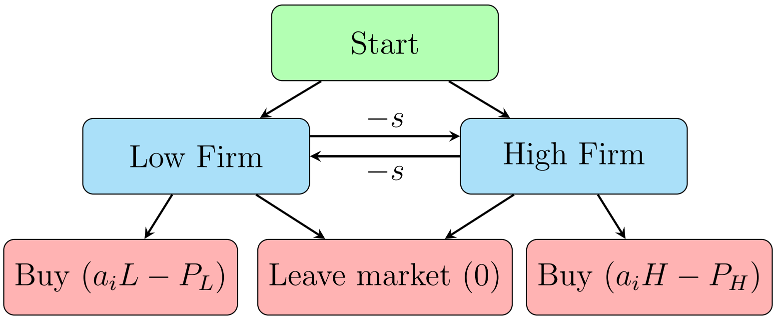

Figure 1: Consumer search paths in the full information setting for the consumer with taste a_i

closure decision of firms is then examined in section 3.3. Extensions are considered in section 4 before section 5 concludes.

3.1 The asymmetric information subgame

In the asymmetric information case, both firms appear identical so both directed and undirected consumers approach firms at random, each of which has an expected quality of $\frac{H+L}{2}$. In equilibrium each firm will be approached by a mass of $\frac{1}{2}$ with tastes uniformly distributed on the unit interval. The condition for a consumer to buy is:

$$a\frac{H+L}{2} - P \geq \max(0, a\frac{H+L}{2} - P^E - s) \tag{3}$$

Where P^E is the consumer's expected price from the other firm. From the Diamond (1971) paradox result the right hand side of this expression will equal 0 for the consumer made (ex post) indifferent in equilibrium. Hence profit for an individual firm can be written as:

$$\pi(P) = \frac{1}{2}P\left[1 - \frac{P}{\frac{H+L}{2}}\right] \tag{4}$$

From the first order conditions the optimal price and profit for each firm is:

$$P = \frac{\frac{H+L}{2}}{2} \qquad\qquad \pi = \frac{\frac{H+L}{2}}{8} \tag{5}$$

Which implies that both firms sell only to consumers with a taste greater than $\frac{1}{2}$.

3.2 The full information subgame

We open our analysis of the full information subgame with a lemma and a corollary that help to narrow the range of putative equilibria considerably:

Lemma 1. *When $\hat{a} > a_*$ and $H > L$ there will not exist any equilibrium where a taste a_* directed consumer goes to the firm with expected quality H whilst a taste $\hat{a}$ directed consumer goes to a firm with expected quality L.*

Proof. In the a space the utility from each good is linear and given by $aL - P_L$ and $aH - P_H$. This implies a single crossing condition that ensures that all consumers with a taste above the intercept point will approach the high firm and all consumers with a lower taste will approach the low firm. The assumed indifference rule ensures this holds in the case of low taste consumers where both $aL - P_L$ and $aH - P_H$ are less than 0. □

Corollary 2. *In any equilibrium, the set of directed consumers that choose to visit firms of a particular quality level will be convex in the "taste" dimension. We will denote the ex ante indifferent consumer's taste as $a_A = \frac{P_H^E - P_L^E}{H-L}$, hence $[0, a_A]$ consumers will visit the low firm and $[a_A, 1]$ consumers will approach the high firm.*

Proof. The ex ante indifferent consumer has equal expected utility from either firm and thus:

$$a_A H - P_H^E = a_A L - P_L^E \tag{6}$$

$$a_A = \frac{P_H^E - P_L^E}{H - L} \tag{7}$$

□

By setting up expressions analogous to equation 6 and rearranging, we can derive expressions for the indifferent taste consumers at each firm. These can be seen in table 1:[9]

[9] It should be noted that at the high firm consumers use their expectation of P_L and at the low firm consumers use their expectation of P_H. Before directed consumers enter the market they use their expectations of both prices. As pricing is simultaneous, each firm prices using an expectation of the other firm's price.

Notation	Location of consumer	Indifferent between	Formula
a_H	High firm	Buying and going to low firm	$\frac{P_H - P_L^E - s}{H-L}$
a_{HL}	High firm	Buying and leaving market	$\frac{P_H}{H}$
a_{HLM}	High firm	Going to low firm and leaving market	$\frac{P_L^E + s}{L}$
a_L	Low firm	Buying and leaving market	$\frac{P_L}{L}$
a_T	Low firm	Buying and going to high firm	$\frac{P_H^E - P_L + s}{H-L}$

Table 1: Indifferent Consumers

These can be used to write the demand function of the high firm:[10]

$$Q_{\text{High}}(P_H) = \overbrace{\frac{\psi}{2}[1 - \max(a_{HL}(P_H), a_H(P_H))]_+}^{\text{Undirected - own}} + \overbrace{\frac{\psi}{2}\left[1 - \max(a_A + \frac{s}{H-L}, a_{HL}(P_H), a_H(P_H))\right]_+}^{\text{Undirected runoff from the } L \text{ firm}} \tag{8}$$

$$+ \overbrace{(1-\psi)\left[1 - \max(a_A, a_{HL}(P_H), a_H(P_H))\right]_+}^{\text{Directed}}$$

In this demand function it can be seen that the high firm receives all directed consumers with a taste greater than a_A and undirected consumers from the low firm with a taste more than $a_A + \frac{s}{H-L}$ (if any exist). Note also that a_A is taken exogenously by the high firm knowing that consumers use their expectation of P_H (rather than the high firm's choice of P_H) to determine their search path. The high firm sells to all consumers that approach it with tastes between 1 and the maximum of a_H and a_{HL}.

The demand function for the low firm can be written as:

$$Q_{\text{Low}}(P_L) = \overbrace{\frac{\psi}{2}\left[\min(1, a_T(P_L)) - a_L(P_L)\right]_+}^{\text{Undirected - own}} + \overbrace{\frac{\psi}{2}\left[\min(a_A - \frac{s}{H-L}, a_T(P_L)) - \max(a_{HLM}, a_L(P_L))\right]_+}^{\text{Undirected Runoff from the } H \text{ firm}} \tag{9}$$

$$+ \overbrace{(1-\psi)\left[\min(a_A, a_T(P_L)) - a_L(P_L)\right]_+}^{\text{Directed}}$$

In this case the low firm receives all directed consumers with a taste less than a_A and undirected consumers from the high firm with tastes between a_H and a_{HLM} (if any exist).

[10]We use here and throughout the notation $[x]_+ = \max(0, x)$.

The low firm sells to all consumers that approach it with tastes between a_T and a_L.[11]

At this point we recall the assumption that search costs are sufficiently small such that the best alternative to buying for the marginal consumer at the high firm is to go to the low firm. Mathematically this translates to the requirement that search costs are sufficiently low that in every equilibrium we always have $a_H > a_{HL}$ which implies that we must have $s < L\left(\frac{P_H}{H} - \frac{P_L}{L}\right)$. We can prove that for a sufficiently low search cost, all equilibria will have the property that equilibrium prices satisfy $\frac{P_H}{H} > \frac{P_L}{L}$ and hence it is possible to find a positive search cost that satisfies this condition.

Lemma 3. *For a sufficiently low search cost, there are no equilibria where* $\frac{P_H}{H} \leq \frac{P_L}{L}$

Proof. See appendix A. □

Mathematically we must have $a_T > a_A > a_H$ and $a_{HLM} > a_L$. As any equilibria will have the property $\frac{P_H}{H} > \frac{P_L}{L}$, we can additionally infer that $a_L > a_{HL}$, $a_T > a_L$ and $a_H > a_{HLM}$.[12] Putting these together leaves the only remaining taste ordering of $a_T > a_A > a_H > a_{HLM} > a_L > a_{HL}$.

At this point it can be noted that the marginal consumer at the high firm has a taste of a_H which is strictly less than the taste of the ex ante indifferent consumer a_A. This has profound implications for the equilibrium as highlighted in the following proposition:

Proposition 1 (Unravelling of equilibrium without undirected consumers). *In the special case where there are no undirected consumers ($\psi \equiv 0$) there cannot exist equilibria where a positive measure of consumers approach both the high and low quality firms.*

Proof. A necessary condition for equilibrium in the absence of undirected consumers is that $a_H = a_A$. If we had $a_H > a_A$ then consumers in the interval $[a_A, a_H)$ will not buy at the high firm and would be better off going to the low firm ex ante. If we had $a_H < a_A$ then the high firm sells to consumers in the interval $[a_A, 1]$ whilst setting a price to make

[11] The fact that $H > L > 0$ ensures that tastes higher than a_T strictly prefer to leave for the high firm and tastes lower than a_L strictly prefer to leave the market and consumers in the interim prefer to buy.

[12] All of these inequalities come immediately from simple algebraic manipulation of the formulae in table 1.

a consumer a_H indifferent. If this firm increased its price to make a_A indifferent it could maintain its quantity at a higher price. □

One way to think about this result is as an unravelling mechanism. Consider if the firm received all consumers with tastes in the interval $[x, 1]$ and x is sufficiently high that the high firm wants to price to sell to all consumers with a taste greater than x.[13] The firm should optimally price to make the consumer with taste x indifferent to buying and walking away to the low firm. This implies a price of $x(H - L) + P_L^E + s$ which leaves the consumer with a taste x as getting utility of $Lx - P_L^E - s$. This consumer would be strictly better off going to the low firm initially however as she would get an expected payoff of $Lx - P_L^E$. If this consumer (and others of similar taste) deviate to the low firm, then the high firm will be approached by consumers with tastes in the interval $[x', 1]$ where $x' > x$. It will again be optimal for the high firm to price high enough so that the consumer with taste x' would be better off ex ante going to the low quality firm. This unravelling would continue until the high firm has no mass of consumers remaining.

This proposition does not hold when there are undirected consumers however as the high firm may set a price to make an undirected consumer with a taste lower than a_A indifferent. The effect of $a_H > a_A$ still has a substantial impact on the resulting equilibrium however as it implies that the optimal price for the high firm will be quite high. To see this informally note that the first order conditions of the general profit equation $\pi = PQ(P)$ imply that the optimal price satisfies $P = \frac{Q(P)}{Q'(P)}$. When there are few undirected consumers then $Q'(P)$ is quite low which implies a high optimal price.

