Do Financing Constraints Kill Creativity? A Bandit Model of Innovation with Endogenous Financing Constraint

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Abstract

Why do small start-ups make most breakthroughs, even though giant enterprises supply most R&D spending? We study a principal-agent model in which the principal chooses the scale of the experiment, and the agent privately observes the outcome realizations and can privately choose the novelty of the project. When the agent has private access to a safe but non-innovative project, the principal starves the agent of funds to incentivise risk-taking. The principal quickly scales up after early successes, and may tolerate early failures. If the principal is equally informed about the outcome, the agent is well-resourced, resembling a large R&D department.

Keywords: innovation, start-ups, financing constraints, moral hazard, bandit problem, principal-agent, exploration, exploitation, arm's length, relationship-based, cash flow diversion, firm dynamics

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1 Introduction

Why do small start-ups make most breakthroughs, even though giant enterprises supply most R&D spending (Baumol, 2010)? Is there such a thing as too much capital when it comes to the financing of innovative projects? Anecdotal evidence suggests that more financial resources do not necessarily lead to more and better innovation. It is surprising how many projects fail, despite pursuing ideas that are eventually successful, with well-resourced competent teams. Nokia, for example, had always been an adaptive company and hardly a technological laggard: its engineers built a prototype of an internet-enabled phone with a touch-screen at the end of the nineties (The New Yorker, 2013). Nokia spent in R&D almost 4 times what Apple spent (The Wall Street Journal, 2012), but saw its market's share fall from 40% in 2007 to 11% in 2014. Similarly, thanks to hefty investment in research, Kodak developed one of the first digital cameras in 1975, and launched a photo-sharing web site in 2001 (The Economist, 2012). However, in January 2012, the company filed for bankruptcy. On the contrary, many successful innovative firms have started small, with limited resources, but enjoyed high growth rates after a few years. Airbnb, Dropbox, and Reddit are among the 1,000 start-ups funded by Y Combinator, an American seed accelerator: Sam Altman, the president, considers frugality "incredibly important for start-ups" (Business Insider UK, 2015) and Jessica Livingston, a co-founder, underlines that "you don't want to give the founders more than they need to survive" (The New Yorker, 2016); Fred Wilson, a venture capitalist with early investments in Twitter and Tumblr, writes on his blog that "less money raised leads to more success" (Wilson, 2013). The reason, Ms. Livingston suggests, is that "being lean forces you to *focus*" (The New Yorker, 2016, emphasis added). In this article, we are interested in understanding the mechanisms through which frugality can help the innovation process. When does an innovative project benefit from a lack of resources?

We offer a two-period principal-agent model of innovation investments, in which a representative investor may use the scale of the experiment to incentivise an entrepreneur with limited liability to be more innovative, and to learn the business potential of a new project. To study how to incentivise the agent to focus on innovation, we assume that the agent privately chooses between *exploring* a new technology that involves a small chance of a breakthrough and a high risk of failure and *exploiting* an old technology with a predictable outcome. To study the trade-off between increasing the *scale* of the project to reap higher profits, and *starving* the agent for incentive reasons, we assume that expected revenues increase in the amount of capital invested. Finally, since entrepreneurs usually have a better idea than investors of whether a new technology works, or whether a new product answers the customers' needs, we assume that the outcome realizations are not directly observable by the principal. We analyse three cases: (i) the full-information case, where the *informed* principal observes both the outcome and the technology used by the agent, (ii) the *relationship-based* financing case, where the *hands-on* principal has the know-how to distinguish between different technologies but cannot observe the outcome realizations, and (iii) the *arm's length* financing case, where the *hands-off* principal can observe neither the type of technology nor the outcome realizations.

We find innovation is best incentivised by *starvation*-contracts when both the agent is better informed than the principal about outcome realizations, and the agent has private access to a safe but non-innovative project. Indeed, providing the agent with less capital (a) minimises the agent's incentives to embezzle revenues, and (b) incentivises risk-taking. A hands-on investor only worries about motive (a): no matter the technology used, truthful reporting on the outcome realizations is incentivised by using a combination of financing constraints, punishments for failures, and rewards for successes. Only a hands-off investor financing an innovative agent needs to consider motive (b) as well: starving an innovative agent minimises her incentives to resort to the more predictable old technology, since this is initially more likely to succeed than the novel one.

We also find that starvation-contracts can be welfare improving. As a measure of social welfare, we consider the total expected payoff of the match between the investor and the entrepreneur. By comparing the optimal innovation contract under arm's length and relationship-based financing, one can notice the following differences. On the one hand, an innovative entrepreneur faces initially stronger credit constraints under arm's length financing than under relationship-based financing, and this decreases social welfare. On the other, to prevent the agent from resorting to the safer project, the hands-off principal needs to tolerate early failures and reward long-term successes, as new technologies are more volatile and may take longer to become successful. As a consequence, entrepreneurs engaging in riskier projects retain access to capital even after a failure in the first period, while they would have been terminated under a hands-on principal; additionally, successful innovative ventures are significantly scaled-up at the beginning of the second period. Both these effects increase social surplus. We show that there exists a parameter space for which the positive effects are greater than the negative one. As a consequence, expected welfare may increase in the degree of informational asymmetry between entrepreneurs and financiers.

We derive some implications of our model for firm dynamics. The relationship between a firm's innovative activities and its growth rate is not straightforward in the empirical literature. Our model suggests that such relationship is influenced by the type of financing the firm has access to, or equivalently by the degree of informational asymmetry between the principal and the agent, and by the riskiness of the project. In particular, we find that innovative firms start smaller and grow faster than conventional firms only under arm's length financing, more significantly so for riskier projects. As far as we know, this finding has not been tested empirically yet.

Our results show that the best way to incentivise innovation varies across different economic environments. When the principal is equally (or better) informed about the outcome of the experiment, the agent is well-resourced, resembling a large R&D department; on the contrary, when the researchers can spend their time on safe but non-innovative alternatives and they have a better idea than investors of whether a new technology answers the customers' needs, lean, fast-growing start-ups are optimal. Consider, for example, the hugely successful Google Maps, and the now scrapped Google Wave. Both were conceived by Lars and Jens Rasmussen. However, Where 2, the start-up that would become Google Maps, was based in the spare bedroom of one of the co-founders (Copeland and Savoia, 2011) and had minimal resources: according to CNN (2009), the Rasmussen brothers only had \$16 between them when they sold their app to Google. After joining Google, they insisted on creating a start-up-like team within Google to develop Wave; moreover, differently from other in-house projects, they were allowed almost limitless autonomy, secrecy, and plentiful resources (CNN, 2009; Copeland and Savoia, 2011). Nevertheless, the project was scrapped after a long runway. Our model suggests that one of the reasons for the success of Maps and the failure of Wave is in the different incentives. The frugal Where 2 provided the optimal high-stakes situation for the development of Maps (CNN, 2009). Conversely, the combination of secrecy and abundant resources did not work well for Wave: perhaps either a large budget coupled with a hands-on management by Google, or the same level of independence but with scarcer resources, would have worked better.

The remainder of the article is organized as follows. Section 2 puts our model in the context of the relevant theoretical literature, and our theoretical predictions in the context of the empirical literature. Section 3 outlines the technical details of the model, and provides the full-information case. Section 4 derives optimal arrangements under the relationship-based financing case. Section 5 considers the full double moral hazard model, and derives the optimal contracts offered by a hands-off principal. Section 6 uses numerical simulations to derive implications for social welfare and firm dynamics. Finally, Section 7 concludes.

2 Related Literature

In this section, we firstly analyse how our model fits in the related theoretical literature on innovation investments, and in particular in the entrepreneurial finance literature and in the optimal experimentation literature. We then show that many of our theoretical results find support in the empirical literature.

This paper is related to two strands of theoretical literature on innovation investments. First, a number of papers in the entrepreneurial finance literature have analysed the financing of a research project under different agency problems. Bergmann and Friedl (2008) provide the optimal contract when an R&D manager has the incentive to under-report the true profitability of an innovative project. In Bergemann and Hege (1998, 2005) and DeMarzo and Sannikov (2017) both sides learn about the quality of the project, but the agent can distort the principal's perception of profitability by shirking or privately reducing the amount invested. Takalo and Tanayama (2010) study the interaction of private and public funding of innovative projects when there are financing constraints arising from adverse selection problems. In all these papers, however, the agent has access to only one project: as a consequence, there is no tension between exploration and exploitation.

Second, following Weitzman (1979), the optimal experimentation literature has incorporated this tension between exploration and exploitation in the innovation process using bandit problems, which allow for knowledge acquisition through experimentation. Manso (2011), whose model is the closest to ours, examines a two-period model where in each period the agent, uncertain about the true distribution of the available actions' payoffs, can choose between shirking, using a new untested action or a well-known process. The agency problem arises because the three actions are associated with different private levels of effort. The optimal contract may reward early failures and compensation depends not only on total performance but also on the path of performance. Ederer (2016) extends this framework to a multi-agent situation. Hörner and Samuelson (2013) and Klein (2016) study related models in continuous-time. In Drugov and Macchiavello (2014) and Halac, Kartik and Liu (2016), the principal-agent model involves both adverse selection on agent's ability and dynamic moral hazard, while Gomes, Gottlieb and Maestri (2016) do not consider moral hazard but two-dimensional adverse selection, as agents have private information about the payoff from both exploration and exploitation.[1]

Whereas Manso's (2011) and related models apply better to mature firms, our focus is on understanding which contracts are better suited to small independent start-ups.[2] The main differences are that start-ups are characterised by minimal resources and hands-off management by investors. In Manso (2011), the size of the project is exogenously assumed away, and agents may be uninformed about their performance. In contrast, in our model the scale of the project is endogenously chosen by the principal, and the termination decision is made partly based on information that the agent provides to the venture capitalists. As a consequence, Manso-style contracts (i.e. well-resourced firms) are optimal when the principal and the agent are equally informed about the outcome realizations, but resource-starved start-ups are optimal when the agent is better informed.

Many of our theoretical results find support in the empirical literature. Firstly, evidence that the combination of tolerance for early failure and reward for long-term success is effective in motivating innovation is provided by Azoulay, Manso and Zivin (2011) and Tian and Wang (2014), using naturally occurring data, and Ederer and Manso (2013), by exogenously varying compensation scheme in an experimental setting.[3] Secondly, Atanassov (2016) finds that firms with a greater proportion of arm's length financing have more and better (i.e. more cited) patents, perhaps because arm's length financing allows greater flexibility and tolerance to experimentation than relationship-based bank financing. Thirdly, while there is no general consensus, many empirical articles support the "less money, better innovation" argument (Hall, Moncada-Paternò-Castello, Montresor and Vezzani, 2016), that financing constraints can have a disciplinary effect on innovative investments, by limiting the moral hazard problem and forcing firms to focus on more productive and value-enhancing innovation. Almeida, Hsu and Li (2013) find that firms that are more likely to be constrained generate more patents and citations

[1] Differently from the above papers, we consider a dynamic double moral hazard problem. On the one hand, the agent may privately choose the project, similarly to Manso (2011). On the other, the agent privately observes the outcomes, similarly to Bolton and Scharfstein (1990) and Clementi and Hopenhayn (2006).

[2] Drugov and Macchiavello (2014) consider the case in which the borrower can experiment by starting small. However, the size of the first experiment is exogenous in their framework.

[3] In the theoretical literature, similar results are reached by Holmstrom (1989), Manso (2011), and Hörner and Samuelson (2013). This is consistent with headlines like *Fail to Succeed* (Wired UK, 2011), *Fail Often, Fail Well* (The Economist, 2011), and *Google's Greatest Strength May Be the Luxury of Failure* (The Wall Street Journal, 2013). Indeed, the tolerance of failure has become a dominant theme in the Silicon Valley, where, for example, FailCon, a conference about embracing failure, was launched in 2009, and while it has been an annual event for several years, it was recently cancelled because "failure chatter is now so pervasive in Silicon Valley that a conference almost seems superfluous" (The New York Times, 2014).

per unit of R&D investment and per employee; in Li (2011) a positive relation between R&D investment and stock returns exists only among financially constrained firms. Dryden, Nickell and Nicolitsas (1997), Nickell and Nicolitsas (1999), and Musso and Schiavo (2008) show that financing constraints are positively related with productivity growth.[4] Keupp and Gassmann (2013) provide partial support for the hypothesis that financial constraints spur radical innovations. Schäfer, Stephan and Mosquera (2015) find that family business are more likely to be constrained but have the same level of innovation outcomes as non family firms, perhaps due to more efficient resource utilization. Lahr and Mina (2013) find that while innovation activities seem to cause financial constraint, the reverse effect appears negligible.

Finally, we find that the relationship between innovation investments and firm growth depends on the type of financing the firm has access to, or equivalently, on the degree of informational asymmetry between the financiers and the entrepreneur. As far as we know, this theoretical prediction has not been tested empirically. The applied literature suggests that the relationship between innovation and firm growth is usually positive but not straightforward (Coad, 2009). For example, Coad and Rao (2008) note that the relationship between innovative activities and firm growth is positive among the fastest growing firms, but it can be negative for others; Demirel and Mazzucato (2012) find that R&D boosts growth only for a subset of small pharmaceutical firms; Segarra and Teruel (2014) and Mazzucato and Parris (2015) argue that the effect of R&D on firm growth differs between industries and competitive environments.

3 The Model

An entrepreneur (the agent) has the ability to operate risky projects but needs a fixed initial investment $I > 0$ to acquire an enabling asset. The agent has no wealth of her own. A representative investor (the principal) has the necessary deep pockets but lacks the entrepreneurial ability. Together they form a firm to run the projects. Both the entrepreneur and the investor are risk neutral and discount future cash flows using the same discount factor, normalized to 1. Both are able to commit to a long-term contract, in the sense that, if they both sign the contract, they abide by it in every circumstance. We assume that the entrepreneur has limited liability and a reservation payoff equal to zero, so that she will never voluntarily quit.

[4]In the theoretical literature, a similar argument is advanced by Jensen (1986) and Aghion, Dewatripont and Rey (1999).

There are two periods and two possible outcomes in each period. In the first period, the agent requires the principal to provide the initial investment I and the investment of working capital. In the second period, only the working capital is required. The collaboration terminates at the end of the second period. Once the firm is formed, the agent can decide whether to rely on a well-known project C (for conventional) or to explore a new technology N (for novel) whose probability of success is unknown.[5] Both work methods yield revenues that are subject to shocks and increase with the amount of working capital invested. In particular, the well-known project is successful with probability π_C and fails with the remaining probability $1 - \pi_C$, yielding zero revenues. If successful, the outcome of the project is positive and given by $R(k_t)$, where k_t represents the amount of working capital invested in the project in period t (its scale) and the function R is assumed continuous, strictly concave and uniformly bounded from above. The results in the text use the functional form $R(k) = Ak^{\alpha}$, with $A > 0$ and $0 < \alpha < 1$.[6]

Similarly, the new work method yields either $R(k_t)$ or 0. However, following Manso (2011), we assume that the novel work method has an exploratory nature: its probability of success π_N is unknown. Moreover, when the agent starts experimenting with this new project, she is less likely to succeed than when she relies on the well-known project. Nevertheless, if the experimentation leads to a success, the new method becomes perceived as better than the conventional work method. This is formalized as follows

Assumption A. *(Explorative project)* $0 \leq E[\pi_N|L, N] < E[\pi_N] < \pi_C < E[\pi_N|H, N] \leq 1$

where $E[\pi_i]$ and $E[\pi_i|s, j]$ denote respectively the unconditional expectation of π_i and the conditional expectation of π_i given outcome $s = \{H, L\}$ on action $j = \{C, N\}$. We indicate success with H (for high outcome) and failure with L (for low). Moreover, we assume that the probability of success of the conventional project is known,

Assumption B. *(Ordinary project)* $\pi_C = E[\pi_C] = E[\pi_C|H, C] = E[\pi_C|L, C]$.