We will show this formally by first rewriting the demand function (8) to incorporate the taste ordering discussed following lemma 3 and noting that the marginal consumer

[13] In the complementary case when x is low there cannot be an equilibrium as some consumers would not buy from the high firm and would instead go to the low firm - thus these consumers would be better off going to the low firm ex ante.

will have a taste between 0 and a_A:

$$Q_{\text{High}}(P_H) = \overbrace{\frac{\psi}{2}\left[1 - a_H(P_H)\right]}^{\text{Undirected - own}} + \overbrace{\frac{\psi}{2}\left[1 - (a_A + \frac{s}{H-L})\right]}^{\text{Undirected runoff from the } L \text{ firm}} + \overbrace{(1-\psi)\left[1 - a_A\right]}^{\text{Directed}} \tag{10}$$

Taking first order conditions for $\pi_{\text{High}} = P_H Q_{\text{High}}(P_H)$ and rearranging yields:

$$P_H = \frac{Ha_A\psi - 2Ha_A + 2H - La_A\psi + 2La_A - 2L + P_L^E\psi}{2\psi} \tag{11}$$

Now looking at the low firm's demand function it is not possible to determine if the firm will lose undirected consumers to the high firm. That is it is unclear if $a_T < 1$ in equilibrium. In the succeeding analysis we will assume $a_T < 1$ and do the complementary case in appendix C. First rewriting the demand function for the low firm (9):

$$Q_{\text{Low}}(P_L) = \overbrace{\frac{\psi}{2}\left[a_T(P_L) - a_L(P_L)\right]}^{\text{Undirected - own}} + \overbrace{\frac{\psi}{2}\left[a_A - \frac{s}{H-L} - a_{HLM}\right]}^{\text{Undirected Runoff from the } H \text{ firm}} + \overbrace{(1-\psi)\left[a_A - a_L(P_L)\right]}^{\text{Directed}} \tag{12}$$

Taking first order conditions for $\pi_{\text{Low}} = P_L Q_{\text{Low}}(P_L)$ and rearranging yields:

$$P_L = \frac{HLa_A\psi - 2HLa_A + HLa_{HLM}\psi - L^2a_A\psi + 2L^2a_A - L^2a_{HLM}\psi - LP_H^E\psi}{2H\psi - 4H - 4L\psi + 4L} \tag{13}$$

In equilibrium we will have $P_H^E = P_H$ and $P_L^E = P_L$. Substituting this, $a_{HLM} = \frac{P_L+s}{L}$ and

$a_A = \frac{P_H - P_L}{H-L}$ into equations 11 and 13 and solving for equilibrium prices yields:

$$P_H = \frac{2\,(H-L)\,(4H-2L-H\psi+2L\psi-\psi s)}{8(H-L)+H\psi(2-\psi)+2L\psi(1+\psi)} \tag{14}$$

$$P_L = \frac{(H-L)\,(4L-\psi^2 s-2\psi s)}{8(H-L)+H\psi(2-\psi)+2L\psi(1+\psi)} \tag{15}$$

$$\pi_H = \frac{2\psi\,(H-L)\,(H\psi-4H-2L\psi+2L+\psi s)^2}{(8(H-L)+H\psi(2-\psi)+2L\psi(1+\psi))^2} \tag{16}$$

$$\pi_L = \frac{(H-L)\,(2H-H\psi+2L\psi-2L)\,(-4L+\psi^2 s+2\psi s)^2}{2L\,(8(H-L)+H\psi(2-\psi)+2L\psi(1+\psi))^2} \tag{17}$$

We can also write an expression for the ex ante indifferent consumer and the marginal undirected consumer who leaves the low firm for the high:

$$a_A = \frac{8(H-L)+2\psi(2L-H)+\psi^2 s}{8(H-L)+H\psi(2-\psi)+2L\psi(1+\psi)} \tag{18}$$

$$a_T = \frac{8H^2-2H^2\psi+6HL\psi-16HL+2H\psi^2 s-2H\psi s-8Hs-4L^2\psi+8L^2-3L\psi^2 s-2L\psi s+8Ls}{(H-L)\,[8(H-L)+H\psi(2-\psi)+2L\psi(1+\psi)]} \tag{19}$$

At this point we can state this paper's second proposition establishing equilibriums for the full information subgame:[14]

Proposition 2. *For any given ψ satisfying $a_T < 1$ (with a_T given by equation 19) there exists a PBE of the full information subgame with firms pricing P_H, P_L according to equations 14 and 15 respectively and directed consumers with tastes $[0, a_A]$ approaching the low firm and consumers with taste $[a_A, 1]$ approaching the high firm with a_A according to equation 18. Firms have correct beliefs over taste distribution of consumers approaching them and consumers have correct beliefs over the quality of goods.*

Of the undirected consumers that initially approach the high firm, consumers with taste $[a_H, 1]$ will buy; consumers with taste $[a_{HLM}, a_H]$ will go the low firm and buy and consumers with taste $[0, a_{HLM}]$ will leave the market. Of the undirected consumers that initially approach the low firm, consumers with taste $[a_T, 1]$ will go to the high firm, consumers with taste $[a_L, a_T]$ will buy from the low firm and consumers with taste $[0, a_L]$

[14]For the equilibrium values in the alternative case where $a_T > 1$ see appendix C.

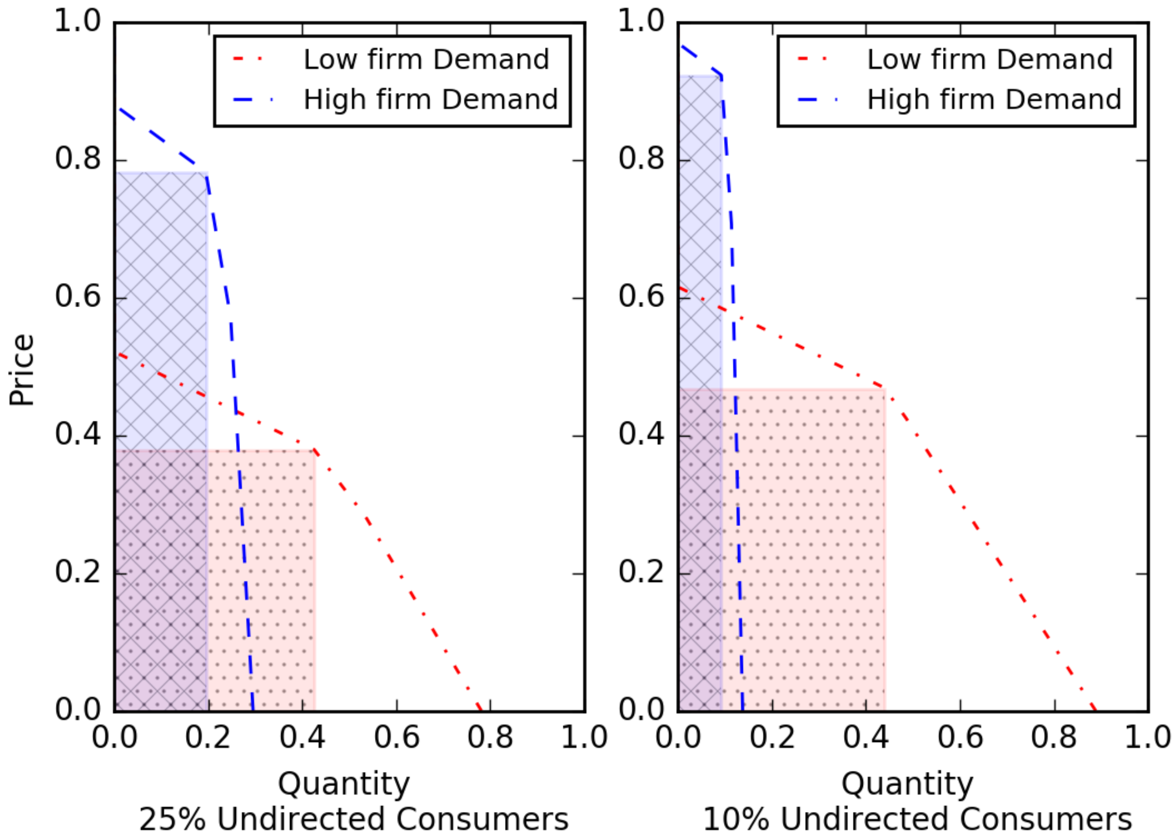

Figure 2: Equilibrium demand curves

will leave the market.

There are a few interesting features of this equilibrium. First we will consider the special case where there are no directed consumers ($\psi = 1$). In this case we get the intuitive result that the high quality firm earns strictly more than the low firm. Dividing equation 16 by equation 17 shows that the high firm's profit is $\frac{4L(3H-s)^2}{H(4L-3s)^2}$ times higher than the profit of the low firm. For a small search cost (in the limit as $s \to 0$) this approaches $\frac{9H}{4L}$. This is a smaller ratio of high to low firm profits however than occurs in the competitive market setting where the high firm earns $\frac{4H}{L}$ times more than the low firm.[15]

Firm profits change markedly however when there are directed consumers in the market. This is illustrated by figure 2 which shows the demand curves faced by the high and low quality firms under two sets of parameters with their optimal prices, quantities and profits being indicated by the shaded rectangles. In both cases there is a search cost of

[15]See appendix B for analysis of the competitive market and comparison with search market profits.

$s = 0.001$ and product qualities of $L = 1$ and $H = 1.5$ for the low and high firms respectively. In the left hand panel[16] 25% of consumers are undirected and so $\psi = 0.25$ while on the right hand panel[17] there are 10% undirected consumers and so $\psi = 0.10$. Considering first the left hand panel it can be seen that both demand curves have a flatter segment at high prices where firms sell to their directed consumers and undirected consumers with the same taste as their directed consumers. In these flatter segments the high firm sells to consumers with tastes in their interval $[a_A, 1]$ while the low quality firm sells to consumers with tastes in the interval $[a_L, a_A]$. As price is lowered however the firms have eventually sold to all consumers in these intervals and start selling to other undirected consumers. The slope of each demand curve becomes steeper reflecting the fact that there are fewer undirected consumers approaching them. The high firm sets its price so that a consumer with a taste of $a_A - \frac{s}{H-L}$ is indifferent to buying and going to the low firm. This price and quantity is slightly to the right and below the point at which the flatter and steeper segments of the demand curve meet which is that point at which consumers with taste a_A are indifferent to buying.

Looking now at the right panel, the only change to the parameters is that the population of undirected consumers has been reduced. In accordance with equation 18, a_A shifts upwards which results in the high firm receiving fewer directed consumers and the flat segment of the demand curve is narrower as a result. As the low firm receives more high taste consumers it raises its price which deteriorates the outside option for directed consumers at the high firm. As a result the high firm can sell to their directed consumers at a higher price. Overall however their profit has deteriorated relative to the case where more consumers were undirected.

The key takeaway from this figure is that even in full information markets where a significant fraction of consumers enter the market randomly, the low firm can earn greater profits than the high firm. In the left panel the low firm earns profits that are around 6% higher than the high firm whilst in the right panel the low firm earns 142% more. The

[16] The equilibrium quantities end up being $P_L = 0.379, P_H = 0.781, \pi_L = 0.161, \pi_H = 0.152, a_A = 0.805$ and $a_T = 0.806$.