Using project i only gives information on the probability of success of the same project, π_i, i.e. $E[\pi_i] = E[\pi_i|H, j] = E[\pi_i|L, j]$ for $j \neq i$.

Following Manso (2011), we define an action plan as $\langle i \rangle_z^j$, where i is the first period action, j is the second period action in case of success and z is the second period action in

[5] In Appendix B we introduce a private cost incurred by the agent when employing the novel approach. This complicates the analysis but does not qualitatively change the conclusions.

[6] The parameter values used to produce the figures in the text are as follows: $\pi_C = 0.4$, $I = 0$, $\alpha = 0.33$, and A is chosen such that the first best scale of the firm when using the conventional project is $k^{FB} = 10,000$. The qualitative results do not hinge on this specific parametrisation.

case of failure. The total expected payoff of the match between investor and entrepreneur by following action plan $\langle i \rangle_z^j$ is given by

$$W\langle i \rangle_z^j = -I + E\left[\pi_i\right] R\left(k_1\langle i \rangle_z^j\right) - k_1\langle i \rangle_z^j + E\left[\pi_i\right]\left\{E\left[\pi_j|H,i\right] R\left(k_H\langle i \rangle_z^j\right) - k_H\langle i \rangle_z^j\right\} + \\ + \left(1 - E[\pi_i]\right)\left\{E\left[\pi_z|L,i\right] R\left(k_L\langle i \rangle_z^j\right) - k_L\langle i \rangle_z^j\right\}$$

where $k_1\langle i \rangle_z^j$ represents the scale of the project in period 1 under the action plan $\langle i \rangle_z^j$ and $k_H\langle i \rangle_z^j$ and $k_L\langle i \rangle_z^j$ are the scales of the project in period 2 contingent on success or failure respectively. In the rest of the article we will refer to $W\langle i \rangle_z^j$ as a measure of social welfare. We consider only two action plans: $\langle C \rangle_C^C$ and $\langle N \rangle_C^N$, which are usually defined as *exploitation* and *exploration* in the bandit problem literature.[7] Exploitation refers to the repetition of the well-known project C, while exploration consists of trying the new project N in the first period, stick to it in case of success, but go back to the conventional project in case of failure.

In what follows, we usually relegate the analytical steps and the proofs to Appendix A.2, together with some more general results.

3.1 Benchmark: the Full-Information Case

Before introducing the moral hazard problems, we solve the bandit problem in isolation. We assume information to be costless and readily available to a benevolent social planner who wishes to maximize the expected net amount of output produced, so that solving this single agent's problem will give us the first best levels of investment and the efficient

[7] In Appendix A.1 we show that any other action plan is dominated by either one of those.

choice between exploration and exploitation.[8]

At the optimum, the expected marginal benefit of providing one unit of working capital must be equal to its constant marginal cost. Under exploitation, the unique solution, $k^{FB}\langle C\rangle_C^C$, is pinned down by the following first order condition (henceforth, FOC)

$$\pi_C R'\left(k^{FB}\langle C\rangle_C^C\right) = 1 \tag{1}$$

where the prime indicates the first derivative. Similarly, under exploration, the first best levels of capital must satisfy the following FOCs

$$E\left[\pi_N\right] R'\left(k_1^{FB}\langle N\rangle_C^N\right) = 1 \tag{2a}$$

$$E\left[\pi_N | H, N\right] R'\left(k_H^{FB}\langle N\rangle_C^N\right) = 1 \tag{2b}$$

$$\pi_C R'\left(k_L^{FB}\langle N\rangle_C^N\right) = 1. \tag{2c}$$

Given Assumption A and the strict concavity of R, it follows that

$$k_1^{FB}\langle N\rangle_C^N < k_L^{FB}\langle N\rangle_C^N = k^{FB}\langle C\rangle_C^C < k_H^{FB}\langle N\rangle_C^N.$$

Under exploitation, the efficient scale of the project is the same across periods and outcomes. When it is socially optimal to explore, the lender provides a lower amount of capital than under exploitation in the first period because the expected probability of success is lower. If the project fails, the agent stops experimenting and reverts to the well-known project in the second period, using the unconstrained efficient amount of capital. On the contrary, after a success, the novel approach becomes perceived as more

[8] The informed social planner's problem is equivalent to a setting in which there is no information asymmetry on the cash flow realization between principal and agent, independently of the investor's ability of observing the action plan chosen by the agent. Consider the case in which the principal can observe and verify both the outcome realizations and the action plan chosen by the agent. Since both agents are risk neutral and have the same discount factor, the optimal contract maximizes the total expected discounted profits of the match and this is achieved by having the investor lends the unconstrained efficient amount of capital in both periods. Any division of the surplus that gives a non negative amount to the entrepreneur is feasible. In Appendix A.3 we show that the contract is first best even if the action plan is not observable by the principal: indeed, there are no profitable deviations for the agent when the contract can be made contingent on outcome realizations. Appendix B shows that the provision of capital is first best even if the agent incurs a private cost when employing the novel approach. In particular, Appendix B.1 tackles a problem similar to Manso (2011), in which the project selection stage is private information of the agent, who incurs a private cost, but the outcome of the project is public information: we confirm Manso's (2011) result that the agent will not be compensated for a success in the first period, but might be for an early failure. In such setting, the scale of the firm does not play a role, as the first best levels of capital are always provided.

likely to succeed than the well-known one, thus providing more capital is efficient.

4 The Relationship-Based Financing Case

Here, we add one moral hazard problem to the model of the previous section. The agency problem arises because it is impossible to make the contract explicitly contingent on realized outcomes, as such outcomes are private information for the agent.[9,10] However, we assume that the investor has the know-how to distinguish between the two technologies: the principal can observe and verify the work method employed by the entrepreneur, and thus the type of project is contractible. Since in this setting the principal acquires significant information about the firm, we will refer to this situation as relationship-based financing, and to such an investor as hands-on. In Section 5 we will relax this assumption and compare the optimal contracts.

At time 0, the investor makes a take-it-or-leave-it offer to the entrepreneur.[11] This offer consists of a contract $\boldsymbol{\sigma}$ which specifies the capital advances $\boldsymbol{k} = \{k_1, k_H, k_L\}$ and the agent's repayment to the lenders $\boldsymbol{\tau}$. Formally, $\boldsymbol{\tau} = \{\tau_H, \tau_L, \tau_{HH}, \tau_{LL}, \tau_{HL}, \tau_{LH}\}$, where τ_i refers to the repayment at the end of period 1 after reported outcome $i = \{H, L\}$ and τ_{ij} indicates the repayment at the end of period 2 after reported outcome $i = \{H, L\}$ in period 1 and $j = \{H, L\}$ in period 2. These terms can be contingent on all information provided by the entrepreneur, which consists of a series of report on the outcome realizations. Since these are privately observed by the agent, she can under-report them, diverting the excess cash flow for her own consumption. While we interpret the diversion as *tunneling* (or stealing), other activities could fit the model, such as the entrepreneur receiving non-monetary benefits from projects that benefit her at the expenses of the principal.

Given that the relationship between the entrepreneur and the investor terminates after period 2, in the second period the entrepreneur will always report the outcome

[9]Another formulation of the problem would be to consider an observable and contractible outcome but a non verifiable use of investment. In Clementi and Hopenhayn (2006) the two formulations (where the entrepreneur faces an outside opportunity that provides strictly increasing and concave utility in the amount of capital diverted) turn out to be identical.

[10]One possible interpretation is that only the entrepreneur can observe the outcome of the project. Another potential interpretation is that the cash flow realizations are observable but not verifiable. However, while in our two-period model the two formulations have identical repercussions, in a multi-period model the former would lead to asymmetric information about the probability of success under N, whereas the second interpretation would not.

[11]Thus, all bargaining power lies in the hand of the investor. While this may not be too far from reality for an emerging firm requiring venture capital, it is more likely that neither part has all the bargaining power. We leave this interesting extension to future research.

associated with the lower cost. This implies that the second period repayment must be independent of the second period realization. Thus, $\tau_{iH} = \tau_{iL}$, $i \in \{L, H\}$. However, the second period repayment can be history dependent, i.e. it can depend on the first period reported outcome: let $\tau^i = \tau_{iH} = \tau_{iL}$ be the second period repayment from the entrepreneur if she has reported state $i \in \{L, H\}$ in period 1. Assuming limited liability protects the agent, the repayment in any period cannot be greater than the current cash flow. It follows that conditional on a low report, the payment demanded to the entrepreneur will always be zero.[12] Formally, $\tau_L = \tau^H = \tau^L = 0$ and

$$R\left(k_1\langle i\rangle_z^j\right) - \tau_H\langle i\rangle_z^j \geq 0. \tag{3}$$

The contracting problem is reminiscent of a truth telling equilibrium of a direct mechanism, and indeed the contract must elicit truthful reporting. The relevant incentive compatibility constraint is the one imposing truthful reporting in the good state at the end of the first period,[13]

$$-\tau_H\langle i\rangle_z^j + E\left[\pi_j|H,i\right] R\left(k_H\langle i\rangle_z^j\right) \geq E\left[\pi_z|H,i\right] R\left(k_L\langle i\rangle_z^j\right). \tag{4}$$

This condition requires the continuation payoff of the entrepreneur when she reports the outcome realization truthfully to be at least as high as the payoff from diversion. We also normalize to zero the outside option of the agent: as a consequence, the participation constraint is implied by the incentive compatibility and limited liability constraints.

The optimal contract under relationship-based financing maximizes the expected profits of the investor

$$S\langle i\rangle_z^j = -I + E\left[\pi_i\right]\tau_H\langle i\rangle_z^j - k_1\langle i\rangle_z^j - E\left[\pi_i\right]k_H\langle i\rangle_z^j - \left(1 - E[\pi_i]\right)k_L\langle i\rangle_z^j \tag{5}$$

subject to the limited liability constraint in (3) and the incentive compatibility constraint in (4). Control variables are the levels of capital and the repayment in period 1 after a

[12]Following Clementi and Hopenhayn (2006), we are excluding from the analysis the possibility for the repayments to be negative. It is likely that this is without loss of generality, as positive repayments would be used anyway for incentive reasons.

[13]Having set the repayments in period 2 independent of the outcome in period 2, we can disregard the incentive compatibility constraint for period 2. We also note that there is no incentive for the agent to report the high outcome when the low outcome is actually realized, as the entrepreneur in that case does not have any fund to make the corresponding transfer to the principal. In Section 5 the actions taken by the agent will not be observable. As a consequence, additional incentive compatibility constraints will ensure than neither tunneling nor the alternative action are chosen by the agent.

success.

Before we start considering the two action plans separately, the following Lemmas underline some characteristics of the optimal contract.

Lemma 1. *In the relationship-based financing case, both the limited liability constraint in* (3) *and the incentive compatibility constraint in* (4) *bind at the optimum.*

Lemma 2. *In the relationship-based financing case, it is optimal to set* $k_L\langle i\rangle_z^j = 0$*.*

Lemma 2 implies termination of the project after a failure. The possibility to terminate the project for incentive reasons is well recognized in the literature, and it is a tool that is used by the principal in both Bolton and Scharfstein (1990) and Clementi and Hopenhayn (2006). Cornelli and Yosha (2003) underline that the option to abandon the project is essential as the entrepreneur will most likely never quit a failing project as long as others are providing the capital; it is also a key component of the relationship between the entrepreneur and the venture capitalist (Sahlman, 1990; Kerr et al., 2014).

The upshot of these Lemmas is that the agent just gets an utility equal to the first period return in case of success, and

$$\tau_H\langle i\rangle_z^j = R\left(k_1\langle i\rangle_z^j\right) = E\left[\pi_j|H,i\right]R\left(k_H\langle i\rangle_z^j\right).$$

Indeed, the optimal contract under unobservable outcome is such that, after a success in the first period, the expected payoff in period 2 will act as the carrot that persuades the agent to part with the first period return, and in fact it is exactly equal to this value.

Proposition 1 and Proposition 2 present respectively the optimal exploitation and exploration contract offered by a hands-on principal. These show that the optimal contracts that motivate exploration and exploitation are substantially similar to each other, once adjusted for the different probabilities, and resemble a standard pay-for-performance incentive scheme. Both requires back-loading of the rewards to the entrepreneur and termination of the firm following a failure. These are characteristics that are common to standard contracts used to motivate agent in costly state verification, cash flow diversion and repeated effort models. However, the optimal contracts under unobservable outcome *and actions* derived in Section 5 will be fundamentally different from each other and from those derived here.

Proposition 1. *(Exploitation contract under a hands-on principal)*

The optimal exploitation contract $\boldsymbol{\sigma}^\star\langle C\rangle_C^C$ under a hands-on principal is such that

$$k_H^\star\langle C\rangle_C^C = \left(\frac{A\alpha\pi_C}{1+\pi_C^{\frac{1-\alpha}{\alpha}}}\right)^{\frac{1}{1-\alpha}} \qquad k_1^\star\langle C\rangle_C^C = \pi_C^{\frac{1}{\alpha}} k_H^\star\langle C\rangle_C^C$$
$$k_L^\star\langle C\rangle_C^C = 0 \qquad \tau_H^\star\langle C\rangle_C^C = R\left(k_1^\star\langle C\rangle_C^C\right) = \pi_C R\left(k_H^\star\langle C\rangle_C^C\right).$$

The total expected payoffs of the principal and the agent are, respectively,

$$S^\star\langle C\rangle_C^C = -I + (1-\alpha)\pi_C^2 R\left(k_H^\star\langle C\rangle_C^C\right) \qquad V^\star\langle C\rangle_C^C = \pi_C^2 R\left(k_H^\star\langle C\rangle_C^C\right)$$

and the total expected payoff of the match is given by

$$W^\star\langle C\rangle_C^C = -I + (2-\alpha)\,\pi_C^2 A\left(\frac{A\alpha\pi_C}{1+\pi_C^{\frac{1-\alpha}{\alpha}}}\right)^{\frac{\alpha}{1-\alpha}}.$$

Proposition 2. *(Exploration contract under a hands-on principal)*
The optimal exploration contract $\boldsymbol{\sigma}^\star\langle N\rangle_C^N$ under a hands-on principal is such that

$$k_H^\star\langle N\rangle_C^N = \left(\frac{A\alpha E\left[\pi_N\right]E\left[\pi_N|H,N\right]}{E\left[\pi_N\right]+E\left[\pi_N|H,N\right]^{\frac{1}{\alpha}}}\right)^{\frac{1}{1-\alpha}}$$
$$k_1^\star\langle N\rangle_C^N = E\left[\pi_N|H,N\right]^{\frac{1}{\alpha}} k_H^\star\langle N\rangle_C^N$$
$$k_L^\star\langle N\rangle_C^N = 0$$
$$\tau_H^\star\langle N\rangle_C^N = R\left(k_1^\star\langle N\rangle_C^N\right) = E\left[\pi_N|H,N\right]R\left(k_H^\star\langle N\rangle_C^N\right).$$

The total expected payoffs of the principal and the agent are, respectively,

$$S^\star\langle N\rangle_C^N = -I + (1-\alpha)E\left[\pi_N\right]E\left[\pi_N|H,N\right]R\left(k_H^\star\langle N\rangle_C^N\right)$$
$$V^\star\langle N\rangle_C^N = E\left[\pi_N\right]E\left[\pi_N|H,N\right]R\left(k_H^\star\langle N\rangle_C^N\right),$$

and total welfare is

$$W^\star\langle N\rangle_C^N = -I + (2-\alpha)E\left[\pi_N\right]E\left[\pi_N|H,N\right]A\left(\frac{A\alpha E\left[\pi_N\right]E\left[\pi_N|H,N\right]}{E\left[\pi_N\right]+E\left[\pi_N|H,N\right]^{\frac{1}{\alpha}}}\right)^{\frac{\alpha}{1-\alpha}}.$$

The amount of working capital provided when information is asymmetric is always lower than the first best levels. We thus define the firm as being *credit constrained* in both periods. It can be shown that the credit constraint is relaxed following a success.

Social welfare is higher under exploration, $W^\star\langle N\rangle_C^N > W^\star\langle C\rangle_C^C$, if and only if

$$\frac{E\left[\pi_N\right] E\left[\pi_N|H,N\right]}{\left(E\left[\pi_N\right]+E\left[\pi_N|H,N\right]^{\frac{1}{\alpha}}\right)^{\alpha}} > \frac{\pi_C^{2-\alpha}}{\left(1+\pi_C^{\frac{1-\alpha}{\alpha}}\right)^{\alpha}}.$$

Hardly surprisingly, the right-hand side is increasing in π_C, the known probability of success when using the conventional project, while the left-hand side is increasing in both probability of success under N. Since we gave all the bargaining power to the principal, the contract offered in equilibrium will be the one that, for given probabilities of success, will maximize the principal's surplus. However, when only the outcome realizations are unobservable by the principal, the surplus of the principal and the agent determined by the optimal contracts are constant shares of the total welfare,

$$S^\star\langle i\rangle_z^j = \frac{1-\alpha}{2-\alpha} W^\star\langle i\rangle_z^j \qquad\qquad V^\star\langle i\rangle_z^j = \frac{1}{2-\alpha} W^\star\langle i\rangle_z^j.$$

As a consequence, the agent prefers to explore when innovation is socially optimal, and, more importantly, the principal always offers the exploration contract when it is socially optimal to do so.

The optimal contracts derived in this section may not be incentive compatible against alternative action plans if the investor can observe neither the output nor the actions taken by the agent. We investigate this issue in Section 5.

5 The Arm's Length Financing Case

In this section we consider the full double moral hazard model: we assume that the principal can observe neither the outcome realization nor agent's work method. Compared to the model in the previous section, the investor has less information: we thus refer to this case as arm's length financing, and to such investor as hands-off. We show that the contracts derived in Section 4 are usually not incentive compatible when the principal can observe neither the outcome nor the work method, as additional incentive compatibility constraints may be binding. We thus derive the optimal arm's length exploitation and exploration contracts. Finally, we discuss the results.

It will prove useful to distinguish between two forms of exploration: we call exploration moderate if the probability of two consecutive successes is higher under the novel approach than under the conventional work method, and radical otherwise.

Definition. *(Moderate Exploration) Exploration is moderate if*

$$E[\pi_N]E[\pi_N|H,N] > \pi_C\pi_C$$

and radical otherwise.

5.1 Optimal Exploitation Contract

Consider the optimal exploitation contract derived in Section 4: following a success, the hands-on principal receives the entire outcome in period 1, and rewards the agent by providing a positive amount of capital to be invested in period 2; the firm is terminated following a failure in the first period. When the work method is not observable, new possible deviations open up for the agent: the optimal exploitation contract must prevent the agent not only from tunneling but also from employing the novel work method.

Under the optimal exploration contract of the previous section, the expected payoff of the agent is $\pi_C\pi_C R(k^\star_H\langle C\rangle^C_C)$. Conversely, imagine an agent who has deviated in period 1 by using the novel approach and has succeeded. At the end of period 1, the deviating agent can either (a) repay the investor with the outcome of the exploration[14] and run N again in period 2, or (b) tunnel the outcome, foregoing any payoff in period 2. Given Assumption A, (a) represents the most profitable deviation, with an expected payoff at time 0 of $E[\pi_N]E[\pi_N|H,N]R(k^\star_H\langle C\rangle^C_C)$. Clearly, the contract of Section 4 is still incentive compatible when innovation is radical: the additional layer of information asymmetry does not matter because the agent does not want to explore anyway. Under moderate exploration, however, the optimal exploitation contract of Section 4 cannot be enforced, as the additional incentive compatibility constraints are not satisfied.

Formally, a hands-off principal proposing an exploitation contract needs to incentivise the agent against three deviations.[15] Firstly, the agent must strictly prefer to report an high outcome in the first period rather than tunneling it,

$$-\tau_H\langle C\rangle^C_C + \pi_C R\left(k_H\langle C\rangle^C_C\right) \geq \pi_C R\left(k_L\langle C\rangle^C_C\right). \tag{6}$$

[14]Note that the fact that $R()$ is the same for both projects plays a role here. For example, if it was higher under C than under the deviation N, the agent would not be able to repay back even if success occurs.

[15]The agent has actually access to other combinations of project, and thus more incentive compatibility constraints should be considered. However, given Assumption A, we can disregard them from the maximization problem as they will never bind.

Secondly, the principal needs to incentivise the agent to exploit rather than exploring by rewarding the agent with an higher expected payoff,

$$(\pi_C - E[\pi_N]) \left\{ R\left(k_1 \langle C \rangle_C^C\right) - \tau_H \langle C \rangle_C^C - \pi_C R\left(k_L \langle C \rangle_C^C\right) \right\} + \\ + \left\{ \pi_C \pi_C - E[\pi_N] E[\pi_N | H, N] \right\} R\left(k_H \langle C \rangle_C^C\right) \geq 0. \quad (7)$$

Lastly, the following constraint ensures that agent does not want to use N in the first period and divert the funds if a success occurs,

$$\pi_C \left\{ R\left(k_1 \langle C \rangle_C^C\right) - \tau_H \langle C \rangle_C^C \right\} - E[\pi_N] R\left(k_1 \langle C \rangle_C^C\right) + \pi_C \pi_C R\left(k_H \langle C \rangle_C^C\right) + \\ - \left\{ \pi_C (\pi_C - E[\pi_N]) + E[\pi_N] E[\pi_N | H, N] \right\} R\left(k_L \langle C \rangle_C^C\right) \geq 0. \quad (8)$$

Two observations simplify the maximization problem. Firstly, note that an increase in $R(k_1\langle C \rangle_C^C) - \tau_H \langle C \rangle_C^C$ relaxes the two constraints associated with exploration, (7) and (8). Indeed, when the principal wants the agent to exploit, she must pay the agent an extra premium in case of success in the first period, since C is initially more likely to succeed. As a consequence, the limited liability constraint does not bind. Secondly, $k_L \langle C \rangle_C^C$ enters with a negative sign in the left-hand side of all constraints, as rewarding the agent for a first period failure incentivises both tunneling and exploration: threat of termination following a failure is a common feature of the optimal exploitation contract, independently of the assumptions on the degree of information asymmetry.

Proposition 3 underlines the main characteristic of the optimal exploitation contract offered by a hands-off principal. The following definition will be useful:

$$w = \left(\frac{E[\pi_N] \left(E[\pi_N | H, N] - \pi_C \right)}{\pi_C - E[\pi_N]} \right)^{\frac{1}{\alpha}}.$$

Proposition 3. *(Exploitation contract under a hands-off principal)*
The optimal exploitation contract under a hands-off principal differs depending on the form of exploration. When exploration is radical, the optimal contract does not depend on the observability of the actions of the agent, $\boldsymbol{\sigma}_{rad}^{\star\star} \langle C \rangle_C^C = \boldsymbol{\sigma}^{\star} \langle C \rangle_C^C$*. When exploration is moderate, the optimal exploitation contract offered by a hands-off principal,* $\boldsymbol{\sigma}_{mod}^{\star\star} \langle C \rangle_C^C$*,*

is such that

$$k^{\star\star}_{H,mod}\langle C\rangle^C_C = \left(\frac{A\alpha\pi_C\pi_C}{\pi_C + w}\right)^{\frac{1}{1-\alpha}} \qquad k^{\star\star}_{1,mod}\langle C\rangle^C_C = w k^{\star\star}_{H,mod}\langle C\rangle^C_C$$

$$k^{\star\star}_{L,mod}\langle C\rangle^C_C = 0 \qquad \tau^{\star\star}_{H,mod}\langle C\rangle^C_C = \pi_C R\left(k^{\star\star}_{H,mod}\langle C\rangle^C_C\right) < R\left(k^{\star\star}_{1,mod}\langle C\rangle^C_C\right)$$

The total expected payoff of the principal and the agent, and social welfare are, respectively,

$$S^{\star\star}_{mod}\langle C\rangle^C_C = -I + (1-\alpha)\pi_C\pi_C R\left(k^{\star\star}_{H,mod}\langle C\rangle^C_C\right) \qquad V^{\star\star}_{mod}\langle C\rangle^C_C = \pi_C w^\alpha R\left(k^{\star\star}_{H,mod}\langle C\rangle^C_C\right)$$

$$W^{\star\star}_{mod}\langle C\rangle^C_C = -I + \left[\pi_C(1-\alpha) + w^\alpha\right]\pi_C A\left(\frac{A\alpha\pi_C\pi_C}{\pi_C + w}\right)^{\frac{\alpha}{1-\alpha}}.$$

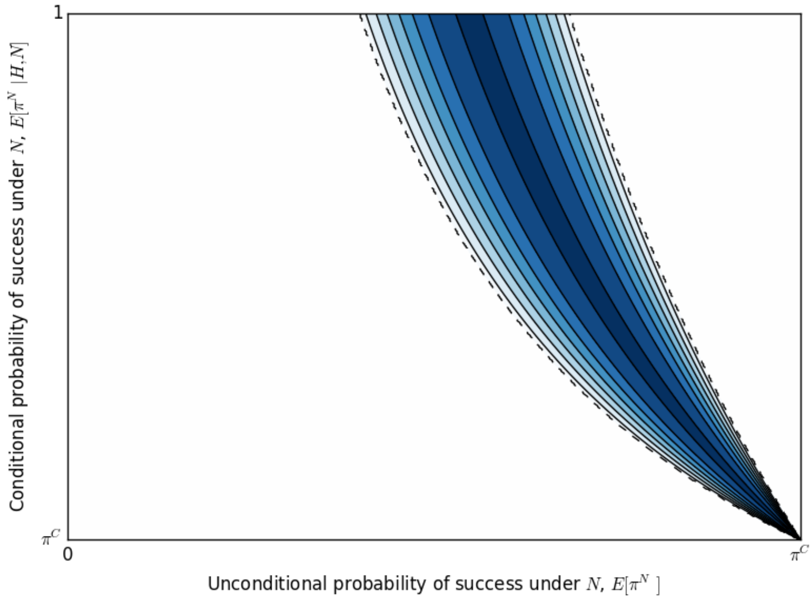
The firm is credit constrained in both periods. However, it may be the case that it receives more capital than from a hands-on principal, as the hands-off investor needs to incentivise the agent to exploit by providing an extra premium for early success. This is done by increasing the level of working capital in the first period, $k^{\star\star}_1\langle C\rangle^C_C$. On the contrary, reward for late success is always discouraged, as it incentivises exploration: the hands-off principal provides less capital in case of success, $k^{\star\star}_{H,mod}$, than a hands-on investor, but still a positive amount which, paired with the threat of termination, is used to avoid tunneling of the outcome after a success. Depending on which of these two effects dominates, social welfare can increase in the degree of informational asymmetry. This is summarized in Figure 1: when exploration is moderate there is a region of the probabilities of success of the novel approach for which welfare is higher under arm's length financing, $W^{\star\star}\langle C\rangle^C_C > W^{\star}\langle C\rangle^C_C$. When the probability of two consecutive successes under the novel approach is very high, this does not happen, as exploration becomes very attractive, and incentives to exploit are provided by reducing the total amount of capital invested.

The increase in social welfare is not a Pareto improvement. Figure 2 shows that, as one would expect, losing information is costly for the principal, and thus the increase in social welfare is just a consequence of the increase in the agent's surplus.

5.2 Optimal Exploration Contract

Consider now the optimal exploration contract derived in Section 4. Given Assumption A, it should be clear than the optimal exploration contract is not incentive compatible if the entrepreneur can rely on the well-known project without getting caught. Indeed,

Figure 1: Welfare from Exploitation - Observable vs Unobservable Actions



Notes. The graph shows contour plots for $\max\{W^{\star\star}\langle C\rangle_C^C - W^{\star}\langle C\rangle_C^C, 0\}$. Darker shades are associated with higher values, meaning that social welfare is higher under unobservability of both outcomes and actions. In the white area, $W^{\star\star}\langle C\rangle_C^C \leq W^{\star}\langle C\rangle_C^C$. Dashed lines represent zero contours and the left one also indicates the separation between moderate exploration (above) and radical exploration (below).

under the putative contract, the expected payoff in period 2 is the carrot that persuades the agent to part with first period return in case of success, and in fact it is exactly equal to this value: the agent gets a utility equal to the first period return in the case of success, but only with the probability of success in period 1. This probability of success is higher under C, and she can clearly get the same payoff by tunneling the outcome if a success occurs, but with a higher probability, so that has to be a profitable deviation.

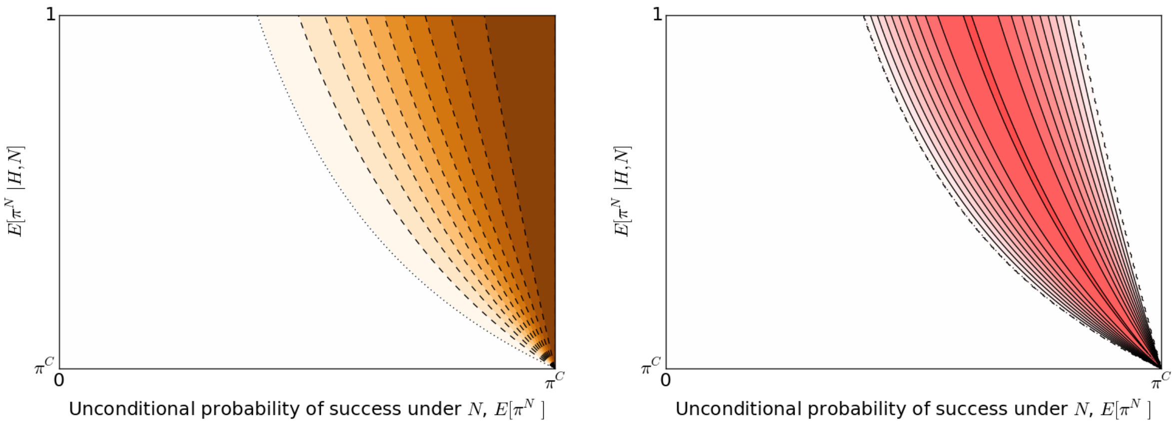
The maximization problem of a hands-off principal wanting to implement exploration must pay cognisance to the following potential deviations.[16] Firstly, the agent can decide to use the novel approach in period 1 but divert the funds in case of success (tunneling); the associated incentive compatibility constraint is

$$E\left[\pi_N|H,N\right] R\left(k_H\langle N\rangle_C^N\right) - \pi_C R\left(k_L\langle N\rangle_C^N\right) \geq \tau_H\langle N\rangle_C^N. \tag{9}$$

Secondly, the following constraint ensures that the agent does not want to deviate by relying on the conventional approach at time 1 with the intention of tunneling the positive

[16]As before, other deviations can be disregarded given Assumption A.