[17] The equilibrium quantities end up being $P_L = 0.468, P_H = 0.922, \pi_L = 0.208, \pi_H = 0.085, a_A = 0.908$ and $a_T = 0.909$.

reduction in the number of undirected consumers allows the low firm to capture market share from the high quality firm. This observation leads to the following proposition for the refined equilibrium of this subgame as the proportion of undirected consumers approaches zero:

Proposition 3. *The sole refined equilibrium of the full information subgame is one where no consumers approach the high firm who make no profits. The low firm sells a quantity of $\frac{1}{2}$ at a price of $\frac{L}{2}$ for a profit of $\frac{L}{4}$.*

Proof. Equation 18 describing the ex ante indifferent consumer taste (a_A) is continuous in ψ and is 1 when $\psi = 0$. This indicates that as $\psi \to 0$ we will have $a_T = a_A + \frac{s}{H-L}$ exceeding 1. Once ψ is sufficiently high that $a_T > 1$ the relevant equation describing a_A is as given in equation C.6 which also exhibits $a_A \to 1$ as $\psi \to 0$.

As in the limit no directed consumers approach the high firm and no undirected consumers exist the high firm will make no profit. The low firm's price, quantity and profit can be found by taking the limit as $\psi \to 0$ for equations C.3 and C.5. $\square$

As a final point before the full game is considered note that in the special case when $\psi \equiv 0$ (as opposed to the refined equilibrium which is a limit as $\psi \to 0$) it is also possible to establish all consumers visiting the low quality firm as an equilibrium of the subgame.[18] In this case however it would be necessary to state beliefs of the high firm over the tastes of consumers who deviate to visit it out of equilibrium. The assumption that the high firm could observe the taste of a deviating consumer before pricing (and assuming a finite amount of consumers so each is of positive mass) or that the high firm believes that deviating consumers are of taste 1 would be sufficient to sustain this equilibrium.

3.3 The disclosure decision

In order to ascertain whether either firm will choose to disclose quality to the market it is necessary to consider each firm's profit in the asymmetric information and full information

[18] For appropriate low firm beliefs on deviating consumers it is also possible to establish an equilibrium of the subgame (with $\psi \equiv 0$) where all consumers approach the high quality firm. As can be seen in the preceding analysis however this alternate equilibrium would not survive trembling hand refinement through trembling the initial market entry strategy of consumers choosing to approach the low or high firm.

cases. Considering the refined equilibrium where all consumers visit the low firm and the high firm makes zero profits it is clear that the high firm will not choose to disclose when ψ is low. Whilst this firm can make $\frac{H+L}{16}$ in the asymmetric information market its profit tends to zero in the full information case as $\psi \to 0$. The condition for the low firm to prefer to disclose is found by comparing its profit in the full information case which tends to $\frac{L}{4}$ as $\psi \to 0$ to its profits in the asymmetric information case $\frac{H+L}{16}$. This shows that the low firm will strictly prefer to disclose if $H < 3L$.

Proposition 4. *If $H > 3L$ then the sole refined equilibrium is one where no firm discloses and the asymmetric information equilibrium described in section 3.1 results. If $H < 3L$ then the sole refined equilibrium is one where the low quality firm discloses and the full information refined equilibrium described by proposition 2 results. If $H = 3L$ then the two aforementioned equilibria are possible.*

Proof. See the paragraph preceding this proposition. □

3.4 Welfare

The profits and surpluses corresponding to the refined PBEs from proposition 4 can be seen in table 2 along with the profits and surpluses that would result from the asymmetric information subgame. It can be seen that the full information refined equilibrium delivers lower surplus than occurs under asymmetric information. Indeed the full information refined equilibrium delivers the same surplus as would occur in a monopolistic market containing only the low quality firm.

	Full Information Refined Equilibrium	Asymmetric Information Refined Equilibrium
Average Purchased Quality	L	$\frac{L+H}{2}$
Low firm Profit	$\frac{L}{4}$	$\frac{L+H}{16}$
High firm Profit	0	$\frac{L+H}{16}$
Sale Quantity	$\frac{1}{2}$	$\frac{1}{2}$
Producer Surplus	$\frac{L}{4}$	$\frac{L+H}{8}$
Consumer Surplus	$\frac{L}{8}$	$\frac{L+H}{8}$
Total Surplus	$\frac{3L}{8}$	$\frac{L+H}{4}$

Table 2: Profits and surpluses in each equilibria

4 Extensions & Discussion

4.1 Endogenous Quality

In the benchmark model quality is exogenous. In this extension we consider the case where quality is endogenous and firms are able to choose their quality level. We assume that firms can costlessly choose any quality level $Q \in [0, \bar{Q}]$ where $\bar{Q}$ is some upper limit set by technology. We consider only the full information subgame where firms choose their quality level concurrently with their pricing decision. Otherwise the game proceeds as per the model of section 3 and we will consider only refined equilibria.

At this point we can state a proposition describing the quality choices in this setting:

Proposition 5. *In all possible refined equilibria of the full information subgame, both firms will choose the minimum quality level* 0 *and no surplus will be generated in the market.*

Proof. Denoting the first and second firm's qualities by Q_1, Q_2, no equilibrium can exist with $Q_1 > Q_2 \geq 0$. In this setting, the second firm could increase their profits by increasing their quality closer to Q_1 . By symmetry no equilibrium can exist with $Q_2 > Q_1 \geq 0$.

In any putative equilibrium with $Q_1 = Q_2 > 0$, one firm can earn a discontinuous increase in profits by reducing their quality by an epsilon and disclosing qualities to the market. This possibility exists until one of the firms offers a quality of zero.

Considering the putative equilibrium where $Q_1 = Q_2 = 0$. Neither firm can increase their profits given the strategy of the opposing firm and this is an equilibrium. □

The intuition here is that in a mechanism similar to the Bertrand (1883) argument, firms can capture the whole market if they undercut the other firm on *quality*. This potential to earn a discontinuous increase in profits through undercutting will persist until one firm has a quality of zero and no further undercutting is possible.

4.2 Signalling

The model presented does not include the possibility of signalling. This section will discuss how the possibility of signalling can affect the outcome of the model. We will consider signalling to occur at the point of the firm (for instance price signalling) rather than occurring before the consumer enters the search market (for instance money-burning advertising).[19]

The addition of signalling at the point of the firm would be a nontrivial change to the model and would require a beliefs refinement as well as cost differences (or some other difference between high and low quality firms) to allow for signalling. For this study on disclosure however only one implication of an expanded model is important. The low quality firm would be worse off (relative to the full information refined equilibrium) in any equilibrium where consumers recognise low quality at the point of sale but do not approach firms endogenously as they are uninformed before approaching firms. Whilst in both cases the low firm is recognised as low quality, in cases where consumers approach firms randomly the low firm does not benefit from higher sale volumes. Furthermore greater equilibrium sales of the high firm means that they are selling to a wider range of high taste consumers. This results in a lower high firm price which means the low firm would lose more high taste consumers to the high firm. These forces mean that the low firm will strictly be worse off in signalling markets (with random market entry) than in the full information disclosure market (with endogenous market entry). Thus signalling could never occur even if allowed as the low quality firm would pre-emptively disclose their quality to induce the full information equilibrium presented in this paper.

On the other hand note that if the low firm were not able to disclose in this market[20] then the high quality firm would also not want to disclose even if it were possible for them to do so. Depending on the costs involved, the high firm may wish to signal at the point of the firm however so consumers recognise its high quality.[21] In this way the high

[19]This temporal relationship where signaling can occur closer to the point of purchase than disclosure is similar in spirit to the assumptions of the model of Jansson & Roy (2015). Signalling which occurs before the consumer enters the search market would be quite similar to disclosure in our model.

[20]For instance consider the case in the United States between 1907 and 1977 where bar association advertising rules prohibited lawyers from advertising.

[21]For instance while lawyer advertising was banned, large law firms invested heavily in opulent offices

firm benefits from a higher perceived value without losing customers through endogenous consumer market entry.

4.3 Multi-product Firms

The endogenous entry of consumers to the market has interesting implications in a setting of multi-product firms. Multi-product firms may also be though to arise in part from high qualities seeking to thwart the unravelling mechanism of this paper by encouraging low taste consumers to approach them. In this extension we consider that there are two firms: one that sells both the high and low quality good (the "*both*" firm) and the other that sells only the low quality good (the "*low*" firm). We will consider only the full information subgame.

This is an interesting extension because a monopolistic screening problem appears for the both firm. The lower is the price they set for the low quality good, the more high taste consumers will opt for it cover the higher margin high quality good. In the case of endogenous market entry, the firm selling both goods will encounter fewer low taste consumers than the low quality firm that specialises in only selling this good. This ratio of high taste to low taste consumers amplifies the monopoly screening effect. The firm selling both goods will not want to price lower than the low firm for fear of high taste consumers opting for it over the high quality good. The implication is that a firm with high quality goods trying to avert the unravelling result by also selling low quality goods will not be successful in this endeavour. Indeed in the full information subgame as $\psi \to 0$, the low firm still takes over the market:

Proposition 6. *In the unique refined equilibrium of the full information subgame, the both firm will price the low good at $P_L + s$ where P_L is the low firm's price for this good. This means the both firm sells to all undirected consumers that visit it but low taste directed consumers still prefer to approach the low firm. As is the case in proposition 3 as $\psi \to 0$ the ex ante indifferent consumer taste will approach 1, $a_A \to 1$. The both firm's profit will tend to zero and the low firm's profit to $\frac{L}{4}$ in this refined equilibrium.*

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and amenities which may have been a mechanism to signal quality.

Proof. The proof is included in appendix D □

4.4 Many Firms

The full information equilibrium result where $a_A \to 1$ as $\psi \to 0$ extends to the setting of many firms. This can be seen in appendix E. It also appears likely that this result would also extend to the whole game depending on how disclosure was modelled in a multifirm setting. This paper has so far modelled disclosure as one firm revealing both qualities. Whilst this is credible in the duopolistic setting as a firm can disclose the relative differences in the two goods, it does not extend easily to the multifirm setting.

A natural way in which disclosure could be added to the model would be to allow firms to choose to disclose their quality or to pool by choosing to stay silent.[22] Depending on what disclosures occur, there could be no disclosures with the asymmetric information subgame occurring, there could be all high quality or all low quality firms disclosing which would result in the full information subgame occuring, there could be some (but not all) of one group of firms disclosing resulting in a pooled quality level and a set of firms of known quality and finally there could be an information setting where three quality levels exist for low firms, high firms and an intermediate group of pooled firms.

With a sufficiently small search cost, the likely equilibrium would exhibit the low taste marginal consumer at a low quality firm indifferent to buying and leaving the market, while the low taste marginal consumer at a pooled firm is indifferent to buying and going to a low quality firm and the marginal consumer at the high quality firm is indifferent to buying and going to a pooled firm. In this setting the same unravelling result will occur as described in proposition 1. Consider a split with consumers of taste $[0, x)$ going to the low firms, consumers of taste $[x, x')$ with $x' > x$ going to the pooled firms and consumers of taste $[x', 1]$ going to the disclosing high firms. In the absence of undirected consumers the high firm will want to price to make the consumer with taste x' indifferent between buying and leaving. This will lead to consumers of taste x' being better off going to a

[22] A second way that disclosure could be modelled is as firms exerting effort to shift the market's ψ value. The low firms may all choose to advertise more which leads to more informed consumers and more directed consumers. Depending on the costs of this advertising to lower ψ and other parameter values, this may be a profitable strategy for low quality firms.

pooled firm ex ante which results in the same unravelling as discussed before. The same unravelling would also occur at the margin of the low firms and the pooled firms.[23] This will lead to low firms disclosing as pooled and high quality firms make no profits in the refined equilibria.