Figure 2: Exploitation Surpluses - Observable vs Unobservable Actions



(a) Principal (b) Agent

Notes. **Left Panel**. It shows contour plots for $S^{\star\star}\langle C\rangle_C^C - S^{\star}\langle C\rangle_C^C$, where darker shades are associated with lower values. In the white area, $S^{\star\star}\langle C\rangle_C^C = S^{\star}\langle C\rangle_C^C$ because the principal offers the same contract. Dashed lines represent negative levels. **Right Panel**. It shows contour plots for $\max\{V^{\star\star}\langle C\rangle_C^C - V^{\star}\langle C\rangle_C^C, 0\}$, where darker shades are associated with higher values. In the white area, $V^{\star\star}\langle C\rangle_C^C \leq V^{\star}\langle C\rangle_C^C$. Dashed lines represent zero contour plots. **Both Panels**. The dotted line separates moderate exploration (above) from radical exploration (below).

outcome in case of success (exploitation and tunneling)

$$E\left[\pi_N\right]\left\{R\left(k_1\langle N\rangle_C^N\right) - \tau_H\langle N\rangle_C^N\right\} - \pi_C R\left(k_1\langle N\rangle_C^N\right) + \\ + E\left[\pi_N\right]E\left[\pi_N|H,N\right]R\left(k_H\langle N\rangle_C^N\right) - E\left[\pi_N\right]\pi_C R\left(k_L\langle N\rangle_C^N\right) \geq 0. \quad (10)$$

Finally, the investor avoids exploitation by imposing

$$\left\{\pi_C - E\left[\pi_N\right]\right\}\left\{\tau_H\langle N\rangle_C^N - R\left(k_1\langle N\rangle_C^N\right) + \pi_C R\left(k_L\langle N\rangle_C^N\right)\right\} + \\ + \left\{E\left[\pi_N\right]E\left[\pi_N|H,N\right] - \pi_C\pi_C\right\}R\left(k_H\langle N\rangle_C^N\right) \geq 0. \quad (11)$$

The following points may be of interest. Firstly, while $k_L\langle N\rangle_C^N$ enters with a negative sign on the left-hand side of the constraint (9) associated with tunneling, it enters with a positive sign in constraint (11), the one associated with exploiting. This is because rewarding the agent for first period failures incentivises the diversion of funds but can be useful to prevent the agent from exploiting as the probability of failure is higher initially when using the novel method. Secondly, $R(k_1\langle N\rangle_C^N) - \tau_H\langle N\rangle_C^N$ enters with a negative sign on the left-hand side of constraint (11), the one associated with exploiting, because early reward for agent's success incentivises exploitation, as success is initially more likely under the conventional work method. Moreover, after a success in the first period, the second period outcome provides additional information about the first period action, since the expected probability of success with the new work method in the second period depends on the action taken by the agent in the first period. It is therefore optimal to delay compensation in order to obtain this additional information. Finally, a similar reasoning can be applied for $k_1\langle N\rangle_C^N$ in constraint (10), the one associated with exploiting and tunneling. By Assumption A, $R(k_1\langle N\rangle_C^N)$ enters with a negative sign on the left-hand side, suggesting that delaying agent's compensation to the second period can help minimising agent's incentives to exploit and tunnel. This motivates the following Lemma.

Lemma 3. *When the investor wants to motivate exploration, delaying any compensation to the agent until the second period is optimal. The limited liability constraint is binding at the optimum i.e.* $R(k_1\langle N\rangle_C^N) = \tau_H\langle N\rangle_C^N$ *is optimal.*

Given the result in Lemma 3, the incentive compatibility constraint in (11) reduces to

$$\left\{\pi_C - E\left[\pi_N\right]\right\}\pi_C R\left(k_L\langle N\rangle_C^N\right) + \left\{E\left[\pi_N\right]E\left[\pi_N|H,N\right] - \pi_C\pi_C\right\}R\left(k_H\langle N\rangle_C^N\right) \geq 0. \quad (12)$$

When exploration is moderate, the inequality in (12) is strictly satisfied by any non negative value of $k_L\langle N\rangle_C^N$ and $k_H\langle N\rangle_C^N$. Given that $k_L\langle N\rangle_C^N$ enters with a negative sign in the right-hand side of all other constraints, it is then optimal to set $k_L\langle N\rangle_C^N = 0$. This suggests that, while under moderate exploration it is optimal to terminate the firm after a failure in the first period, $k_{L,mod}^{\star\star}\langle N\rangle_C^N = 0$, it may be optimal to tolerate (or even reward) a failure under radical innovation. We therefore consider the cases of moderate and radical exploration in turn in Proposition 4 and 5. The following definitions will be used:

$$z = \left(\frac{E\left[\pi_N\right]E\left[\pi_N|H,N\right]}{\pi_C}\right)^{\frac{1}{\alpha}}$$
$$x = \frac{\pi_C\pi_C - E\left[\pi_N\right]E\left[\pi_N|N,H\right]}{\pi_C\left(\pi_C - E\left[\pi_N\right]\right)}$$
$$y = \left(\frac{E\left[\pi_N\right]E\left[\pi_N|N,H\right] - E\left[\pi_N\right]\pi_C x}{\pi_C}\right)^{\frac{1}{\alpha}}.$$

Proposition 4. *(Moderate exploration contract under a hands-off principal)*
The optimal moderate exploration contract under a hands-off principal, $\boldsymbol{\sigma}_{mod}^{\star\star}\langle N\rangle_C^N$*, is*

such that

$$k^{\star\star}_{H,mod}\langle N\rangle^N_C = \left(\frac{A\alpha E\left[\pi_N\right]z^{\alpha}}{z+E\left[\pi_N\right]}\right)^{\frac{1}{1-\alpha}} \qquad k^{\star\star}_{1,mod}\langle N\rangle^N_C = zk^{\star\star}_{H,mod}\langle N\rangle^N_C$$

$$k^{\star\star}_{L,mod}\langle N\rangle^N_C = 0 \qquad \tau^{\star\star}_{H,mod}\langle N\rangle^N_C = R\left(k^{\star\star}_{1,mod}\langle N\rangle^N_C\right).$$

The total expected payoffs of the principal and the agent are, respectively

$$S^{\star\star}_{mod}\langle N\rangle^N_C = -I+(1-\alpha)E[\pi_N]z^{\alpha}R\left(k^{\star\star}_{H,mod}\langle N\rangle^N_C\right)$$
$$V^{\star\star}_{mod}\langle N\rangle^N_C = E\left[\pi_N\right]E\left[\pi_N|H,N\right]R\left(k^{\star\star}_{H,mod}\langle N\rangle^N_C\right)$$

while social welfare is

$$W^{\star\star}_{mod}\langle N\rangle^N_C = -I+\left\{z^{\alpha}\left(1-\alpha\right)+E\left[\pi_N|H,N\right]\right\}E[\pi_N]A\left(\frac{A\alpha E[\pi_N]z^{\alpha}}{z+E\left[\pi_N\right]}\right)^{\frac{\alpha}{1-\alpha}}.$$

Proposition 5. *(Radical exploration contract under a hands-off principal)*
The optimal radical exploration contract under a hands-off principal, $\boldsymbol{\sigma}^{\star\star}_{rad}\langle N\rangle^N_C$, is such that

$$k^{\star\star}_{H,rad}\langle N\rangle^N_C = \left(\frac{A\alpha E[\pi_N]y^{\alpha}}{y+E[\pi_N]+(1-E[\pi_N])\,x^{\frac{1}{\alpha}}}\right)^{\frac{1}{1-\alpha}} \qquad k^{\star\star}_{1,rad}\langle N\rangle^N_C = yk^{\star\star}_{H,rad}\langle N\rangle^N_C$$

$$k^{\star\star}_{L,rad}\langle N\rangle^N_C = x^{\frac{1}{\alpha}}k^{\star\star}_{H,rad}\langle N\rangle^N_C \qquad \tau^{\star\star}_{H,rad}\langle N\rangle^N_C = R\left(k^{\star\star}_{1,rad}\langle N\rangle^N_C\right).$$

The total expected payoffs of the principal and the agent are, respectively

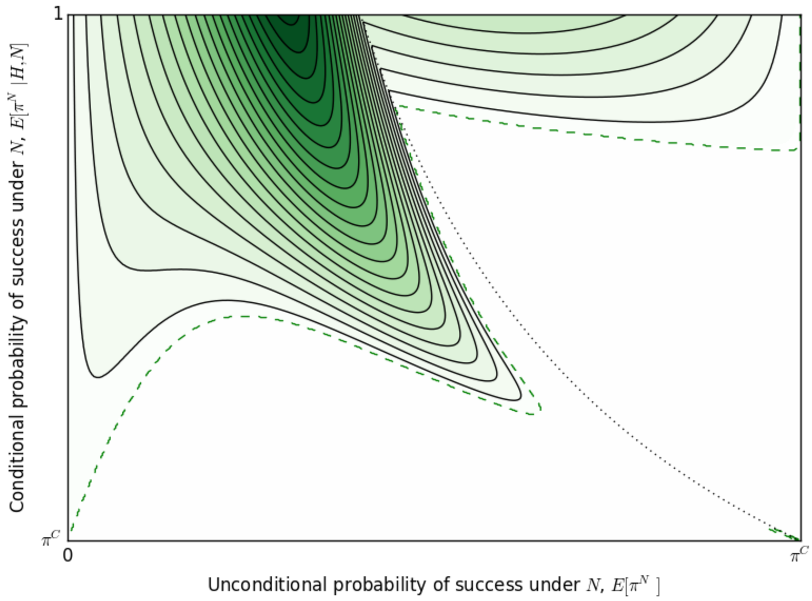
$$S^{\star\star}_{rad}\langle N\rangle^N_C = -I+(1-\alpha)E[\pi_N]y^{\alpha}R\left(k^{\star\star}_{H,rad}\langle N\rangle^N_C\right)$$
$$V^{\star\star}_{rad}\langle N\rangle^N_C = \left\{E[\pi_N]E[\pi_N|H,N]+(1-E[\pi_N])\pi_C x\right\}R\left(k^{\star\star}_{H,rad}\langle N\rangle^N_C\right).$$

Total welfare is

$$W^{\star\star}_{rad}\langle N\rangle^N_C = -I+\left\{E[\pi_N]y^{\alpha}(1-\alpha)+\pi_C\left(x+y^{\alpha}\right)\right\}A\left(\frac{A\alpha E[\pi_N]y^{\alpha}}{y+E[\pi_N]+(1-E[\pi_N])\,x^{\frac{1}{\alpha}}}\right)^{\frac{\alpha}{1-\alpha}}.$$

As in Manso (2011), agent's reward is contingent on the performance path and not only on the number of successes. In particular, if we compare total agent's compensation when performance is LH rather than HL, we see than an agent who recovers from a failure receives a compensation at least as high as one who obtains a short-lived success.[17] As before, the firm is credit constrained in both periods but the scale of the firm increases following a success. Moreover, the increased information asymmetry can be socially valuable, as the fact that the principal must provide incentives to the agent to avoid exploitation can increase social welfare. In Figure 3, dark areas represent the region of the probabilities of success of the novel approach for which social welfare is higher when the informational asymmetry has increased, $W^{\star\star}\langle N\rangle_C^N > W^{\star}\langle N\rangle_C^N$.

Figure 3: Welfare from Exploration - Observable vs Unobservable Actions



Notes. The graph shows contour plots for $\max\{W^{\star\star}\langle N\rangle_C^N - W^{\star}\langle N\rangle_C^N, 0\}$. Darker shades are associated with higher values, meaning that social welfare is higher under unobservability of both outcomes and actions. In the white area, $W^{\star\star}\langle N\rangle_C^N < W^{\star}\langle N\rangle_C^N$. The dashed line represents the zero contour. The dotted line indicates the separation between moderate exploration (above) and radical exploration (below).

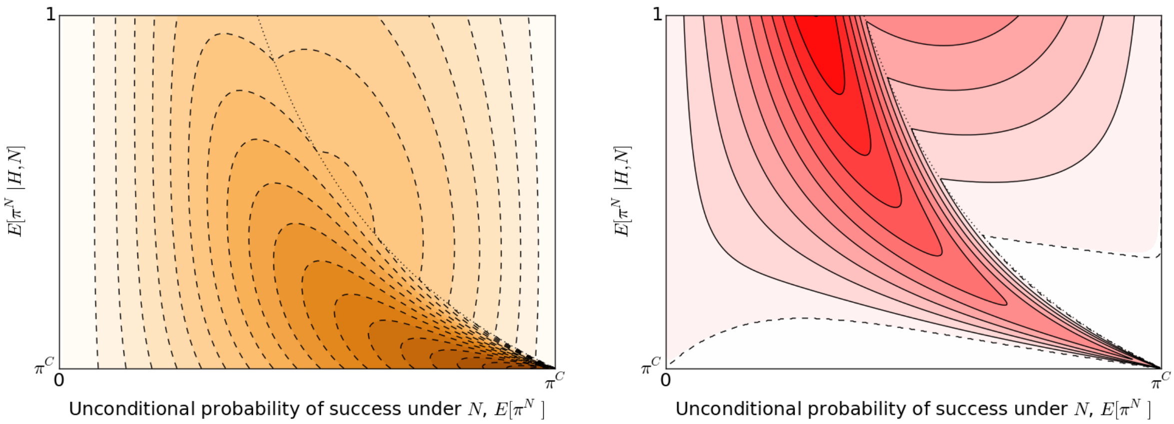
This result is driven by increases of $k_H^{\star\star}\langle N\rangle_C^N$ over $k_H^{\star}\langle N\rangle_C^N$ and, under radical exploration, by positive values of $k_L^{\star\star}\langle N\rangle_C^N$. Conversely, $k_1^{\star\star}\langle N\rangle_C^N$ is always lower than $k_1^{\star}\langle N\rangle_C^N$. The principal prefers to start small not only to minimise potential losses, but also because starving the agent of funds incentivises risk-taking, since the conventional approach is

[17] When performance is LH, the agent receives 0 under moderate exploration (since firm is terminated) or $R(k_L)$ under radical exploration. When performance is HL, the agent receives 0 under both moderate and radical exploration, as the agent repays the entire outcome in the first period, and produces zero in the second. In Manso (2011), it may be the case that even an agent who fails twice receives a higher compensation than one who succeeds only in the first period. Here, both receives zero since there is no outcome is case of failure.

initially more likely to succeed. The firm is significantly scaled-up after a success in the first period: this prevents tunneling and also allows the principal to delay compensation to the second period. To prevent the agent from resorting to the safer project, the principal provides rewards the agent for early failure and long-term success, balancing the use of k_L and k_H depending on the form of exploration. When exploration is moderate, the principal prefers to incentivise exploration through k_H, because two consecutive successes are a clear signal of the use of N. When exploration is radical, the principal uses a combination of k_H and k_L. The latter is used because failure in the first period is a strong signal that the agent explored, even stronger than two consecutive successes. However, using only k_L is not optimal because it incentivises the agent to tunnel the funds.

Figure 4 shows that a hands-off principal is always worse off than a hands-on investor, given the additional informational asymmetry, but the presence of additional deviations means that the agent may require an higher surplus to implement the action plan chosen by the principal.

Figure 4: Exploration Surpluses - Observable vs Unobservable Actions



(a) Principal (b) Agent

Notes. **Left Panel**. It shows contour plots for $S^{\star\star}\langle N\rangle_C^N - S^{\star}\langle N\rangle_C^N$, where darker shades are associated with lower values. Dashed lines represent negative contours. **Right Panel**. It shows contour plots for $\max\{V^{\star\star}\langle N\rangle_C^N - V^{\star}\langle N\rangle_C^N, 0\}$, where darker shades are associated with higher values. In the white area, $V^{\star\star}\langle N\rangle_C^N \leq V^{\star}\langle N\rangle_C^N$. Dashed lines represent zero contour levels. **Both Panels**. The dotted line separates moderate exploration (above) from radical exploration (below).

6 Implications: A Numerical Example

This section complements the above analysis with some numerical results, providing implications of the model for firm dynamics and social welfare.

6.1 Firm Dynamics

We start by highlighting some of the implications of our model for firm size and growth. As said before, we refer to the level of working capital invested as a measure of the scale of the firm. We compare the scale of the firm in the first period and in the second period after a success under different contracts and types of exploration.[18]

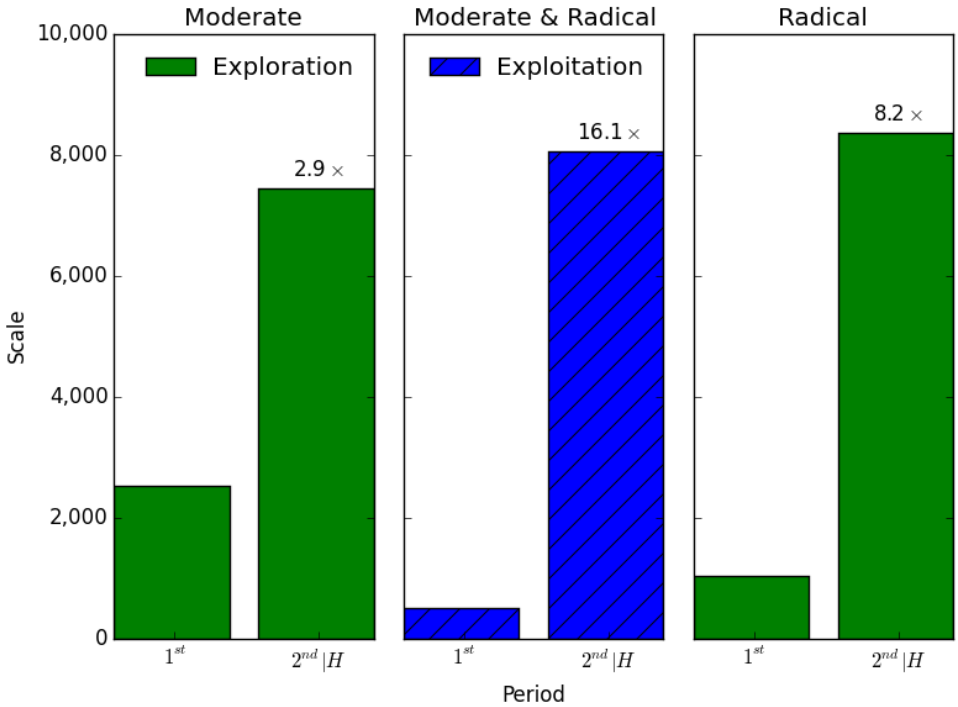
On the one hand, Figure 5 shows that, under a hands-on principal, the dynamics of the two types of firm, conventional and innovative, are substantially similar to one another. Both a moderate and a radical innovative project start small, and are substantially scaled-up only after a success is reported. Similarly, the optimal exploitation contract (which does not depend on the probabilities of success of the novel approach) involves a relaxation of the financing constraints following a success. Indeed, stricter credit constraints at the beginning of the relationship, together with a reward for success and a punishment for failure, minimize agent's incentives to embezzle revenues. Interestingly, a conventional project under a hands-on principal seems to start smaller and to grow faster than an innovative venture.

On the other hand, Figure 6 shows that the implications for firm dynamics are substantially different if both the outcomes and the action plan are private information of the agent. Under both radical and moderate exploration, an innovative firm receives less capital in the first period than the corresponding conventional one: starving an innovative agent not only limits losses and disincentives stealing, but also minimises agent's incentive to resort to the conventional project (thus incentivising risk-taking). Moreover, following a success, growth rates are substantially greater for small successful innovative firms than for conventional ones.

As far as we know, the theoretical prediction that the effect of innovation investments on firm growth differs depending on the type of financing the firm has access to has not been tested empirically. However, the empirical literature has recognised that the impact of innovation on growth is indeed different for different types of firms (Del Monte and

[18] The unconditional probability of success of the novel approach used to generate the figures in this subsection is $E[\pi_N] = 0.3$. For Moderate, the conditional probability of success of the novel approach following a success in the first period is $E[\pi_N|H, N] = 0.7$, while for Radical is $E[\pi_N|H, N] = 0.5$.

Figure 5: Firm size and growth under a hands-on principal



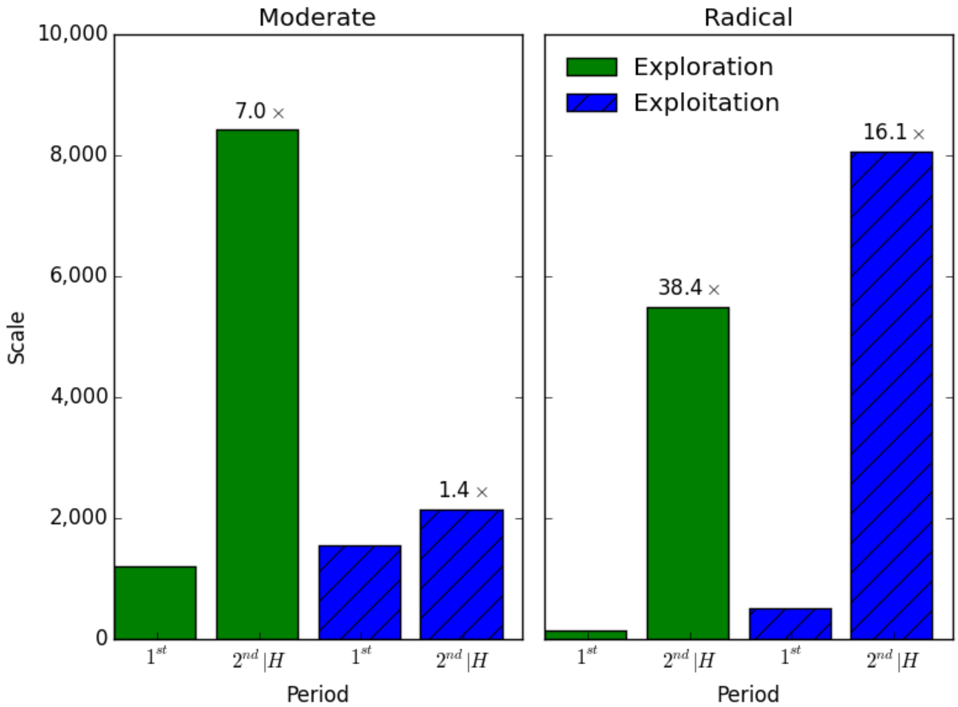
Notes. The number above the second bar in each panel represents the scale in the second period after a success relative to the scale in the first period.

Papagni, 2003; Coad and Rao, 2008; Demirel and Mazzucato, 2012; Segarra and Teruel, 2014; Mazzucato and Parris, 2015).

6.2 Welfare Implications

Since we gave all the bargaining power to the investor, the contract offered in equilibrium is the one which maximizes their discounted total profits. The first panel of Figure 7 shows that a hands-off principal's surplus from offering an exploration contract exceed the surplus from an exploitation contract only when the probability of two consecutive successes under N is quite high. It also shows that the region of probabilities for which an exploration contract is offered by the principal is smaller when the informational asymmetry increases. The reverse is obviously true with regards to the exploitation contract, as shown in panel (b). This suggests that under arm's length financing it is relatively more difficult to finance innovations with respects to the relationship-based financing case, as only those innovative projects perceived as more likely to succeed (less risky) have access to fund.

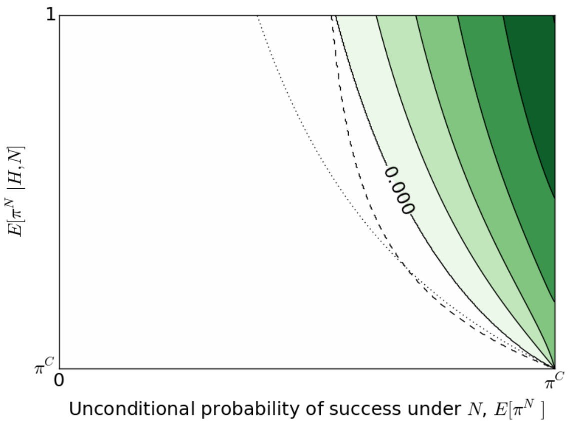
Figure 6: Firm size and growth under a hands-off principal



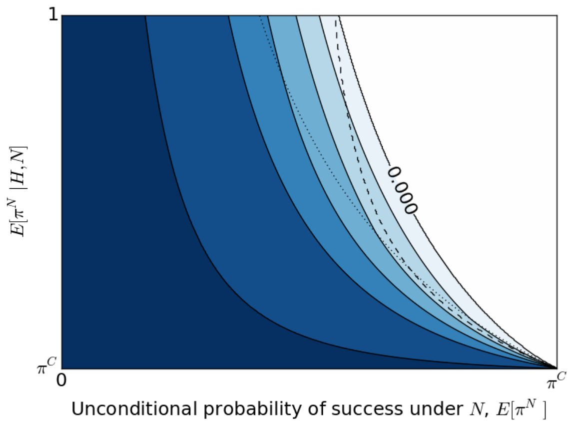
Notes. The number above the second bar in each panel represents the scale in the second period after a success relative to the scale in the first period.

However, when offering an exploration contract, a hands-off principal may need to provide more capital to prevent the agent not only from tunneling but also from resorting to the more predictable old technology. Therefore, if on the one hand innovation is harder to finance when financiers have less information, on the other hand the increased degree of informational asymmetry means that entrepreneurs whose projects are perceived as more productive not only are allowed to explore but also they may be less constrained in the amount they can borrow in the long-run, and this is socially efficient. This is summarized in Figure 8: the green set on the right is the intersection between the set of probabilities for which an exploration contract delivers higher social welfare under a hands-off principal than under a hands-on principal (Figure 3) with the set of probabilities for which an exploration contract is always offered (the right panel of Figure 7). It thus shows that there exists a region of probabilities for which an exploration contract is offered in equilibrium and for which the social welfare is higher when the informational asymmetry increases. One can also see that an equivalent set exists when the principal wants the agent to exploit: for the blue set on the left, the equilibrium exploitation contract is such that social welfare is higher when the information asymmetry is greater.

Figure 7: Which contract does the principal offer?



(a) Exploration



(b) Exploitation

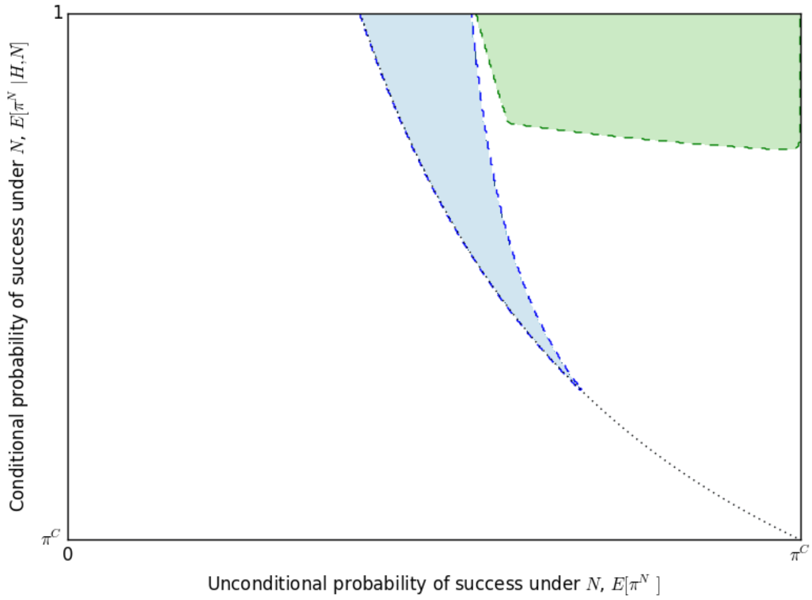
Notes. **Left Panel**. It shows contour plots for $\max\{S^{\star\star}\langle N\rangle_C^N - S^{\star\star}\langle C\rangle_C^C, 0\}$, where darker shades are associated with higher values. **Right Panel**. It shows contour plots for $\max\{S^{\star\star}\langle C\rangle_C^C - S^{\star\star}\langle N\rangle_C^N, 0\}$, where darker shades are associated with higher values. **Both Panels**. The dotted line separates moderate exploration (above) from radical exploration (below). The "0.000" line indicates the zero contour. The dashed line represents the threshold level above (below) which the hands-on principal offers an exploration (exploitation) contract.

7 Conclusions

When does an innovative project benefit from a lack of resources? This article has offered a two-period principal-agent model, where innovation is modelled as experimentation of untested actions, riskier than a conventional approach. We studied the relationship between an entrepreneur and a potential financier under different degrees of information asymmetry. We have considered and compared three cases: the full-information case, in which the researcher and the investor have access to the same information; the relationship-based financing case, in which the researcher is better informed than the principal on the outcome of the production process; and the arm's length financing case, in which the agent privately chooses the novelty of the project and can embezzle revenues for her own consumption. We focused on the trade-off between increasing the scale of the project to reap higher profits and starving the agent for incentive reasons.