5 Conclusion

This paper has presented a consumer search model with two key features: consumers have heterogeneous marginal utility from quality and enter the market endogenously. It is shown that when consumers have full information in this simple setting the only equilibrium that survives refinement is one where only the low quality firms sell goods. This result comes about because the effect of quality on pricing decisions can be anticipated by consumers. The high firm will price to make one of the consumers approaching it indifferent between buying and searching further after this consumer has arrived at the firm. Anticipating this the consumer would be better off going a low quality firm ex ante. Having lost such consumers the high quality firm will want to raise its price to make a higher taste consumer indifferent. And thus there is an unravelling result where all consumers end up shopping at the low quality firm.[24]

In an asymmetric information setting a pooling equilibrium may be sustainable if the quality of the high firm's goods are sufficiently higher than the quality of the low firm's goods. In such cases the low firm will prefer to pool with a higher perceived value of their good than disclose and get a higher quantity. On the other hand if the high firm started signalling or disclosing after consumers approach the firm then the low firm would not benefit from pooling and would be strictly better of disclosing. Thus we can offer an argument for why a high quality firm may choose costly signalling (at the point of the

[23] The calculations for the case of unravelling with 3 levels of quality and many firms is shown in appendix F.

[24] At a methodological level, consumer search models often have equilibria where no consumer searches beyond their initial firm. A further contribution of this paper is that it highlights that in such a market with heterogeneity, the assignment of consumers to their initial firm is important. In this paper's model the equilibrium induced by endogenous market entry differs markedly from the equilibrium induced by random assignment of consumers to firms (which can be seen by considering equations 14 to 19 when $\psi = 1$).

firm) over costless disclosure (before consumers approach firms) in search markets. The caveat is that where a high quality firm separates itself from a low quality firm through signalling there is no disincentive for the low quality firm to proactively disclose its quality. The low firm can disclose their quality and benefit from a greater number of consumers visiting the firm in equilibrium.

The analysis indicates that information provision is not necessarily harmful to a low quality firm in markets exhibiting search frictions. It may indeed be in the best interests of a low quality firm to disclose its low quality. Similarly it may not be in the interests of a high quality firm to disclose its high quality as that may deter consumers from approaching the firm for fear of high prices.

Whilst quality disclosure in this paper's model can boost a low firm's profits it is generally bad for welfare. Both consumer surplus and producer surplus are decreased by the disclosure of information to the market. In addition when quality is endogenised in the model it is shown that competition can lead to firms reducing the quality they offer consumers even when both firms can produce a good of any quality at the same cost. This implies that it may be welfare improving for regulators to impose minimal quality standards in the market. It may also be welfare improving for regulators to stop a low quality firm from voluntarily disclosing its quality to consumers until consumers are at the point of sale. This suggestion may be problematic in many cases as it could be argued firms and consumers have a moral right to voluntarily exchange truthful information before trading. On the other hand such disclosures adversely impact consumers and the high quality firm in the refined equilibrium presented in this paper. This is in addition to the issue in some markets (such as the United States legal services market) where the mechanisms for low quality disclosure produces undesirable externalities for other participants in the market.[25] Thus in certain cases it may on balance be better for a regulator or professional body to intervene to prevent low quality firm disclosures.

[25]For instance the American Bar Association regularly states that the antics of some advertising lawyers brings the legal profession into disrepute and it has been argued that such advertising has adversely impact the respect with which the public affords lawyers (Cebula 1998).

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Appendices

A Proof of lemma 3

To prove lemma 3 we will show first that given sufficiently small search costs, no equilibrium can satisfy $\frac{P_H}{H} = \frac{P_L}{L}$ before showing that given small search costs no equilibrium can exhibit $\frac{P_H}{H} < \frac{P_L}{L}$.

Lemma 4. *For a sufficiently low search cost, there are no equilibriums where $\frac{P_H}{H} = \frac{P_L}{L}$:*

Proof. In the first case note that in an equilibrium with $\frac{P_H}{H} = \frac{P_L}{L} = 0$ then neither firm is making any profits. The high firm could earn positive profits by setting a small price and selling to high taste consumers.

In the second case we consider a putative equilibria where $\frac{P_H}{H} = \frac{P_L}{L} > 0$. We will use the notation $x = \frac{P_H}{H} = \frac{P_L}{L}$ and note that $a_A = x$ and the low firm gets all consumers with a lower taste and the high firm gets all consumers with a higher taste. At first we will assume $a_T \leq 1$. Using the demand function (9) we can write the profit for the low firm pricing at xL:

$$\begin{aligned}\pi_{\text{Low}}(xL) &= xL\frac{\psi}{2}\left[x + \frac{s}{H-L} - x\right] \\ &= \frac{sxL\psi}{2(H-L)}\end{aligned} \tag{A.1}$$

where all sales are to undirected consumers that visit the low firm initially. Note that sales are entirely dependent on the extent of the search cost s. Now if the low firm instead reduced their price to $xL - \epsilon$ where $xL > \epsilon > 0$ then their profit would be:

$$\begin{aligned}\pi_{\text{Low}}(xL-\epsilon) &= [xL-\epsilon]\left[\frac{\psi}{2}\left[x + \frac{\epsilon+s}{H-L} - (x - \frac{\epsilon}{L})\right] + (1-\psi)\left[x - (x - \frac{\epsilon}{L})\right]\right] \\ &= xL\left[\frac{\psi(\epsilon+s)}{2(H-L)} + \frac{\epsilon}{L}(1-\frac{\psi}{2})\right] - \epsilon\left[\frac{\psi(\epsilon+s)}{2(H-L)} + \frac{\epsilon}{L}(1-\frac{\psi}{2})\right] \\ &= \pi_{\text{Low}}(xL) + xL\left[\frac{\psi(\epsilon)}{2(H-L)} + \frac{\epsilon}{L}(1-\frac{\psi}{2})\right] - \epsilon\left[\frac{\psi(\epsilon+s)}{2(H-L)} + \frac{\epsilon}{L}(1-\frac{\psi}{2})\right]\end{aligned}$$

Where sales are to undirected consumers who visit the firm initially as well as directed consumers with a taste less than a_A. As $xL > \epsilon$ we will have $xL\left[\frac{\psi(\epsilon)}{2(H-L)} + \frac{\epsilon}{L}(1-\frac{\psi}{2})\right] - \epsilon\left[\frac{\psi(\epsilon+s)}{2(H-L)} + \frac{\epsilon}{L}(1-\frac{\psi}{2})\right]$ positive for a sufficiently small s.

Now considering the case where $a_T > 1$. Using the demand function (9) we can write the profit for the low firm pricing at xL:

$$\begin{aligned}\pi_{\text{Low}}(xL) &= xL\frac{\psi}{2}\left[1-x\right] \\ &= \frac{xL\psi}{2}(1-x)\end{aligned}$$

where all sales are to undirected consumers that visit the low firm initially. Note that as $a_T > 1$ then $(1-x) < \frac{s}{H-L}$. Now if the low firm instead reduced their price to $xL-\epsilon$ where $xL > \epsilon > 0$ and ϵ is sufficiently small that a_T is still greater than one then their profit would be:

$$\begin{aligned}\pi_{\text{Low}}(xL-\epsilon) &= [xL-\epsilon]\left[\frac{\psi}{2}\left[1-(x-\frac{\epsilon}{L})\right] + (1-\psi)\left[x-(x-\frac{\epsilon}{L})\right]\right] \\ &= [xL-\epsilon]\left[\frac{\psi}{2}(1-x) + (1-\frac{\psi}{2})\frac{\epsilon}{L}\right] \\ &= \pi_{\text{Low}}(xL) + xL\left[(1-\frac{\psi}{2})\frac{\epsilon}{L}\right] - \epsilon\left[\frac{\psi}{2}(1-x) + (1-\frac{\psi}{2})\frac{\epsilon}{L}\right]\end{aligned}$$

Where sales are to undirected consumers who visit the firm initially as well as directed consumers with a taste less than a_A. As $(1-x)$ is small for a sufficiently small s and $xL > \epsilon$ we will get $xL\left[(1-\frac{\psi}{2})\frac{\epsilon}{L}\right] - \epsilon\left[\frac{\psi}{2}(1-x) + (1-\frac{\psi}{2})\frac{\epsilon}{L}\right]$ as positive and this is a profitable deviation. □

Lemma 5. *For a sufficiently low search cost, there are no equilibria where* $\frac{P_H}{H} < \frac{P_L}{L}$*:*

Proof. Suppose that $\frac{P_H}{H} < \frac{P_L}{L}$. We will denote $x_L = \frac{P_L}{L}$ and $x_H = \frac{P_H}{H}$. In this case $a_A = \frac{Hx_H - Lx_L}{H-L}$.

We can note this implies we must have $a_T < 1$ if the search cost is small. This is because $a_T > 1$ would imply:

$$\frac{Hx_H - Lx_L + s}{H - L} > 1$$
$$L(1 - x_L) + s > H(1 - x_H)$$

Now as $H > L$ and $(1 - x_H) > (1 - x_L)$, it is not possible for this to hold if the search cost is sufficiently small.

Now examining the case where $a_T \leq 1$, the case profit for the low firm is:

$$\pi_{\text{Low}}(x_L L) = x_L L \left[\frac{\psi}{2} \left[\frac{Hx_H - Lx_L + s}{H - L} - x_L \right]_+ \right]$$
$$\pi_{\text{Low}}(x_L L) = \frac{\psi x_L L}{2} \left[\frac{H(x_H - x_L) + s}{H - L} \right]_+$$

Where the firm only sells to undirected consumers that approach it initially if the search cost is sufficiently high (as $x_H < x_L$). Note for a sufficiently small s this profit is zero. Now if the low firm lowers its price to $x_L L - L\epsilon$,[26] it gets a profit of:

$$\pi_{\text{Low}}(x_L L - L\epsilon) = (x_H L - \epsilon L)\frac{\psi}{2} \left[\frac{x_H(H - L) + \epsilon L + s}{H - L} - (x_H - \epsilon L) \right]$$
$$\pi_{\text{Low}}(x_L L - L\epsilon) = \frac{\psi(x_H L - \epsilon L)}{2} \left[\frac{\epsilon L + s}{H - L} \right]$$

Which is greater than $\pi_{\text{Low}}(x_L L)$ for a sufficiently small value of s. □

B Equilibrium in a competitive market

In this appendix the benchmark model will be analysed in the absence of search frictions. All consumers buy one good knowing the two prices. There is no search cost and as a result the concept of directed/undirected consumers is not used here.

With similar logic as earlier in this paper it can be noted that the set of consumers who will buy from a given firm will be a convex set. It can also be noted that the high firm will not sell to all consumers as this would require a price of 0 which would result in

[26]Note that this is always possible unless $x_H = 0$ which is not possible from lemma 4.

no profits.

We will use the notation of $[0, a_L]$ to describe the consumers who will not buy but will take the outside option; $[a_L, a_A]$ is the set of consumers who will buy from the low firm and $[a_A, 1]$ is the set of consumers who will buy from the high firm. Considering the condition for a consumer to be indifferent between the high and low firm's offering we get $a_A = \frac{P_H - P_L}{H-L}$. Considering a consumer indifferent between buying at the low firm and the ouside option yields $a_L = \frac{P_L}{L}$. These can be used to write the firm profit functions:

$$\pi_H(P_L) = \max_{P_H} P_H \left[1 - [\frac{P_H - P_L}{H - L}]_+\right]_+ \tag{B.1}$$

$$\pi_L(P_L) = \max_{P_L} P_L \left[[\frac{P_H - P_L}{H - L}]_+ - [\frac{P_L}{L}]_+\right]_+ \tag{B.2}$$

Taking first order conditions of equations B.1 and B.2 yields the reaction functions $P_H(P_L) = \frac{H-L+P_L}{2}$ and $P_L(P_H) = \frac{LP_H}{2}$ which can be used to find the equilibrium price and profits:

$$P_L = \frac{L(H-L)}{4H-L} \qquad P_H = \frac{2H(H-L)}{4H-L} \tag{B.3}$$

$$\pi_{\text{L, Comp}} = \frac{HL(H-L)}{(4H-L)^2} \qquad \pi_{\text{H, Comp}} = \frac{4H^2(H-L)}{(4H-L)^2} \tag{B.4}$$

Examining the profit expressions (B.4), we can note in this setting the high firm's profit is always strictly higher than low firm's profit by a factor of $\frac{4H}{L}$.

Comparing the high firms profit in the search model (16) with no directed consumers ($\psi = 1$) to the high firm profit in the competitive model (B.1) we find that in the search case profits are $\frac{(3H-s)^2(4H-L)^2}{2H^2(9H-4L)^2}$ times higher. Taking the limit as $s \to 0$ this ratio translates to $\frac{9(4H-L)^2}{2(9H-4L)^2} = [\frac{3(4H-L)}{\sqrt{2}(9H-4L)}]^2$, If $\frac{H}{L} \leq \frac{4\sqrt{2}-3}{9\sqrt{2}-12} \approx 3.65$ then this profit ratio is greater than one. Now examining the ratio of low firm profits it can be calculated that the low firms profits are $\frac{(4L-3s)^2(4H-L)^2}{2L^2(9H-4L)^2}$. In the limit as $s \to 0$ this ratio approaches $\left[\frac{8\sqrt{2}H-2\sqrt{2}L}{9H-4L}\right]^2$. This is more than one if $\frac{H}{L} > \frac{2\sqrt{2}-4}{8\sqrt{2}-9} \approx -0.5$ which is satisfied.

C Derivation of Equilibrium when $a_T > 1$

We derive the equilibrium in the case when $a_T > 1$ which is likely when ψ is low and a_A is close to 1. First rewriting $Q_{\text{Low}}(P_L)$ (equation 12):

$$Q_{\text{Low}}(P_L) = \overbrace{\frac{\psi}{2}\left[1 - a_L(P_L)\right]}^{\text{Undirected - own}} + \overbrace{\frac{\psi}{2}\left[a_A - \frac{s}{H-L} - a_{HLM}\right]}^{\text{Undirected Runoff from the } H \text{ firm}} + \overbrace{(1-\psi)\left[a_A - a_L(P_L)\right]}^{\text{Directed}} \tag{C.1}$$

With the same solution steps as in section 3.2 we can derive the following expressions:

$$P_H = \frac{2H^2\psi - 8H^2 - 4HL\psi + 12HL + 2H\psi s + 2L^2\psi - 4L^2}{H\psi^2 - 2H\psi - 8H + 8L} \tag{C.2}$$

$$P_L = \frac{-HL\psi^2 - 4HL + H\psi^2 s + 2H\psi s + L^2\psi^2 + 4L^2}{H\psi^2 - 2H\psi - 8H + 8L} \tag{C.3}$$

$$\pi_H = \frac{2\psi\left(H^2\psi - 4H^2 - 2HL\psi + 6HL + H\psi s + L^2\psi - 2L^2\right)^2}{(H-L)\left(H\psi^2 - 2H\psi - 8H + 8L\right)^2} \tag{C.4}$$

$$\pi_L = \frac{(2-\psi)\left(-HL\psi^2 - 4HL + H\psi^2 s + 2H\psi s + L^2\psi^2 + 4L^2\right)^2}{2L\left(H\psi^2 - 2H\psi - 8H + 8L\right)^2} \tag{C.5}$$

$$a_A = \frac{8H^2 - 2H^2\psi - HL\psi^2 + 4HL\psi - 16HL + 2H\psi^2 s - 2H\psi s - 8Hs + L^2\psi^2 - 2L^2\psi + 8L^2 + 8Ls}{(H-L)\left(8(H-L) + 2H\psi - H\psi^2\right)} \tag{C.6}$$

$$a_T = \frac{8H^2 - 2H^2\psi - HL\psi^2 + 4HL\psi - 16HL + 2H\psi^2 s - 2H\psi s - 8Hs + L^2\psi^2 - 2L^2\psi + 8L^2 + 8Ls}{(H-L)\left(8(H-L) + 2H\psi - H\psi^2\right)} \tag{C.7}$$

D Multiproduct firms

Now consider that there is one firm that sells both high and low quality goods (the "both" firm) and one that only sells the low quality firm (the "low" firm). P_H is the price of the high good (sold by the both firm), P_B is the price of the low good from the both firm and P_L is the price of the low good from the low firm. The profits of the low and both firm are given by π_L and π_B respectively. We will consider only the full information subgame. We will redefine the ex ante indifferent consumer's taste of a_A to be:

$$a_A = \frac{P_H - \min(P_L, P_B)}{H - L} \tag{D.1}$$

We will use λ to denote the fraction of directed consumers with tastes lower than a_A that go to the low firm upon entering the market. Clearly $\lambda = 1$ if in an equilibrium $P_L < P_B$ and $\lambda = 0$ if $P_L > P_B$. A consumer indifferent between buying and leaving a firm will choose to buy at the current firm.

We can note that no equilibrium can take place where $P_B > P_L + s$. In this case the both firm could reduce their price to $P_L + s$ and sell to low taste undirected consumers at the both firm while the outside option for these consumers (of buying the low good at total cost $P_L + s$) would stay the same. There can also be no equilibrium where $P_L > P_B + s$. In this case the low firm would sell no quantity and make no profit. They could make strictly positive profits by pricing at another level (for instance this firm can guarantee a positive profit by selling to undirected consumers at a price of $\frac{s}{2}$).

Now writing the firm profit functions for a putative equilibrium where $|P_B - P_L| \leq s$. In this case a consumer will never leave one firm to buy the low quality good from the other. The profit functions for each firm can be written as:

$$\pi_L = \max_{P_L} \overbrace{\frac{\psi}{2} P_L \left[\min(1, \frac{P_H^E - P_L + s}{H - L}) - \frac{P_L}{L} \right]}^{\text{Undirected}} + \overbrace{(1-\psi)\lambda P_L \left[a_A - \frac{P_L}{L} \right]}^{\text{Directed}} \tag{D.2}$$

$$\pi_B = \max_{P_B, P_H} \overbrace{\frac{\psi}{2} \left[P_B(\frac{P_H - P_B}{H - L} - \frac{P_B}{L}) + P_H(1 - \frac{P_H - P_B}{H - L}) \right]}^{\text{Undirected}} \tag{D.3}$$

$$+ \overbrace{(1-\psi) \left[(1-\lambda) P_B(a_A - \frac{P_B}{L}) + P_H(1 - a_A) \right]}^{\text{Directed}} + \overbrace{\frac{\psi}{2} P_H [1 - (a_A + \frac{s}{H - L})]_+}^{\text{High taste runoff from low firm}}$$

At this point it can be seen that intuitively as the both firm lowers the price of the low quality good (within $P - s \leq P_B \leq P_L + s$) it reduces the amount of the high quality good it can sell. The low firm faces no such problem. This leads to the following lemma:

Lemma 6. *For a small search cost and a nonzero fraction of undirected consumers, no equilibrium exists where $P_B \leq P_L$.*

Proof. We will show this by supposing the contrary such that $P_B \leq P_L$ and hence $a_A = \frac{P_H - P_B}{H - L}$. We will first guess that in equilibrium $\frac{P_H - P_L + s}{H - L} < 1$ Taking first order conditions of equations D.2, D.3 and rearranging

$$P_L = \frac{L\left(-2Ha_A\lambda\psi + 2Ha_A\lambda + 2La_A\lambda\psi - 2La_A\lambda + P_H^E\psi + \psi s\right)}{4H\lambda - 4H\lambda\psi + 2H\psi + 4L\lambda\psi - 4L\lambda} \tag{D.4}$$

$$P_H = \frac{H}{2} - \frac{L}{2} + p_B - \frac{1}{2\psi}\left(\psi\left(-H + L + a_A\left(H - L\right) + s\right) - 2\left(H - L\right)\left(a_A - 1\right)\left(\psi - 1\right)\right) \tag{D.5}$$

$$P_B = \frac{L\left(Ha_A\lambda\psi - Ha_A\lambda - Ha_A\psi + Ha_A - La_A\lambda\psi + La_A\lambda + La_A\psi - La_A + P_H^E\psi\right)}{2H\lambda\psi - 2H\lambda - H\psi + 2H - 2L\lambda\psi + 2L\lambda + 2L\psi - 2L} \tag{D.6}$$

Substituting in that in equilibrium $P_H^E = P_H$ and a_A must satisfy $a_A = \frac{P_H - P_B}{H-L}$ and solving these equations yields expressions for the prices of the putative equilibrium:[27]

$$P_H = \frac{\left(2H - 2L - \psi s\right)\left(2H\lambda\psi - 2H\lambda - H\psi + 2H - L\lambda\psi + L\lambda + L\psi - L\right)}{\left(H - L\right)\left(\psi + 2\right)\left(2\lambda\psi - 2\lambda - \psi + 2\right)} \tag{D.7}$$

$$P_B = \frac{L\left(2H - 2L - \psi s\right)\left(\lambda\psi - \lambda + 1\right)}{\left(H - L\right)\left(\psi + 2\right)\left(2\lambda\psi - 2\lambda - \psi + 2\right)} \tag{D.8}$$

$$a_A = \frac{2H - 2L - \psi s}{\left(H - L\right)\left(\psi + 2\right)} \tag{D.9}$$

Now taking the difference between P_L and P_B and taking s to zero in the limit. This should be nonnegative by assumption.

$$P_L - P_B = \frac{L\psi\left(2H\lambda\psi - 2H\lambda - H\psi - 3L\lambda\psi + 3L\lambda + L\psi - L\right)}{\left(\psi + 2\right)\left(2\lambda\psi - 2\lambda - \psi + 2\right)\left(2H\lambda - 2H\lambda\psi + H\psi + 2L\lambda\psi - 2L\lambda\right)} \tag{D.10}$$

First considering the case where $P_L > P_B$ and hence $\lambda = 0$ this difference should be strictly positive. In this case it simplifies to:

$$P_L - P_B = \frac{L\left(-L - (H - L)\psi\right)}{H\left(2 - \psi\right)\left(\psi + 2\right)} \tag{D.11}$$

Which is strictly negative. Thus we cannot have an equilibrium where $P_L > P_B$ for a small search cost. Now considering the other case where $P_L = P_B$ and hence $1 \geq \lambda \geq 0$.