In the full-information case, the agent is well-resourced, and experimentation occurs unless the agents are sufficiently pessimistic about the probability of success of the novel approach. An innovative project starts smaller than a conventional one, given that the probability of success is lower. In the relationship-based financing case, the amount of working capital that the principal is willing to provide is reduced, as financing constraints

Figure 8: Increased information asymmetry can be valuable



Notes. **The set on the right** (green) is the intersection of Panel a of Figure 7 with Figure 3, or $\{E[\pi_N] \in [0, \pi_C], E[\pi_N|H,N] \in [\pi_C, 1] : S^{\star\star}\langle N\rangle_C^N \geq S^{\star\star}\langle C\rangle_C^C \wedge W^{\star\star}\langle N\rangle_C^N \geq W^{\star}\langle N\rangle_C^N\}$. It represents probabilities for which the principal always offers the exploration contract and welfare is higher under a hands-off principal. **The set on the left** (blue) is $\{E[\pi_N] \in [0, \pi_C], E[\pi_N|H,N] \in [\pi_C, 1] : S^{\star}\langle C\rangle_C^C \geq S^{\star}\langle N\rangle_C^N \wedge W^{\star\star}\langle C\rangle_C^C \geq W^{\star}\langle C\rangle_C^C\}$. It represents probabilities for which the hands-on principal always offers the exploitation contract and welfare is higher under a hands-off principal. **The dotted line** separates moderate exploration (above) from radical exploration (below).

arise endogenously to minimise agent's incentives to divert the outcome realizations. In the arm's length financing case, fewer innovative projects are funded, and the principal further starves an innovative agent, as this incentivises risk-taking and minimises the agent's incentive to resort to the safer conventional approach. Moreover, under the optimal contract, the innovative firm is significantly scaled-up after a success in the first period, and an innovative entrepreneur may retain its access to capital even after a failure. This has the counter-intuitive consequence that decreasing the principal's information (while keeping agent's information constant) can potentially increase social welfare: resource-starved innovative start-ups can be socially optimal when researchers are better informed.

Our results show that innovation can be incentivised both in mature firms and small independent start-ups, but the best way to do so varies across different economic environments. A large R&D department is optimal when the principal is equally (or better) informed about the outcome of the innovation process; on the contrary, when the researchers can focus on safer but non-innovative alternatives and have a better idea than investors of whether a new technology answers the customers' needs, fast-growing start-ups are optimal.

In this article we have neglected many potential distortions in order to maintain tractability, and future works could try to incorporate them (e.g. the presence of limited commitment, the possibility that principal and agent have different discount factors, or different degrees of risk aversion). More interestingly, one could remove the assumption that the functional form for the outcome in case of success is the same between conventional and novel projects: a perhaps more natural way of modelling the problem would be to let the successful outcome increase with the riskiness of the project. This is particular relevant when studying the incentives for truly radical innovations, but one may argue that in such cases, innovation should be perceived as *ambiguous*. Moreover, our two-period model is not well suited to study the optimal number of experiments: one could try to develop a fully dynamic problem, where the firm operates for multiple periods, capital is long-lived and potentially irreversible, and the innovative projects evolve through different phases, with research at each step depending on the outcomes of previous phases. Additionally, by including adverse selection, one could provide the entrepreneur with the possibility to signal his ability, perhaps allowing firms' past patenting activities to alleviate the presence of financing constraints. Finally, following Kiyotaki and Moore (1997), there has been an increasing interest in incorporating the optimal contract framework into a general equilibrium model. We leave these interesting extensions to future research.

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A Mathematical Appendix

A.1 Action Plans

With two possible projects, there are 8 potential three-permutations with repetition: $\langle C\rangle_C^C$, $\langle C\rangle_N^C$, $\langle C\rangle_C^N$, $\langle C\rangle_N^N$, $\langle N\rangle_C^C$, $\langle N\rangle_N^C$, $\langle N\rangle_C^N$ and $\langle N\rangle_N^N$. However, one can restrict his attention to $\langle C\rangle_C^C$ and $\langle N\rangle_C^N$.

Given Assumption A, every action plan using the conventional project in the first period and the new project in the second one is dominated by $\langle C\rangle_C^C$. Thus $\langle C\rangle_N^C, \langle C\rangle_C^N$ and $\langle C\rangle_N^N$ are dominated by $\langle C\rangle_C^C$. After a success using the new project, the probability of success in the second period is higher under N, therefore $\langle N\rangle_C^C$ is dominated by $\langle N\rangle_C^N$. After a failure, the conventional project is perceived as more effective so $\langle N\rangle_N^C$ is dominated by $\langle N\rangle_C^C$ and $\langle N\rangle_N^N$ is dominated by $\langle N\rangle_C^N$.

A.2 Proofs

Proof of Lemma 1. Suppose that the limited liability constraint in (3) and the incentive compatibility constraint in (4) are both slack. The principal can do better by increasing $\tau_H\langle i\rangle_z^j$ without changing the amount of capital she has to provide, until one of the two constraints is binding. Now consider a binding limited liability constraint but a slack incentive compatibility constraint: the principal would be better off by decreasing $k_H\langle i\rangle_z^j$ until also the incentive compatibility constraint binds. Finally, consider a binding incentive compatibility constraint and a slack limited liability constraint: the principal can do better by decreasing $k_1\langle i\rangle_z^j$ until the latter binds as well. $\square$

Proof of Lemma 2. Suppose $k_L\langle i\rangle_z^j > 0$ and both constraints are binding. The principal could offer a new contract which only differs in $\tilde{k}_L\langle i\rangle_z^j = 0$. This alternative contract would satisfy both constraints but the objective function of the investor would be higher. As a consequence, $k_L\langle i\rangle_z^j > 0$ cannot be optimal. $\square$

Proof of Proposition 1 (Exploitation contract under a hands-on principal). Using Lemmas 1 and 2 we can write the incentive compatibility constraint and the limited liability constraint as $\pi_C R(k_H\langle C\rangle_C^C) = \tau_H\langle C\rangle_C^C$ and $R(k_1\langle C\rangle_C^C) = \tau_H\langle C\rangle_C^C$. The maximization problem thus simplifies to

$$\max_{\left\{k_1\langle C\rangle_C^C,\ k_H\langle C\rangle_C^C\right\}\in\mathbb{R}^2_{\geq}} -I - k_1\langle C\rangle_C^C + \pi_C\left\{\pi_C R(k_H\langle C\rangle_C^C) - k_H\langle C\rangle_C^C\right\}$$
$$\text{s.t.}\quad R(k_1\langle C\rangle_C^C) = \pi_C R(k_H\langle C\rangle_C^C).$$

The FOC with respect to $k_1\langle C\rangle_C^C$ gives us the value of the Lagrangian multiplier as $\lambda\langle C\rangle_C^C = 1/R'(k_1\langle C\rangle_C^C)$ which is positive and increasing in k_1, confirming that the limited liability constraint is binding at the optimum. The remaining FOC is

$$R'(k_H\langle C\rangle_C^C) = \frac{1}{\pi_C - \lambda\langle C\rangle_C^C} = \frac{1}{\pi_C - 1/R'(k_1\langle C\rangle_C^C)},$$

which, together with the two constraints, indirectly gives us the amounts of working capital provided following a success, $k_H^\star\langle C\rangle_C^C$, and in the first period, $k_1^\star\langle C\rangle_C^C$, under the optimal exploitation contract. □

Proof of Proposition 2 (Exploration contract under a hands-on principal). Using the binding incentive compatibility and limited liability constraints, the maximization problem simplifies to

$$\max_{\left\{k_1\langle N\rangle_C^N,\ k_H\langle N\rangle_C^N\right\}\in\mathbb{R}^2_{\geq}} -I - k_1\langle N\rangle_C^N + E[\pi_N]\left\{E[\pi_N|H,N]R(k_H\langle N\rangle_C^N) - k_H\langle N\rangle_C^N\right\}$$
$$\text{s.t.}\quad R(k_1\langle N\rangle_C^N) = E[\pi_N|H,N]R(k_H\langle N\rangle_C^N).$$

We can recover the value of the Lagrangian multiplier from the first FOC, $\lambda\langle N\rangle_C^N = 1/R'(k_1\langle N\rangle_C^N)$, while from the second FOC

$$R'(k_H\langle N\rangle_C^N) = \frac{E[\pi_N]}{E[\pi_N]E[\pi_N|H,N] - E[\pi_N|H,N]/R'(k_1\langle C\rangle_C^C)}.$$

The two constraints, and the above FOC, indirectly give us the amounts of working capital provided following a success, $k_H^\star\langle N\rangle_C^N$, and in the first period, $k_1^\star\langle N\rangle_C^N$, under the optimal exploration contract. □

Proof of Proposition 3 (Exploitation contract under a hands-off principal). We only focus on the moderate exploration case, since the optimal contract under radical exploration is equivalent to the one offered by a hands-on principal. From the text, $k_L^{\star\star}\langle C\rangle_C^C = 0$. One can note that the incentive compatibility constraint associated with "exploration" in

(7) is stricter than both the anti "exploration and tunneling" one in (8) and the limited liability constraint, so that the maximization problem simplifies to

$$\max_{\{k_1\langle C\rangle_C^C,\ k_H\langle C\rangle_C^C,\ \tau_H\langle C\rangle_C^C\}\in\mathbb{R}^3_{\geq 0}} -I + \pi_C \tau_H\langle C\rangle_C^C - k_1\langle C\rangle_C^C - \pi_C k_H\langle C\rangle_C^C$$

$$\text{s.t.}\quad \pi_C R(k_H\langle C\rangle_C^C) \geq \tau_H\langle C\rangle_C^C$$

$$\big(\pi_C - E[\pi_N]\big)\{R(k_1\langle C\rangle_C^C) - \tau_H\langle C\rangle_C^C\} \geq \big\{E[\pi_N]E[\pi_N|H,N] - \pi_C\pi_C\big\}R(k_H\langle C\rangle_C^C).$$

From the FOCs

$$\lambda_{ICE} = \frac{1}{(\pi_C - E[\pi_N])R'(k_1\langle C\rangle_C^C)}$$

$$\lambda_{ICT} = \pi_C - \lambda_{ICE}\{\pi_C - E[\pi_N]\}$$

where λ_{ICT} and λ_{ICE} are the Lagrangian multipliers associated with the first and second constraints. Given Assumption A, and taking as given the presence of financing constraint, these are positive, and thus both constraints bind. With further manipulation,

$$R'(k_H\langle C\rangle_C^C) = \frac{\pi_C}{\pi_C\pi_C - \frac{(E[\pi_N|H,N]-\pi_C)E[\pi_N]}{(\pi_C - E[\pi_N])R'(k_1\langle C\rangle_C^C)}}.$$

This can be used to find the optimal levels of capital, together with the binding incentive compatibility constraint associated with exploitation. □

Proof of Lemma 3. Let $\boldsymbol{\sigma}$ be an optimal contract with $R(k_1\langle N\rangle_C^N) - \tau_H\langle N\rangle_C^N = \epsilon > 0$. Let $\hat{\boldsymbol{\sigma}}$ be the same as $\boldsymbol{\sigma}$ except that $R(\hat{k}_1\langle N\rangle_C^N) = R(k_1\langle N\rangle_C^N) - \epsilon = \tau_H\langle N\rangle_C^N$. Such contract satisfies all incentive compatibility constraints but the expected profits of the investor is higher. Thus, $\hat{\boldsymbol{\sigma}}$ must be preferred to $\boldsymbol{\sigma}$. □

Proof of Proposition 4 (Moderate exploration contract under a hands-off principal). From Lemma 3, we can use the binding limited liability constraint to substitute $\tau_H\langle N\rangle_C^N$ out of the maximization problem. Moreover, from the text, $k_L\langle N\rangle_C^N = 0$ at the optimum. In the maximization problem, we can thus disregard the "anti exploitation" constraint in (12), as this is satisfied for any non-negative value of $k_H\langle N\rangle_C^N$, and the "anti tunneling" constraint in (9), as this will be slack for any values satisfying the "anti exploitation and tunneling" constraint in (10). The simplified maximization problem is

$$\max_{\{k_1\langle N\rangle_C^N,\ k_H\langle N\rangle_C^N\}\in\mathbb{R}^2_{\geq 0}} - I - k_1\langle N\rangle_C^N + E[\pi_N]R(k_1\langle N\rangle_C^N) - E[\pi_N]k_H\langle N\rangle_C^N$$
$$\text{s.t.}\quad E[\pi_N]\,E[\pi_N|H,N]\,R\left(k_H\langle N\rangle_C^N\right) - \pi_C R\left(k_1\langle N\rangle_C^N\right) \geq 0.$$

From the first FOC,

$$\lambda_{ICET} = \frac{E[\pi_N]R'(k_1\langle N\rangle_C^N) - 1}{\pi_C R'(k_1\langle N\rangle_C^N)}.$$

Assuming that the optimal level of capital in period 1 is lower than the corresponding first best level, the Lagrange multiplier is positive, and thus the remaining incentive compatibility constraint binds. It follows that

$$R'(k_H\langle N\rangle_C^N) = \frac{\pi_C R'(k_1\langle N\rangle_C^N)}{\{E[\pi_N]R'(k_1\langle N\rangle_C^N) - 1\}E[\pi_N|H,N]},$$

together with the binding "exploitation and tunneling" constraint, gives us the solution.

□

Proof of Proposition 5 (Radical exploration contract under a hands-off principal). From Lemma 3, we can use the binding limited liability constraint to substitute $\tau_H\langle N\rangle_C^N$ out of the maximization problem. It is also easy to show that we can disregard the "anti tunneling" constraint in (9) as this will be slack for any values satisfying the constraint in (10). The maximization problem is thus

$$\max_{\{k_1\langle N\rangle_C^N,\ k_H\langle N\rangle_C^N,\ k_L\langle N\rangle_C^N\}\in\mathbb{R}^3_{\geq 0}} -I - k_1\langle N\rangle_C^N + E[\pi_N]R(k_1\langle N\rangle_C^N) - E[\pi_N]k_H\langle N\rangle_C^N +$$
$$- (1 - E[\pi_N])\,k_L\langle N\rangle_C^N$$
$$\text{s.t.}\quad - \pi_C R\left(k_1\langle N\rangle_C^N\right) + E[\pi_N]\,E[\pi_N|H,N]\,R\left(k_H\langle N\rangle_C^N\right) - E[\pi_N]\,\pi_C R\left(k_L\langle N\rangle_C^N\right) \geq 0.$$
$$\{\pi_C - E[\pi_N]\}\,\pi_C R\left(k_L\langle N\rangle_C^N\right) + \{E[\pi_N]\,E[\pi_N|H,N] - \pi_C\pi_C\}\,R\left(k_H\langle N\rangle_C^N\right) \geq 0.$$

The FOC with respect to $k_1\langle N\rangle_C^N$ is

$$\lambda_{ICET} = \frac{E[\pi_N]R'(k_1\langle N\rangle_C^N) - 1}{\pi_C R'(k_1\langle N\rangle_C^N)}$$

where the right-hand side is positive (assuming the presence of financing constraint).