[27] The expression for P_L is omitted as it is long but it can be found by substituting equations D.9 and D.7 into equation D.4.

The difference in equation D.10 must be equal to zero. This is the case if either $\psi = 0$[28] or $(2H\lambda\psi - 2H\lambda - H\psi - 3L\lambda\psi + 3L\lambda + L\psi - L) = 0$. This second condition can only hold if $\lambda = \frac{(H-L)\psi+L}{(3L-2H)(1-\psi)}$. The numerator of this equation is always greater than L and the denominator is always less than L. As λ can only be in the range $[0, 1]$ this indicates there is no equilibrium with equal low quality good prices.

Now we guess that $\frac{P_H - P_L + s}{H-L} \geq 1$. Taking first order conditions of equation D.2 and rearranging:

$$P_L = \frac{L\left(2a_A\lambda\psi - 2a_A\lambda - \psi\right)}{4\lambda\psi - 4\lambda - 2\psi} \tag{D.12}$$

With the reaction functions for P_H and P_B being identical to equations D.5 and D.6. As P_L does not enter these other reaction functions the prices P_H,P_L and a_A in this equilibrium are identical to those given in equations D.7, D.8 and D.9 respectively. P_L can be found by substituting these expressions into equation D.12. Now taking the difference between P_L and P_B and taking s to zero in the limit. This should be nonnegative by assumption.

$$P_L - P_B = -\frac{L\psi\left(\psi^2 + 4\lambda - 2\lambda\psi^2 - 2\lambda\psi\right)}{2\left(\psi+2\right)\left(\psi + 2\lambda(1-\psi)\right)\left(2 - 2\lambda(1-\psi) - \psi\right)} \tag{D.13}$$

Which is negative for all λ. Hence we cannot have an equilibrium where $\frac{P_H - P_L + s}{H-L} \geq 1$ and $P_B \leq P_L$. □

With $\lambda = 0$ the first order condition for the both firm's low quality good pricing decision becomes:

$$\frac{\partial \pi_B}{\partial P_B} = \psi\left[\frac{P_H - P_B}{H-L} - \frac{P_B}{L}\right] \tag{D.14}$$

Which will be positive for a small search cost and $P_B < L$.[29]. This implies that the optimal P_B will be equal to $P_L + s$ which is the top of the domain of possible prices

[28] This can be seen intuitively by substituting $\psi = 0$ into eqations D.2 and D.3 and taking the first order conditions which are scalar multiples of each other.

[29] This can be seen by substituting taking the first order condition for the optimal P_H (identical to equation D.5) and substituting this into $\frac{P_H - P_B}{H-L} - \frac{P_B}{L}$ which simplifies to $\frac{1}{\psi} - \frac{P_B}{L} + \frac{a_A}{2} - \frac{s}{2(H-L)}$ which is positive as $\frac{1}{\psi} > 1 > \frac{P_B}{L}$ and s is small.

satisfying $|P_B - P_L| \leq s$. Note that if the both firm posted a higher price they would experience a discontinuous in sales while the outside option of their consumers would stay the same.

Now using $\lambda = 1$, $a_A = \frac{P_H - P_L}{H-L}$ and $P_B = P_L + s$ and using first order conditions to solve for equilibrium quantities for the case where $\frac{P_H - P_L + s}{H-L} < 1$ as:

$$P_H = \frac{4H^2\psi - 8H^2 - 8HL\psi + 12HL + 2H\psi^2 s - 4H\psi s + 4L^2\psi - 4L^2 - 3L\psi^2 s}{(\psi+2)(2H\psi - 4H - 3L\psi + 4L)} \tag{D.15}$$

$$P_L = \frac{2L(-H\psi + 2H + L\psi - 2L + 2\psi s)}{(\psi+2)(4H - 2H\psi + 3L\psi - 4L)} \tag{D.16}$$

$$P_B = \frac{2HL\psi - 4HL + 2H\psi^2 s - 8Hs - 2L^2\psi + 4L^2 - 3L\psi^2 s - 6L\psi s + 8Ls}{(\psi+2)(2H\psi - 4H - 3L\psi + 4L)} \tag{D.17}$$

$$a_A = \frac{2H - 2L + \psi s}{(H-L)(\psi+2)} \tag{D.18}$$

Where a_A approaches 1 as $\psi \to 0$. This will imply that at some level of ψ we have $\frac{P_H - P_L + s}{H-L} \geq 1$. Now considering this case we can get the equilibrium quantities:

$$P_H = \frac{4H^2\psi - 8H^2 - HL\psi^2 - 6HL\psi + 12HL + 2H\psi^2 s - 4H\psi s + L^2\psi^2 + 2L^2\psi - 4L^2 + 2L\psi s}{2(H-L)(\psi-2)(\psi+2)} \tag{D.19}$$

$$P_L = \frac{L(-H\psi^2 + 2H\psi - 4H + L\psi^2 - 2L\psi + 4L + 2\psi^2 s - 2\psi s)}{2(H-L)(\psi-2)(\psi+2)} \tag{D.20}$$

$$P_B = \frac{(-HL\psi^2 + 2HL\psi - 4HL + 2H\psi^2 s - 8Hs + L^2\psi^2 - 2L^2\psi + 4L^2 - 2L\psi s + 8Ls)}{2(H-L)(\psi-2)(\psi+2)} \tag{D.21}$$

$$a_A = \frac{2H - 2L + \psi s}{(H-L)(\psi+2)} \tag{D.22}$$

Where again a_A approaches 1 as $\psi \to 0$.

E For Online Appendix: Many Firms in a full information consumer search

There are $h+l$ firms in a market, h of which sell a product of quality H (the high firms) and l of which sell goods with a lower quality of L (the low firms), where $H > L$. We assume $h > 1$ and $l > 1$ and that all firms produce their goods costlessly. We make the assumption that all firms of a particular quality level sell at the same price. This is done in order for this analysis to arrive at a similar equilibrium to that of section 3.2. With these assumption lemmas 1, corollary 2 and lemma 3 carry over with the only modification that directed consumers approach a quality level rather than a specific firm. As before we will denote the ex ante indifferent consumer a_A so consumers with a lower taste approach a low firm and higher taste consumers approach high firms.

$$\begin{aligned} Q_{\text{High}}(P_H) = & \overbrace{\frac{\psi}{h+l}[1-\max(a_{HL}(P_H), a_H(P_H))]_+}^{\text{Undirected - own}} + \overbrace{\frac{\psi l}{h(h+L)}\left[1-\max(a_A+\frac{s}{H-L}, a_{HL}(P_H), a_H(P_H))\right]_+}^{\text{Undirected runoff from the } L \text{ firm}} \\ & + \overbrace{\frac{1-\psi}{h}[1-\max(a_A, a_{HL}(P_H), a_H(P_H))]_+}^{\text{Directed}} \end{aligned} \tag{E.1}$$

The demand function for the low firm can be written as:

$$\begin{aligned} Q_{\text{Low}}(P_L) = & \overbrace{\frac{\psi}{h+l}[\min(1, a_T(P_L)) - a_L(P_L)]_+}^{\text{Undirected - own}} + \overbrace{\frac{\psi l}{h(h+l)}\left[\min(a_A-\frac{s}{H-L}, a_T(P_L)) - \max(a_{HLM}, a_L(P_L))\right]_+}^{\text{Undirected Runoff from the } H \text{ firm}} \\ & + \overbrace{\frac{1-\psi}{h}[\min(a_A, a_T(P_L)) - a_L(P_L)]_+}^{\text{Directed}} \end{aligned} \tag{E.2}$$

As before we examine the case when the search costs are sufficiently low that the high firm's marginal consumer is indifferent to the low firm.

$$\begin{aligned} Q_{\text{High}}(P_H) = & \overbrace{\frac{\psi}{h+l}[1-a_H(P_H)]}^{\text{Undirected - own}} + \overbrace{\frac{\psi l}{h(h+l)}\left[1-(a_A+\frac{s}{H-L})\right]}^{\text{Undirected runoff from the } L \text{ firm}} \\ & + \overbrace{\frac{1-\psi}{h+l}[1-a_A]}^{\text{Directed}} \end{aligned} \tag{E.3}$$

E.1 Case A: $a_T < 1$

Now assuming $a_T < 1$ we can rewrite the low firm's demand function as:

$$Q_{\text{Low}}(P_L) = \overbrace{\frac{\psi}{2}\left[a_T(P_L) - a_L(P_L)\right] +}^{\text{Undirected - own}} \overbrace{\frac{\psi}{2}\left[a_A - \frac{s}{H-L} - a_{HLM}\right]}^{\text{Undirected Runoff from the } H \text{ firm}} \tag{E.4}$$

And we can solve to get:

$$\begin{aligned}
P_H &= \frac{2H^2h^2 + 3H^2hl\psi + H^2l^2\psi^2 + HLh^2\psi - 3HLh^2 + HLhl\psi^2 - 4HLhl\psi - HLl^2\psi^2 + 2Hh^2\psi s + Hhl\psi^2 s - 3Hhl\psi s - 2Hl^2\psi^2 s - L^2h^2\psi + L^2h^2 - L^2hl\psi^2 + L^2hl\psi + Lh^2\psi^2 s + Lhl\psi s}{2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + Hl^2\psi^2 + Lh^2\psi^2 - 2Lh^2 + Lhl\psi^2 - 3Lhl\psi - Ll^2\psi^2} \\
P_L &= \frac{HLh^2 + 2HLhl\psi + HLl^2\psi^2 - Hhl\psi^2 s - Hhl\psi s - Hl^2\psi^2 s - L^2h^2 - 2L^2hl\psi - L^2l^2\psi^2 + Lh^2\psi^2 s + 2Lh^2\psi s + 2Lhl\psi^2 s - Lhl\psi s - Ll^2\psi^2 s}{2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + Hl^2\psi^2 + Lh^2\psi^2 - 2Lh^2 + Lhl\psi^2 - 3Lhl\psi - Ll^2\psi^2} \\
\pi_H &= \frac{\psi\left(2H^2h^2 + 3H^2hl\psi + H^2l^2\psi^2 + HLh^2\psi - 3HLh^2 + HLhl\psi^2 - 4HLhl\psi - HLl^2\psi^2 + 2Hh^2\psi s + Hhl\psi^2 s - 3Hhl\psi s - 2Hl^2\psi^2 s - L^2h^2\psi + L^2h^2 - L^2hl\psi^2 + L^2hl\psi + Lh^2\psi^2 s + Lhl\psi s\right)^2}{(H-L)(h+l)\left(2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + Hl^2\psi^2 + Lh^2\psi^2 - 2Lh^2 + Lhl\psi^2 - 3Lhl\psi - Ll^2\psi^2\right)^2} \\
\pi_L &= \frac{(H + L\psi - L)\left(HLh^2 + 2HLhl\psi + HLl^2\psi^2 - Hhl\psi^2 s - Hhl\psi s - Hl^2\psi^2 s - L^2h^2 - 2L^2hl\psi - L^2l^2\psi^2 + Lh^2\psi^2 s + 2Lh^2\psi s + 2Lhl\psi^2 s - Lhl\psi s - Ll^2\psi^2 s\right)^2}{L(H-L)(h+l)\left(2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + Hl^2\psi^2 + Lh^2\psi^2 - 2Lh^2 + Lhl\psi^2 - 3Lhl\psi - Ll^2\psi^2\right)^2} \\
a_A &= \frac{2Hh^2 + 3Hhl\psi + Hl^2\psi^2 + Lh^2\psi - 2Lh^2 + Lhl\psi^2 - 3Lhl\psi - Ll^2\psi^2 + 2h^2\psi s + 2hl\psi^2 s - 2hl\psi s - l^2\psi^2 s}{2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + Hl^2\psi^2 + Lh^2\psi^2 - 2Lh^2 + Lhl\psi^2 - 3Lhl\psi - Ll^2\psi^2} \\
a_T &= -\frac{HLh^2(4-\psi) - H^2l\psi(3h + l\psi) - 2H^2h^2 + HLhl\psi(6-\psi) + 2H(Ll^2\psi^2 + h^2 s) - Hhl\psi s(\psi - 5) + \psi(2Hl^2\psi s + L^2h^2) - L^2h(2h - l\psi^2) - L^2l\psi(3h + l\psi) + Lh^2\psi s(\psi + 2) + Lhs(3l\psi^2 - 2h) - Ll\psi s(5H + 2l\psi)}{(H-L)\left(2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + Hl^2\psi^2 + Lh^2\psi^2 - 2Lh^2 + Lhl\psi^2 - 3Lhl\psi - Ll^2\psi^2\right)}
\end{aligned} \tag{E.5}$$

E.2 Case B: $a_T \geq 1$

Now assuming $a_T \geq 1$ we can rewrite the low firm's demand function as:

$$Q_{\text{Low}}(P_L) = \overbrace{\frac{\psi}{2}\left[1 - a_L(P_L)\right]}^{\text{Undirected - own}} + \overbrace{\frac{\psi}{2}\left[a_A - \frac{s}{H-L} - a_{HLM}\right]}^{\text{Undirected Runoff from the } H \text{ firm}} \tag{E.6}$$

And we can solve to get:

$$
\begin{aligned}
P_H &= \frac{2H^2h^2 + 3H^2hl\psi + H^2l^2\psi^2 - 3HLh^2 - 4HLhl\psi - HLl^2\psi^2 + 2Hh^2\psi s + Hhl\psi^2 s - 3Hhl\psi s - 2Hl^2\psi^2 s + L^2h^2 + L^2hl\psi - Lh^2\psi^2 s - Lh^2\psi s + Lhl\psi^2 s + Lhl\psi s}{2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + Hl^2\psi^2 - Lh^2\psi^2 - Lh^2\psi - 2Lh^2 - 3Lhl\psi - Ll^2\psi^2} \\
P_L &= -\frac{-HLh^2\psi^2 - HLh^2 - 2HLhl\psi - HLl^2\psi^2 + Hhl\psi^2 s + Hhl\psi s + Hl^2\psi^2 s + L^2h^2\psi^2 + L^2h^2 + 2L^2hl\psi + L^2l^2\psi^2 + Lh^2\psi^2 s - Lh^2\psi s - 2Lhl\psi^2 s + Lhl\psi s + Ll^2\psi^2 s}{2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + Hl^2\psi^2 - Lh^2\psi^2 - Lh^2\psi - 2Lh^2 - 3Lhl\psi - Ll^2\psi^2} \\
\pi_H &= \frac{\psi\left(2H^2h^2 + 3H^2hl\psi + H^2l^2\psi^2 - 3HLh^2 - 4HLhl\psi - HLl^2\psi^2 + 2Hh^2\psi s + Hhl\psi^2 s - 3Hhl\psi s - 2Hl^2\psi^2 s + L^2h^2 + L^2hl\psi - Lh^2\psi^2 s - Lh^2\psi s + Lhl\psi^2 s + Lhl\psi s\right)^2}{(H-L)(h+l)\left(2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + Hl^2\psi^2 - Lh^2\psi^2 - Lh^2\psi - 2Lh^2 - 3Lhl\psi - Ll^2\psi^2\right)^2} \\
\pi_L &= \frac{\left(-HLh^2\psi^2 - HLh^2 - 2HLhl\psi - HLl^2\psi^2 + Hhl\psi^2 s + Hhl\psi s + Hl^2\psi^2 s + L^2h^2\psi^2 + L^2h^2 + 2L^2hl\psi + L^2l^2\psi^2 + Lh^2\psi^2 s - Lh^2\psi s - 2Lhl\psi^2 s + Lhl\psi s + Ll^2\psi^2 s\right)^2}{L(h+l)\left(2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + Hl^2\psi^2 - Lh^2\psi^2 - Lh^2\psi - 2Lh^2 - 3Lhl\psi - Ll^2\psi^2\right)^2} \\
a_A &= \frac{2H^2h^2 + 3H^2hl\psi + H^2l^2\psi^2 - HLh^2\psi^2 - 4HLh^2 - 6HLhl\psi - 2HLl^2\psi^2 + 2Hh^2\psi s + 2Hhl\psi^2 s - 2Hhl\psi s - Hl^2\psi^2 s + L^2h^2\psi^2 + 2L^2h^2 + 3L^2hl\psi + L^2l^2\psi^2 - 2Lh^2\psi s - Lhl\psi^2 s + 2Lhl\psi s + Ll^2\psi^2 s}{(H-L)\left(2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + Hl^2\psi^2 - Lh^2\psi^2 - Lh^2\psi - 2Lh^2 - 3Lhl\psi - Ll^2\psi^2\right)} \\
a_T &= -\frac{HLh(4h + 6l\psi) - H\psi^2(Hl^2 - Lh^2) - H^2h(2h + 3l\psi) + 2H(Ll^2\psi^2 + h^2 s) + Hhl\psi s(5-\psi) + 2Hl^2\psi^2 s - L^2h^2\psi^2 - 2L^2h^2 - 3L^2hl\psi - L^2l^2\psi^2 - Lh^2\psi^2 s + Lh^2\psi s - 2Lh^2 s + Lhl\psi^2 s - 5Lhl\psi s - 2Ll^2\psi^2 s}{(H-L)\left(2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + Hl^2\psi^2 - Lh^2\psi^2 - Lh^2\psi - 2Lh^2 - 3Lhl\psi - Ll^2\psi^2\right)}
\end{aligned}
\tag{E.7}
$$

E.3 As $\psi \to 0$

As $\psi \to 0$ we get a_A as defined by equation E.5 approaching 1.[30] This indicates that as $\psi \to 0$ we get a_T going above a_A and hence the second case will arise. As $\psi \to 0$ we get a_A as defined by equation E.7 approaching 1.

Thus in a similar result to proposition 3, the low quality firms take over the market in the full information as the number of directed consumers increases.

[30] This can be seen by substituting $\psi = 0$ into the equation and noting the feasible range of a_A is the closed interval between 0 and 1.

F For Online Appendix: Three levels of quality in the market

The top firm has a quality of H. The middle firm has a quality of M and the lower firm has a quality of L. The ex ante indifferent consumers between the low and middle quality firms have a taste denoted a_A while the ex ante indifferent consumers between the high and middle quality firms have a taste denoted a_{A2}. There are γ firms in total, β lower quality firms and α middle quality firms and so $\gamma - \alpha - \beta$ high quality firms.

By setting up expressions similar to equation 6 and rearranging, we can derive expressions for the indifferent taste consumers at each firm. These can be seen in table F.1:

Notation	Location of consumer	Indifferent between	Formula
a_A	Ex Ante	Going to medium and low firm	$\frac{P_M^E - P_L^E}{M-L}$
a_{A2}	Ex Ante	Going to high and medium firm	$\frac{P_H^E - P_M^E}{H-M}$
a_H	High firm	Buying and going to medium firm	$\frac{P_H - P_M^E - s}{H-M}$
a_{HLeave}	High firm	Going to low firm and leaving market	$\frac{P_L^E + s}{L}$
a_{MH}	Medium firm	Buying and going to high firm	$\frac{P_H^E - P_M + s}{H-M}$
a_M	Medium firm	Buying and going to low firm	$\frac{P_M - P_L^E - s}{M-L}$
a_{MLeave}	Medium firm	Going to low firm and leaving market	$\frac{P_L^E + s}{L}$
a_{LM}	Low firm	Buying and going to medium firm	$\frac{P_M^E - P_L + s}{M-L}$
a_L	Low firm	Buying and leaving market	$\frac{P_L}{L}$

Table F.1: Indifferent Consumers for model with three quality levels

Now we will examine the putative equilibria where s is small. We shall suppose that the search cost is sufficiently low that all of the following hold. The best outside for the marginal consumer at a high firm is to go to a medium firm, the best outside option for the highest taste marginal consumer at a medium firm is to go to a high firm. The best outside option for the bottom taste marginal consumer at a medium firm is to go to a low firm. The top and bottom marginal consumers at the low firm will go to a medium firm/ leave market respectively.