From the FOC with respect to $k_L\langle N\rangle_C^N$,

$$\lambda_{ICE} = \frac{1 - E[\pi_N] + \lambda_{ICET} E[\pi_N]\pi_C R'(k_L\langle N\rangle_C^N)}{\{\pi_C - E[\pi_N]\}\pi_C R'(k_L\langle N\rangle_C^N)}$$

where the right-hand side is always positive given Assumption A. Both incentive compatibility constraints bind at the optimum and we can use the remaining FOC together with the constraints to solve for the optimal level of capitals. □

A.3 Unobservable Project Selection, But Observable Outcomes

In this section, we assume that the outcome is observable and contractible but the principal cannot observe the action plan chosen by the agent. As a consequence, we can disregard the incentive compatibility constraints associated with tunneling but the optimal contract still needs to disincentivise the agent to use alternative action plans. Assuming limited liability protects the agent, the repayment in any contingency cannot be greater than the current cash flow: following a low outcome, the repayment will always be zero i.e. $\tau_L = \tau_{HL} = \tau_{LL} = 0$. The relevant limited liability constraints are

$$R(k_1\langle i\rangle_z^j) - \tau_H\langle i\rangle_z^j \geq 0 \tag{A.1a}$$

$$R(k_H\langle i\rangle_z^j) - \tau_{HH}\langle i\rangle_z^j \geq 0 \tag{A.1b}$$

$$R(k_L\langle i\rangle_z^j) - \tau_{LH}\langle i\rangle_z^j \geq 0. \tag{A.1c}$$

Exploitation. Suppose the principal wants the agent to exploit but cannot observe the actual action plan chosen by the agent. The optimal exploitation contract under unobservable work method maximizes the expected profits of the investor

$$-I + \pi_C\tau_H\langle C\rangle_C^C - k_1\langle C\rangle_C^C + \pi_C\left\{\pi_C\tau_{HH}\langle C\rangle_C^C - k_H\langle C\rangle_C^C\right\} + (1-\pi_C)\left\{\pi_C\tau_{LH}\langle C\rangle_C^C - k_L\langle C\rangle_C^C\right\}$$

subject to the limited liability constraints

$$R\left(k_1\langle C\rangle_C^C\right) - \tau_H\langle C\rangle_C^C \geq 0 \tag{A.2a}$$

$$R\left(k_H\langle C\rangle_C^C\right) - \tau_{HH}\langle C\rangle_C^C \geq 0 \tag{A.2b}$$

$$R\left(k_L\langle C\rangle_C^C\right) - \tau_{LH}\langle C\rangle_C^C \geq 0 \tag{A.2c}$$

and the incentive compatibility constraint

$$
\begin{aligned}
(\pi_C - E[\pi_N]) \left\{R(k_1\langle C\rangle_C^C) - \tau_H\langle C\rangle_C^C\right\} + \\
+ (\pi_C\pi_C - E[\pi_N]E[\pi_N|H,N]) \left\{R(k_H\langle C\rangle_C^C) - \tau_{HH}\langle C\rangle_C^C\right\} + \\
+ \pi_C\left(E[\pi_N] - \pi_C\right) \left\{R(k_L\langle C\rangle_C^C) - \tau_{LH}\langle C\rangle_C^C\right\} \geq 0. \quad \text{(A.3)}
\end{aligned}
$$

Given Assumption A and the limited liability constraints in (A.2), the third line in (A.3) is non positive as rewarding the agent for first period failure gives the agent incentives to employ the novel approach in the first period. Therefore, the associated limited liability constraint in (A.2c) is binding i.e. $R(k_L\langle C\rangle_C^C) = \tau_{LH}\langle C\rangle_C^C$. As a consequence, it is optimal to set $k_L\langle C\rangle_C^C$ at its first best value.

Under moderate exploration, the second line in (A.3) is negative, and the limited liability constraint in (A.2b) is binding, thus $k_H\langle C\rangle_C^C$ is optimally set. This reduces the incentive compatibility constraint to just the first line, which is satisfied for any non negative value of $R(k_1\langle C\rangle_C^C) - \tau_H\langle C\rangle_C^C$. It is then optimal for the principal to set $R(k_1\langle C\rangle_C^C) = \tau_H\langle C\rangle_C^C$, and $k_1\langle C\rangle_C^C$ at its first best value. As a consequence, the principal appropriates the outcomes under any circumstance so that it is optimal for her to always provide the first best level of capital. Entrepreneur gets zero payoff.

Under radical exploration, the incentive compatibility constraint is satisfied for any non negative values of $R(k_H\langle C\rangle_C^C) - \tau_{HH}\langle C\rangle_C^C$ and $R(k_1\langle C\rangle_C^C) - \tau_H\langle C\rangle_C^C$. It is then optimal for the principal to drive the repayments from the agent up to their maximum. As a consequence, the optimal levels of capital are provided in any contingencies.

Exploration. Suppose the principal wants the agent to explore but cannot observe the actual action plan chosen by the agent. The optimal exploration contract under unobservable work method maximizes the expected profits of the investor

$$
\begin{aligned}
-I + E[\pi_N]\tau_H\langle N\rangle_C^N - k_1\langle N\rangle_C^N + E[\pi_N]\left\{E[\pi_N|H,N]\tau_{HH}\langle N\rangle_C^N - k_H\langle N\rangle_C^N\right\} + \\
+ (1 - E[\pi_N])\left\{\pi_C\tau_{LH}\langle N\rangle_C^N - k_L\langle N\rangle_C^N\right\}
\end{aligned}
$$

subject to the limited liability constraints

$$R\left(k_1\langle N\rangle_C^N\right) - \tau_H\langle N\rangle_C^N \geq 0 \quad \text{(A.4a)}$$

$$R\left(k_H\langle N\rangle_C^N\right) - \tau_{HH}\langle N\rangle_C^N \geq 0 \quad \text{(A.4b)}$$

$$R\left(k_L\langle N\rangle_C^N\right) - \tau_{LH}\langle N\rangle_C^N \geq 0 \quad \text{(A.4c)}$$

and the incentive compatibility constraint

$$\begin{aligned}(E[\pi_N]-\pi_C)\left\{R\left(k_1\langle N\rangle_C^N\right)-\tau_H\langle N\rangle_C^N\right\}+\\+\left(E[\pi_N]E[\pi_N|H,N]-\pi_C\pi_C\right)\left\{R(\,k_H\langle N\rangle_C^N\,)-\tau_{HH}\langle N\rangle_C^N\right\}+\\+\pi_C\left(\pi_C-E[\pi_N]\right)\left\{R\left(k_L\langle N\rangle_C^N\right)-\tau_{LH}\langle N\rangle_C^N\right\}\geq 0.\end{aligned} \tag{A.5}$$

The first line in (A.5) is non positive as delaying the agent's reward to the second period is optimal since the conventional method is initially more likely to succeed. As a consequence, the associated limited liability constraint in (A.2a) is binding i.e. $R(k_1\langle N\rangle_C^N) = \tau_H\langle N\rangle_C^N$: $k_1\langle N\rangle_C^N$ does not enter into the incentive compatibility constraint, so it is optimal to set it at its first best level.

Under moderate exploration, the limited liability constraint is satisfied by any non negative value of $R(k_L\langle N\rangle_C^N)-\tau_{LH}\langle N\rangle_C^N$ and $R(k_H\langle N\rangle_C^N)-\tau_{HH}\langle N\rangle_C^N$. It is then optimal for the principal to provide the first best levels of capital.

Under radical exploration, the second line in (A.5) is non positive, thus the limited liability constraint in (A.4b) is binding. This reduces the incentive compatibility constraint to just the third line, which is satisfied for any non negative value of $R(k_L\langle N\rangle_C^N)-\tau_{LH}\langle N\rangle_C^N$. It is then optimal for the principal to set $R(k_L\langle N\rangle_C^N) = \tau_{LH}\langle N\rangle_C^N$. As a consequence, the principal appropriates the outcomes under any circumstance and thus it is optimal for her to always provide the first best levels of capital.

B Costly Exploration

We here assume that the agent incurs a private cost $c_N \geq 0$ to perform the novel project (i.e. the agent dislikes the new work method, perhaps because it requires more effort), while the conventional method's cost is normalized to zero.[19] For this Appendix, it will prove useful to strengthen the definition of moderate exploration, as shown in Figure B.1.

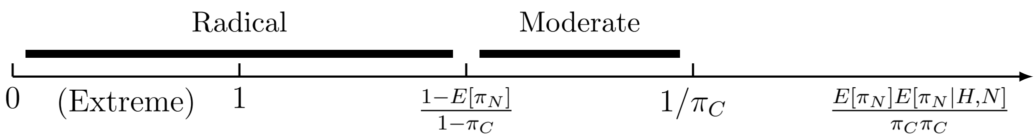

Figure B.1: Types of exploration

Definition. *Exploration is moderate if*

$$\frac{E[\pi_N]E[\pi_N|H,N]}{\pi_C\pi_C} > \frac{1-E[\pi_N]}{1-\pi_C}$$

and radical otherwise. Within the realm of radical exploration, we define it to be extreme if

$$\frac{E[\pi_N]E[\pi_N|H,N]}{\pi_C\pi_C} < 1.$$

As in Manso (2011), here we define exploration as moderate if the likelihood ratio between exploration and exploitation of two consecutive successes is higher than the one of a failure in the first period. Note that this is a stricter definition than the one we used in the main text.

Since the private cost does not multiply capital, the first best exploitation and exploration contracts are unchanged from the main section.

B.1 Unobservable project selection, but observable outcome

In this section, we assume that the principal cannot observe the project selection stage but the outcome of the project is contractible. This section therefore considers a setting

[19] Manso (2011) also considers the case in which the agent dislikes monotonous work and thus the private cost associated with the conventional work method is higher than the private cost associated with the novel approach.

very similar to the problem analysed by Manso (2011), who however does not consider capital in the production process. We show that the optimal contracts are similar to his, and the scale of the firm does not play a role, since the first best levels are always provided.

Assuming limited liability protects the agent, the repayment in any contingency cannot be greater than the current cash flow: following a low outcome, the repayment will always be zero i.e. $\tau_L = \tau_{HL} = \tau_{LL} = 0$. The relevant limited liability constraints thus read as follows

$$R(k_1\langle i\rangle_z^j) - \tau_H\langle i\rangle_z^j \geq 0 \tag{B.1a}$$

$$R(k_H\langle i\rangle_z^j) - \tau_{HH}\langle i\rangle_z^j \geq 0 \tag{B.1b}$$

$$R(k_L\langle i\rangle_z^j) - \tau_{LH}\langle i\rangle_z^j \geq 0. \tag{B.1c}$$

Exploitation. Suppose the principal wants the agent to exploit but cannot observe the actual action plan chosen by the agent. The optimal exploitation contract under unobservable work method maximizes the expected profits of the investor

$$-I+\pi_C\tau_H\langle C\rangle_C^C-k_1\langle C\rangle_C^C+\pi_C\left\{\pi_C\tau_{HH}\langle C\rangle_C^C - k_H\langle C\rangle_C^C\right\}+(1-\pi_C)\left\{\pi_C\tau_{LH}\langle C\rangle_C^C - k_L\langle C\rangle_C^C\right\}$$

subject to the limited liability constraints

$$R\left(k_1\langle C\rangle_C^C\right) - \tau_H\langle C\rangle_C^C \geq 0 \tag{B.2a}$$

$$R\left(k_H\langle C\rangle_C^C\right) - \tau_{HH}\langle C\rangle_C^C \geq 0 \tag{B.2b}$$

$$R\left(k_L\langle C\rangle_C^C\right) - \tau_{LH}\langle C\rangle_C^C \geq 0 \tag{B.2c}$$

and the incentive compatibility constraint

$$\begin{aligned} c_N\left(1+E[\pi_N]\right) + \left(\pi_C - E[\pi_N]\right)\left\{R(k_1\langle C\rangle_C^C) - \tau_H\langle C\rangle_C^C\right\} + \\ + \left(\pi_C\pi_C - E[\pi_N]E[\pi_N|H,N]\right)\left\{R(k_H\langle C\rangle_C^C) - \tau_{HH}\langle C\rangle_C^C\right\} + \\ + \pi_C\left(E[\pi_N] - \pi_C\right)\left\{R(k_L\langle C\rangle_C^C) - \tau_{LH}\langle C\rangle_C^C\right\} \geq 0. \end{aligned} \tag{B.3}$$

The optimal contract is summarized in the following Proposition B.1.

Proposition B.1. *(Exploitation contract under unobservable costly action)*
The optimal exploitation contract when the actions are unobservable and the agent incurs

a private cost when employing the new work method is first best. The agent has a zero payoff.

$$k_1^\star\langle C\rangle_C^C = k_H^\star\langle C\rangle_C^C = k_L^\star\langle C\rangle_C^C = k^{FB}\langle C\rangle_C^C = (A\alpha\pi_C)^{\frac{1}{1-\alpha}}$$
$$\tau_H^\star\langle C\rangle_C^C = \tau_{HH}^\star\langle C\rangle_C^C = \tau_{LH}^\star\langle C\rangle_C^C = R\left(k^{FB}\langle C\rangle_C^C\right).$$

The total expected payoffs of the principal and the agent are, respectively,

$$S^\star\langle C\rangle_C^C = W^\star\langle C\rangle_C^C \qquad V^\star\langle C\rangle_C^C = 0$$

where the total expected payoff of the match is given by

$$W_{FB}^\star\langle C\rangle_C^C = -I + 2\,(1-\alpha)\,A\pi_C\,(A\alpha\pi_C)^{\frac{\alpha}{1-\alpha}}.$$

Proof of Proposition B.1. Given Assumption A and the limited liability constraints in (A.2), the following things can be noticed. Firstly, the first line in (B.3) is non negative as paying the agent for first period success incentivises her to use the conventional method which is initially more likely to succeed. Secondly, the sign of the second line in (B.3) depends on the form of exploration: it is non negative if exploration is extreme, while it is non positive under radical and moderate exploration. Finally, the third line in (B.3) is non positive as rewarding the agent for first period failure gives the agent incentives to employ the novel approach in the first period. Therefore, the associated limited liability constraint in (B.2c) is binding i.e. $R(k_L\langle C\rangle_C^C) = \tau_{LH}\langle C\rangle_C^C$, and it is optimal to set $k_L\langle C\rangle_C^C$ at its first best value.

Under moderate and (non extreme) radical exploration, the limited liability constraint in (A.2b) is binding, thus $k_H\langle C\rangle_C^C$ is optimally set. This reduces the incentive compatibility constraint to just the first line, which is satisfied for any non negative value of $R(k_1\langle C\rangle_C^C) - \tau_H\langle C\rangle_C^C$. It is then optimal for the principal to set $R(k_1\langle C\rangle_C^C) = \tau_H\langle C\rangle_C^C$, and $k_1\langle C\rangle_C^C$ at its first best value. As a consequence, the principal appropriates the outcomes under any circumstance so that it is optimal for her to always provide the first best level of capital. Entrepreneur gets zero payoff.