We write profit functions for each firm starting with the low firms:

$$
\begin{aligned}
\text{Quantity}_L(P_L) = & \overbrace{\frac{\psi}{\gamma}[\min(1, a_{LM}(P_L)) - a_L(P_L)]_+}^{\text{Undirected - own}} + \overbrace{\frac{\psi\frac{\alpha}{\gamma}}{\beta}\left[a_A - \frac{s}{M-L} - a_L(P_L)\right]_+}^{\text{Undirected Runoff from the middle firms}} \\
& + \overbrace{\frac{\psi\frac{\gamma-\beta-\alpha}{\gamma}}{\beta}[a_A - a_L(P_L)]_+}^{\text{Undirected Runoff from the top firms}} + \overbrace{\frac{(1-\psi)}{\beta}[a_A - a_L(P_L)]_+}^{\text{Directed}}
\end{aligned}
$$

Now writing the demand function for the medium firms:

$$
\begin{aligned}
\text{Quantity}_M(P_M) = & \overbrace{\frac{\psi}{\gamma}[\min(1, a_{MH}(P_M)) - a_M(P_M)]_+}^{\text{Undirected - own}} + \overbrace{\frac{\psi\frac{\beta}{\gamma}}{\alpha}\left[a_{A2} - \max(a_A + \frac{s}{M-L}, a_M(P_M))\right]_+}^{\text{Undirected Runoff from the bottom firms}} \\
& + \overbrace{\frac{\psi\frac{(\gamma-\alpha-\beta)}{\gamma}}{\alpha}\left[a_{A2} - \frac{s}{H-M} - \max(a_A, a_M(P_M))\right]_+}^{\text{Undirected Runoff from the top firms}} \\
& + \overbrace{\frac{(1-\psi)}{\alpha}[a_{A2} - \max(a_A, a_M(P_M))]_+}^{\text{Directed}}
\end{aligned}
$$

Now writing the demand function for the high firms:

$$
\begin{aligned}
\text{Quantity}_H(P_H) = & \overbrace{\frac{\psi}{\gamma}[1 - a_H(P_H)]_+}^{\text{Undirected - own}} + \overbrace{\frac{\psi\frac{\beta}{\gamma}}{\gamma-\alpha-\beta}[1 - \max(a_{A2}, a_H(P_H))]_+}^{\text{Undirected Runoff from the bottom firms}} \\
& + \overbrace{\frac{\psi\frac{\alpha}{\gamma}}{\gamma-\alpha-\beta}\left[1 - \max(a_{A2} + \frac{s}{H-M}, a_H(P_H))\right]_+}^{\text{Undirected Runoff from the medium firms}} \\
& + \overbrace{\frac{(1-\psi)}{\gamma-\alpha-\beta}[1 - \max(a_{A2}, a_H(P_H))]_+}^{\text{Directed}}
\end{aligned}
$$

F.1 Algebra

The sympy package of python was used to solve for equilibrium values $P_L, P_M, P_H, a_A, a_{A2}$. The code to do this can be seen on the following pages. It can be found that in the case $a_{MH} < 1$ & $a_{LM} < 1$, the case $a_{MH} < 1$ & $a_{LM} < 1$ and the case $a_{MH} > 1$ & $a_{LM} > 1$ we have $a_A \to 1$ and $a_{A2} \to 1$ as $\psi \to 0$.[31]

[31] Output of the code is not shown as the expressions are complicated and occupy many pages. Should you wish to replicate these results please find the python code on the author's (Baumann) website.

```
'''
Paper: It's Good to be Bad: A model of low quality dominance in a full information consumer search
    market
Author: Stuart Baumann & Margaryta Klymak
Date: 6th of April 2017
Comments: This code solves the model where there are three quality levels as occurs when there are many
firms only some of which may disclose. Version 1.0 of sympy is used.
'''
# Preparing workspace and declaring variables
%reset -f
import sympy as sy
H, M, L, psi, a_A, a_A2, s, p_L, p_M, p_H, alpha, beta, gamma = sy.symbols('H M L psi a_A a_{A2} s p_L
    p_M p_H alpha beta gamma')

# Writing Function to solve for equilibrium.
def GetEquilibrium(LowDemandExpr, MedDemandExpr, HighDemandExpr):
    global H, M, L, psi, a_A, a_A2, s, p_L, p_M, p_H, alpha, beta, gamma
    # Getting Reaction Functions
    LowPi                  = LowDemandExpr*p_L
    LowReactionFunction  = next(iter(sy.solveset(sy.diff(LowPi, p_L),p_L)))
    MedPi                  = MedDemandExpr*p_M
    MedReactionFunction  = next(iter(sy.solveset(sy.diff(MedPi, p_M),p_M)))
    HighPi                 = HighDemandExpr*p_H
    HighReactionFunction = next(iter(sy.solveset(sy.diff(HighPi, p_H),p_H)))

    # Ex Ante Indifferent Consumers
    ExAnteIndifferent = {a_A: (p_M-p_L)/(M-L),  a_A2: (p_H-p_M)/(H-M)  }
    # Sub in a_A and a_A2
    LowFunction2  = LowReactionFunction.subs(ExAnteIndifferent)
    MedFunction2  = MedReactionFunction.subs(ExAnteIndifferent)
    HighFunction2 = HighReactionFunction.subs(ExAnteIndifferent)

    # Solving to get each as function of other two
    LowFunction3  = next(iter(sy.solveset(LowFunction2  - p_L,p_L)))
    MedFunction3  = next(iter(sy.solveset(MedFunction2  - p_M,p_M)))
    HighFunction3 = next(iter(sy.solveset(HighFunction2 - p_H,p_H)))

    # Eliminate Low price from other two.
    MedFunction4  = next(iter(sy.solveset(MedFunction3.subs(p_L, LowFunction3)  - p_M,p_M)))
    HighFunction4 = next(iter(sy.solveset(HighFunction3.subs(p_L, LowFunction3)  - p_H,p_H)))

    # Sub MedFunction4 into HighFunction4 to get expression for high price. Then get medium and low
        prices
    RawHighPrice = next(iter(sy.solveset(HighFunction4.subs(p_M, MedFunction4)                      - p_H
        ,p_H)))
    RawMedPrice  = next(iter(sy.solveset( MedFunction4.subs(p_H, RawHighPrice)                       - p_M
        ,p_M)))
    RawLowPrice  = next(iter(sy.solveset( LowFunction3.subs({p_H:RawHighPrice, p_M:RawMedPrice})  - p_L
        ,p_L)))

    # Simplifying prices
    LowPrice = sy.factor(sy.simplify(RawLowPrice))
    MedPrice = sy.factor(sy.simplify(RawMedPrice))
    HighPrice = sy.factor(sy.simplify(RawHighPrice))

    # Getting expressions for ex ante indifferent consumers
    a_AExpr = sy.factor(sy.simplify((MedPrice - LowPrice)/(M-L)))
    a_A2Expr = sy.factor(sy.simplify((HighPrice - MedPrice)/(H-M)))
```

```
    # What do a_A and a_A2 tend to as $psi \rightarrow 0$. Note that direct substitution is ok
    # as expressions will be continuous
    a_ALowPsi = sy.factor(sy.simplify(a_AExpr.subs(psi,0)))
    a_A2LowPsi = sy.factor(sy.simplify(a_A2Expr.subs(psi,0)))

    # Dictionary To Return
    EquilibriumValues = {'p_L': LowPrice, 'p_M': MedPrice, 'p_H': HighPrice, 'a_A': a_AExpr, 'a_A2':
        a_A2Expr,
    'a_ALimit': a_ALowPsi, 'a_A2Limit': a_A2LowPsi  }

    # Returning Equilibrium Values
    return(EquilibriumValues)

# Case 1: This is guessing for the low and medium firms that the 1 never binds in the min function.
LowDemandExpr  = (psi/gamma)*((p_M - p_L + s)/(M-L) - p_L/L ) + ((psi*alpha)/(gamma*beta))*(a_A - s/(M-
    L)- p_L/L) + ((psi*(gamma-beta-alpha))/(gamma*beta))*(a_A - p_L/L) + ((1-psi)/(beta))*(a_A - p_L/L
    )
MedDemandExpr  = (psi/gamma)*((p_M - p_H - s)/(M-H) - (p_M - p_L - s)/(M-L)) + ((psi*beta)/(gamma*alpha
    ))*(a_A2 -a_A - s/(M-L) ) + ((psi*(gamma-beta-alpha))/(gamma*alpha))*(a_A2 - s/(H-M) - a_A) + ((1-
    psi)/(alpha))*(a_A2 - a_A)
HighDemandExpr = (psi/gamma)*(1 - (p_H - p_M - s)/(H-M)) + ((psi*beta)/(gamma*(gamma-alpha-beta)))*(1 -
     a_A2 ) + ((psi*alpha)/(gamma*(gamma-alpha-beta)))*(1- a_A2 - s/(H-M)) + ((1-psi)/(gamma-alpha-
    beta))*(1 - a_A2)
EqCase1 = GetEquilibrium(LowDemandExpr, MedDemandExpr, HighDemandExpr)

# Case 2: We have a_{MH} > 1 but a_{LM} < 1
LowDemandExpr  = (psi/gamma)*((p_M - p_L + s)/(M-L) - p_L/L ) + ((psi*alpha)/(gamma*beta))*(a_A - s/(M-
    L)- p_L/L) + ((psi*(gamma-beta-alpha))/(gamma*beta))*(a_A - p_L/L) + ((1-psi)/(beta))*(a_A - p_L/L
    )
MedDemandExpr  = (psi/gamma)*(1 - (p_M - p_L - s)/(M-L)) + ((psi*beta)/(gamma*alpha))*(a_A2 -a_A - s/(M
    -L) ) + ((psi*(gamma-beta-alpha))/(gamma*alpha))*(a_A2 - s/(H-M) - a_A) + ((1-psi)/(alpha))*(a_A2
    - a_A)
HighDemandExpr = (psi/gamma)*(1 - (p_H - p_M - s)/(H-M)) + ((psi*beta)/(gamma*(gamma-alpha-beta)))*(1 -
     a_A2 ) + ((psi*alpha)/(gamma*(gamma-alpha-beta)))*(1- a_A2 - s/(H-M)) + ((1-psi)/(gamma-alpha-
    beta))*(1 - a_A2)
EqCase2 = GetEquilibrium(LowDemandExpr, MedDemandExpr, HighDemandExpr)

# Case 3: We have a_{MH} > 1 and a_{LM} > 1
LowDemandExpr  = (psi/gamma)*(1 - p_L/L ) + ((psi*alpha)/(gamma*beta))*(a_A - s/(M-L)- p_L/L) + ((psi*(
    gamma-beta-alpha))/(gamma*beta))*(a_A - p_L/L) + ((1-psi)/(beta))*(a_A - p_L/L)
MedDemandExpr  = (psi/gamma)*(1 - (p_M - p_L - s)/(M-L)) + ((psi*beta)/(gamma*alpha))*(a_A2 -a_A - s/(M
    -L) ) + ((psi*(gamma-beta-alpha))/(gamma*alpha))*(a_A2 - s/(H-M) - a_A) + ((1-psi)/(alpha))*(a_A2
    - a_A)
HighDemandExpr = (psi/gamma)*(1 - (p_H - p_M - s)/(H-M)) + ((psi*beta)/(gamma*(gamma-alpha-beta)))*(1 -
     a_A2 ) + ((psi*alpha)/(gamma*(gamma-alpha-beta)))*(1- a_A2 - s/(H-M)) + ((1-psi)/(gamma-alpha-
    beta))*(1 - a_A2)
EqCase3 = GetEquilibrium(LowDemandExpr, MedDemandExpr, HighDemandExpr)
```