Under extreme exploration, the incentive compatibility constraint is satisfied for any non negative values of $R(k_H\langle C\rangle_C^C) - \tau_{HH}\langle C\rangle_C^C$ and $R(k_1\langle C\rangle_C^C) - \tau_H\langle C\rangle_C^C$. It is then optimal for the principal to drive the repayments from the agent up to their maximum. As a consequence, the optimal levels of capital are provided in any contingencies. □

Exploration. Suppose the principal wants the agent to explore but cannot observe the actual action plan chosen by the agent. The optimal exploration contract under unobservable work method maximizes the expected profits of the investor

$$
\begin{aligned}
&- I + E[\pi_N]\tau_H\langle N\rangle_C^N - k_1\langle N\rangle_C^N + E[\pi_N]\left\{E[\pi_N|H,N]\tau_{HH}\langle N\rangle_C^N - k_H\langle N\rangle_C^N\right\} + \\
&\qquad\qquad\qquad\qquad + (1-E[\pi_N])\left\{\pi_C\tau_{LH}\langle N\rangle_C^N - k_L\langle N\rangle_C^N\right\}
\end{aligned}
$$

subject to the limited liability constraints

$$R\left(k_1\langle N\rangle_C^N\right) - \tau_H\langle N\rangle_C^N \geq 0 \tag{B.4a}$$

$$R\left(k_H\langle N\rangle_C^N\right) - \tau_{HH}\langle N\rangle_C^N \geq 0 \tag{B.4b}$$

$$R\left(k_L\langle N\rangle_C^N\right) - \tau_{LH}\langle N\rangle_C^N \geq 0 \tag{B.4c}$$

and the incentive compatibility constraint

$$
\begin{aligned}
&- c_N\left(1+E[\pi_N]\right) + \left(E[\pi_N]-\pi_C\right)\left\{R\left(k_1\langle N\rangle_C^N\right) - \tau_H\langle N\rangle_C^N\right\} + \\
&\qquad + \left(E[\pi_N]E[\pi_N|H,N] - \pi_C\pi_C\right)\left\{R(\,k_H\langle N\rangle_C^N\,) - \tau_{HH}\langle N\rangle_C^N\right\} + \\
&\qquad\qquad + \pi_C\left(\pi_C - E[\pi_N]\right)\left\{R\left(k_L\langle N\rangle_C^N\right) - \tau_{LH}\langle N\rangle_C^N\right\} \geq 0.
\end{aligned} \tag{B.5}
$$

Proposition B.2 shows that the optimal exploration contract is also first best. However, if the principal wants the agent to explore, it must compensate the agent for the private costs of effort. When exploration is radical, the agent is compensated at the end of the relationship after a failure in the first period and a success in the second period, because a failure in the first period is a strong signal of exploration. When exploration is moderate, the agent is compensated only after two consecutive successes, which now constitutes a stronger signal of exploration. The following definition will be useful,

$$\Pi = \left\{(\pi_C)^{\frac{1}{1-\alpha}}\left(1-E[\pi_N]\right) + \left(E[\pi_N]\right)^{\frac{1}{1-\alpha}} + E[\pi_N]\left(E[\pi_N|H,N]\right)^{\frac{1}{1-\alpha}}\right\}.$$

Proposition B.2. *(Exploration contract under unobservable costly action)*
The optimal exploration contract when the actions are unobservable and the agent incurs

a private cost when employing the new work method is first best.

$$\begin{aligned}
k^{\star}_{1,mod}\langle N\rangle^N_C = k^{\star}_{1,rad}\langle N\rangle^N_C = k^{FB}_1\langle N\rangle^N_C = (A\alpha E[\pi_N])^{\frac{1}{1-\alpha}}\\
k^{\star}_{H,mod}\langle N\rangle^N_C = k^{\star}_{H,rad}\langle N\rangle^N_C = k^{FB}_H\langle N\rangle^N_C = (A\alpha E[\pi_N|H,N])^{\frac{1}{1-\alpha}}\\
k^{\star}_{L,mod}\langle N\rangle^N_C = k^{\star}_{L,rad}\langle N\rangle^N_C = k^{FB}\langle C\rangle^C_C = (A\alpha \pi_C)^{\frac{1}{1-\alpha}}\\
\tau^{\star}_{H,mod}\langle N\rangle^N_C = \tau^{\star}_{H,rad}\langle N\rangle^N_C = R\left(k^{FB}_1\langle N\rangle^N_C\right).
\end{aligned}$$

Total welfare is

$$W^{\star}_{FB}\langle N\rangle^N_C = -\left\{I + c_N\left(1+E[\pi_N]\right)\right\} + (1-\alpha)A\left(A\alpha\right)^{\frac{\alpha}{1-\alpha}}\Pi.$$

The agent's payoff depends on the type of exploration. When exploration is radical,

$$\begin{aligned}
\tau^{\star}_{HH,rad}\langle N\rangle^N_C = R\left(k^{FB}_H\langle N\rangle^N_C\right)\\
\tau^{\star}_{LH,rad}\langle N\rangle^N_C = A\left(A\alpha\pi_C\right)^{\frac{\alpha}{1-\alpha}} - \frac{c_N\left(1+E[\pi_N]\right)}{\pi_C\left(\pi_C - E[\pi_N]\right)}
\end{aligned}$$

and the total expected payoffs of the principal and the agent are, respectively,

$$\begin{aligned}
S^{\star}_{rad}\langle N\rangle^N_C = -\left\{I + \frac{c_N\left(1-E[\pi_N]E[\pi_N]\right)}{\pi_C - E[\pi_N]}\right\} + (1-\alpha)\,A\left(A\alpha\right)^{\frac{\alpha}{1-\alpha}}\Pi\\
V^{\star}_{rad}\langle N\rangle^N_C = \frac{(1-\pi_C)c_N\left(1+E[\pi_N]\right)}{\pi_C - E[\pi_N]}.
\end{aligned}$$

When exploration is moderate,

$$\begin{aligned}
\tau^{\star}_{HH,mod}\langle N\rangle^N_C = R\left(k^{FB}_H\langle N\rangle^N_C\right) - \frac{c_N\left(1+E[\pi_N]\right)}{E[\pi_N]E[\pi_N|H,N] - \pi_C\pi_C}\\
\tau^{\star}_{LH,mod}\langle N\rangle^N_C = R\left(k^{FB}_L\langle N\rangle^N_C\right)
\end{aligned}$$

and the total expected payoffs of the principal and the agent are, respectively,

$$\begin{aligned}
S^{\star}_{mod}\langle N\rangle^N_C = -\left\{I + \frac{E[\pi_N]E[\pi_N|H,N]c_N\left(1+E[\pi_N]\right)}{E[\pi_N]E[\pi_N|H,N] - \pi_C\pi_C}\right\} + (1-\alpha)\,A\left(A\alpha\right)^{\frac{\alpha}{1-\alpha}}\Pi\\
V^{\star}_{mod}\langle N\rangle^N_C = \frac{\pi_C\pi_C c_N\left(1+E[\pi_N]\right)}{E[\pi_N]E[\pi_N|H,N] - \pi_C\pi_C}
\end{aligned}$$

Proof of Proposition B.2. The first line in (B.5) is non positive as delaying the agent's reward to the second period is optimal since the conventional method is initially more likely to succeed. As a consequence, the associated limited liability constraint in (B.4a) is binding i.e. $R(k_1\langle N\rangle_C^N) = \tau_H\langle N\rangle_C^N$. Therefore $k_1\langle N\rangle_C^N$ does not enter into the incentive compatibility constraint, so it is optimal to set it at its first best level.

The Lagrangian is

$$\begin{aligned}
&- I + E[\pi_N]\tau_H\langle N\rangle_C^N - k_1\langle N\rangle_C^N + E[\pi_N]\left\{E[\pi_N|H,N]\tau_{HH}\langle N\rangle_C^N - k_H\langle N\rangle_C^N\right\} + \\
&+ (1 - E[\pi_N])\left\{\pi_C\tau_{LH}\langle N\rangle_C^N - k_L\langle N\rangle_C^N\right\} + \\
&+ \lambda_{IC}[-c_N(1 + E[\pi_N]) + (E[\pi_N]E[\pi_N|H,N] - \pi_C\pi_C)\left\{R(\,k_H\langle N\rangle_C^N\,) - \tau_{HH}\langle N\rangle_C^N\right\} + \\
&+ \pi_C(\pi_C - E[\pi_N])\left\{R\left(k_L\langle N\rangle_C^N\right) - \tau_{LH}\langle N\rangle_C^N\right\}] + \\
&+ \lambda_{LLCH}\left\{R\left(k_H\langle N\rangle_C^N\right) - \tau_{HH}\right\} + \lambda_{LLCL}\left\{R\left(k_L\langle N\rangle_C^N\right) - \tau_{LH}\right\}
\end{aligned}$$

whose relevant FOCs are

$$\begin{aligned}
\tau_{HH}: &\quad E[\pi_N]E[\pi_N|H,N] - \lambda_{IC}(E[\pi_N]E[\pi_N|H,N] - \pi_C\pi_C) - \lambda_{LLCH} = 0 \\
\tau_{LH}: &\quad (1 - E[\pi_N])\pi_C - \lambda_{IC}\pi_C(\pi_C - E[\pi_N]) - \lambda_{LLCL} = 0 \\
k_H: &\quad -E[\pi_N] + \lambda_{IC}(E[\pi_N]E[\pi_N|H,N] - \pi_C\pi_C)R'\left(k_H\langle N\rangle_C^N\right) + \lambda_{LLCH}R'\left(k_H\langle N\rangle_C^N\right) = 0 \\
k_L: &\quad -(1 - E[\pi_N]) + \lambda_{IC}\pi_C(\pi_C - E[\pi_N])R'\left(k_L\langle N\rangle_C^N\right) + \lambda_{LLCL}R'\left(k_L\langle N\rangle_C^N\right) = 0
\end{aligned}$$

There are two cases. Suppose exploration to be moderate. In this case, the incentive compatibility constraint is binding, $\lambda_{IC} > 0$, as is the limited liability constraint after a failure, $\lambda_{LLCL} > 0$, while the remaining limited liability constraint is slack, $\lambda_{LLCH} = 0$. The first two FOCs confirm this

$$\begin{aligned}
\lambda_{IC} &= \frac{E[\pi_N]E[\pi_N|H,N]}{E[\pi_N]E[\pi_N|H,N] - \pi_C\pi_C} \\
\lambda_{LLCL} &= \pi_C\frac{E[\pi_N]E[\pi_N|H,N](1 - \pi_C) - \pi_C\pi_C(1 - E[\pi_N])}{E[\pi_N]E[\pi_N|H,N] - \pi_C\pi_C}
\end{aligned}$$

Since $\lambda_{LLCL} > 0$, $R\left(k_L\langle N\rangle_C^N\right) = \tau_{LH}$ and thus $k_L\langle N\rangle_C^N$ is set at its first best level (this can be confirmed using the last FOC). From the third FOC, one can see that also $k_H\langle N\rangle_C^N$ is set at the first best level. We can find the optimal repayment after two consecutive successes from the binding incentive compatibility constraint,

$$\tau_{HH}\langle N\rangle_C^N = R\left(k_H\langle N\rangle_C^N\right) - \frac{c_N(1 + E[\pi_N])}{E[\pi_N]E[\pi_N|H,N] - \pi_C\pi_C}.$$

Now, suppose exploration to be radical. In this case, the incentive compatibility constraint is binding, $\lambda_{IC} > 0$, as is the limited liability constraint after a double success, $\lambda_{LLCH} > 0$, while the remaining limited liability constraint is slack, $\lambda_{LLCL} = 0$. The first two FOCs confirm this

$$\lambda_{IC} = \frac{1 - E[\pi_N]}{\pi_C - E[\pi_N]}$$
$$\lambda_{LLCH} = \frac{\pi_C \pi_C \left(1 - E[\pi_N]\right) - E[\pi_N] E[\pi_N|H,N] \left(1 - \pi_C\right)}{\pi_C - E[\pi_N]},$$

and thus it must be the case that $k_H \langle N \rangle_C^N$ is set at its first best level (as the principal appropriates the entire outcome). From the last FOC, we see that $k_L \langle N \rangle_C^N$ also is at its first best level. Using the binding incentive compatibility constraint gives us

$$\tau_{LH} \langle N \rangle_C^N = R\left(k_L \langle N \rangle_C^N\right) - \frac{c_N \left(1 + E[\pi_N]\right)}{\pi_C \left(\pi_C - E[\pi_N]\right)}.$$

□

B.2 Observable project selection, but unobservable outcome

In this section, the agency problem arises because it is impossible to make the contract explicitly contingent on realized outcome, as such outcome is private information for the agent. However, the principal can observe and verify the work method employed by the entrepreneur. It is thus the equivalent of Section 4 but with a private cost for running the novel approach. The aim of this section is to show that introducing a private cost only complicates the analysis, without qualitatively changing the conclusions.

Similar to the analysis in the main text, the relevant incentive compatibility constraint is $R\left(k_1 \langle i \rangle_z^j\right) - \tau_H \langle i \rangle_z^j \geq 0$. However, the incentive compatibility constraint (imposing truthful reporting in the good state at the end of the first period) is modified to

$$-\tau_H \langle i \rangle_z^j + E\left[\pi_j | H, i\right] R\left(k_H \langle i \rangle_z^j\right) - c_j \geq E\left[\pi_z | H, i\right] R\left(k_L \langle i \rangle_z^j\right) - c_z. \qquad \text{(B.6)}$$

Using the same reasoning as in the no-cost scenario, it can be shown that both the limited liability and the incentive compatibility constraints must bind at the optimum, and that it is optimal to set $k_L \langle i \rangle_z^j = 0$. The agent just gets an utility equal to the first period return in case of success, and $\tau_H \langle i \rangle_z^j = R\left(k_1 \langle i \rangle_z^j\right) = E\left[\pi_j | H, i\right] R\left(k_H \langle i \rangle_z^j\right) - c_j + c_z$.

Exploitation. Since the private cost associated with the conventional approach is normalized to zero, the optimal contract is the same as in Proposition 1.

Exploration. Using the binding incentive compatibility constraint, the problem simplifies to

$$\max_{\{k_1\langle N\rangle_C^N,\ k_H\langle N\rangle_C^N\}\in\mathbb{R}^2_{\geq}} -I - k_1\langle N\rangle_C^N + E[\pi_N]\left\{E[\pi_N|H,N]R(k_H\langle N\rangle_C^N) - c_N - k_H\langle N\rangle_C^N\right\}$$
$$\text{s.t.}\quad R(k_1\langle N\rangle_C^N) = E[\pi_N|H,N]R(k_H\langle N\rangle_C^N) - c_N$$

From the FOC with respect to $k_1\langle N\rangle_C^N$, we find the value of the Lagrangian multiplier, $\lambda\langle N\rangle_C^N = 1/R'(k_1\langle N\rangle_C^N)$, which is positive and increasing in $k_1\langle N\rangle_C^N$, confirming that the limited liability constraint is binding at the optimum. The remaining FOC becomes

$$R'(k_H\langle N\rangle_C^N) = \frac{E[\pi_N]}{E[\pi_N]E[\pi_N|H,N] - E[\pi_N|H,N]/R'(k_1\langle N\rangle_C^N)}$$

Under the selected functional forms, the optimal levels of capital are given by the solution of the following system

$$\begin{cases} k_1\langle N\rangle_C^N = \left(E[\pi_N|H,N]k_H^\alpha\langle N\rangle_C^N + \frac{c_N}{A}\right)^{\frac{1}{\alpha}} \\ A\alpha k_H^{\alpha-1}\langle N\rangle_C^N = \frac{E[\pi_N]}{E[\pi_N]E[\pi_N|H,N] - E[\pi_N|H,N]\left(A\alpha k_1^{\alpha-1}\langle N\rangle_C^N\right)^{-1}} \end{cases}$$

Since we cannot find a closed-form solution, we resort to numerical solutions. Figure B.2 shows that, when c_N increases, $k_1^\star\langle N\rangle_C^N$ (and thus $\tau_H\langle N\rangle_C^N$) linearly decreases, while $k_H^\star\langle N\rangle_C^N$ linearly increases. Combine the incentive compatibility and limited liability constraint as $E[\pi_N|H,N]R(k_H\langle N\rangle_C^N) - R(k_1\langle N\rangle_C^N) = c_N$: increases in the right hand side are compensated by a mixture of decreases in $R(k_1\langle N\rangle_C^N)$ and increases in $R(k_H\langle N\rangle_C^N)$. The former corresponds to decreases in the repayment demanded by the principal after a success in the first period, and the latter to an increases in the long-term reward to the agent. The magnitude of these changes, however, is not overwhelming.

Figure B.2: Changes in the optimal exploration contract under a hands-on principal when exploration is costly